

NAME: _____

TIME ALLOWED: 75 minutes

INSTRUCTIONS AND INFORMATION

General

- 1 Do not open the booklet until told to do so by your teacher.
- 2 NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
- 3 Diagrams are NOT drawn to scale. They are intended only as aids.
- 4 There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
- 5 This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
- 6 Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
- 7 When your teacher gives the signal, begin working on the problems.

The answer sheet

- 1 Use only lead pencil.
- 2 Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
- 3 Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder: You may sit this competition once, in one division only, or risk no score.

1-10 题, 每题 3 分
Questions 1 to 10, 3 marks each

1. 请问算式 201×9 之值等于多少?

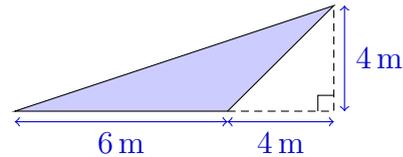
What is the value of 201×9 ?

- (A) 189 (B) 1809 (C) 1818 (D) 2001 (E) 2019

2. 请问右图中涂上阴影的三角形之面积为何?

What is the area of the shaded triangle?

- (A) 8 m^2 (B) 12 m^2 (C) 14 m^2
(D) 20 m^2 (E) 24 m^2



3. 请问 \$20 的 19% 是多少?

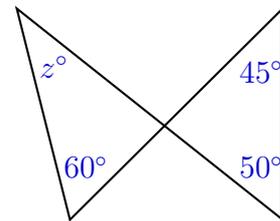
What is 19% of \$20?

- (A) \$20.19 (B) \$1.90 (C) \$0.19 (D) \$3.80 (E) \$0.38

4. 请问 z 之值是什么?

What is the value of z ?

- (A) 30 (B) 35 (C) 45 (D) 50 (E) 55



5. 请问算式 $2^0 + 1^9$ 之值等于多少?

The value of $2^0 + 1^9$ is

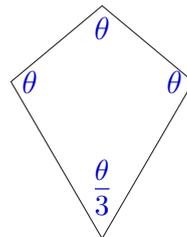
- (A) 1 (B) 2 (C) 3 (D) 10 (E) 11

6. 令 $f(x) = 3x^2 - 2x$, 请问 $f(-2)$ 等于多少?

Let $f(x) = 3x^2 - 2x$. Then $f(-2) =$

- (A) -32 (B) -8 (C) 16 (D) 32 (E) 40

7. 一个筝形的内角为 θ 、 θ 、 θ 与 $\frac{\theta}{3}$ 。
请问角 θ 的大小为何？



This kite has angles θ , θ , θ and $\frac{\theta}{3}$.
What is the size of the angle θ ?

- (A) 120° (B) 105° (C) 90° (D) 112° (E) 108°

8. 考虑一个震荡数列

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \dots,$$

它每四项循环。最前 3 项的累计总和是 12，最前 7 项的累计总和是 28。
请问下列哪一个选项内的值可能是这个数列的累计总和？

Consider the *undulating* number sequence

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \dots,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.

Which one of the following is also a running total of this sequence?

- (A) 61 (B) 62 (C) 67 (D) 66 (E) 65

9. 小咪每秒钟行走 1.5 m，她的朋友每秒钟行走 2 m。
她们沿着她们喜爱的圆形步道从同地点出发反向而行。在 20 分钟后，她们首度再相遇。
请问这条步道的总长度为多少 km？

Mia walks at 1.5 metres per second. Her friend Crystal walks at 2 metres per second. They walk in opposite directions around their favourite bush track, starting together from the same point. They first meet again after 20 minutes.

How long, in kilometres, is the track?

- (A) 3.5 (B) 4.2 (C) 6 (D) 7 (E) 8.4

10. 算式 $\frac{1^1 + 2^2 + 3^3 + 4^4}{1^1 + 2^2 + 3^3} =$

$$\frac{1^1 + 2^2 + 3^3 + 4^4}{1^1 + 2^2 + 3^3} =$$

- (A) 2^3 (B) 3^2 (C) 11 (D) 4^3 (E) 259

11-20 题, 每题 4 分
Questions 11 to 20, 4 marks each

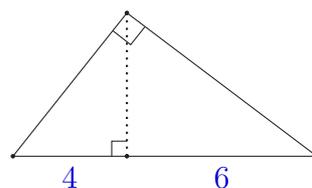
11. 五位数 $P679Q$ 可被 72 整除, 请问数字 P 等于什么?

The 5-digit number $P679Q$ is divisible by 72. The digit P is equal to

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

12. 一个直角三角形的高将斜边分成长度为 4 与 6 的两段。请问这个三角形的面积为何?

The altitude of a right-angled triangle divides the hypotenuse into lengths of 4 and 6. What is the area of the triangle?



- (A) $10\sqrt{6}$ (B) 24 (C) 25 (D) 12 (E) $6\sqrt{10}$

13. 在一箱苹果中, 有 $\frac{3}{7}$ 的苹果是红色的, 其它的则是绿色的。若将五颗绿色苹果加入箱子内, 则现在绿色的苹果占 $\frac{5}{8}$ 。
请问现在的箱子中总共有多少颗苹果?

In a box of apples, $\frac{3}{7}$ of the apples are red and the rest are green. Five more green apples are added to the box. Now $\frac{5}{8}$ of the apples are green.

How many apples are there now in the box?

- (A) 32 (B) 33 (C) 38 (D) 40 (E) 48

14. 请问下列哪一个选项内的数超过它的平方最多?

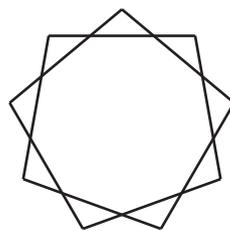
Which number exceeds its square by the greatest possible amount?

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$ (E) $\frac{\sqrt{2}}{2}$

15. 一个正九角星形是由九个点相连而成，如图所示。请问这九个点上的每个内角之大小为何？

A regular nonagram is a nine-pointed star drawn as shown. What is the angle at each of the nine points?

- (A) 100° (B) 110° (C) 120° (D) 130° (E) 140°



16. 构造两个各有 900 项的数列：

5, 8, 11, 14, ... (公差为 3)

3, 7, 11, 15, ... (公差为 4)

请问这两个数列中有多少项的数是相同的？

Two sequences are constructed, each with 900 terms:

5, 8, 11, 14, ... (increasing by 3)

3, 7, 11, 15, ... (increasing by 4)

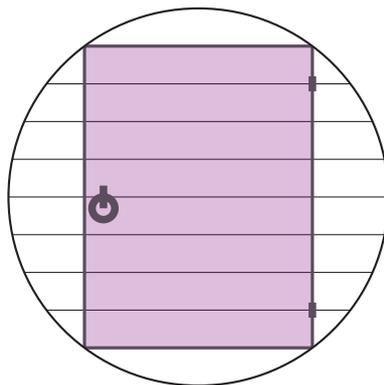
How many terms do these two sequences have in common?

- (A) 400 (B) 300 (C) 275 (D) 225 (E) 75

17. 有一个圆形的钢制出入口内接一个矩形门，其设计图如图所示。这个出入口被等距离的水平横杠分割成 10 个等宽的区域。请问这个矩形门的面积占整个出入口面积的几分之几？

A circular steel gateway surrounding a rectangular gate is designed as shown. The total height of the gateway is divided into 10 equal intervals by equally-spaced horizontal bars.

The rectangular gate is what fraction of the area of the entire circular gateway?

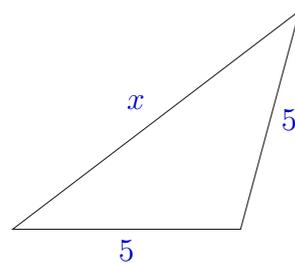


- (A) $\frac{48}{25\pi}$ (B) $\frac{\sqrt{3}}{\pi}$ (C) $\frac{2}{\pi}$ (D) $\frac{8\sqrt{2}}{25\pi}$ (E) $\frac{8}{5\pi}$

18. 请问当 x 等于什么值时，边长为 5、5、 x 的三角形是个钝角三角形？

For what values of x does the triangle with side lengths 5, 5 and x have an obtuse angle?

- (A) $0 < x \leq 5\sqrt{2}$ (B) $5 < x \leq 5\sqrt{2}$
 (C) $5 < x < 10$ (D) $0 < x < 10$
 (E) $5\sqrt{2} < x < 10$



19. 一个矩形的面积为 20，它的周长为 22。请问它的每条对角线之长度为何？

A rectangle has area 20 and perimeter 22. How long is each of its diagonals?

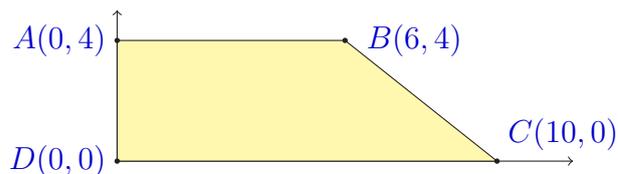
- (A) $4\sqrt{5}$ (B) 10 (C) $\sqrt{29}$ (D) $2\sqrt{26}$ (E) 9

20. 直线 $y = mx$ 将四边形 $ABCD$ 分割成相等面积的两块。
 请问 m 之值是什么？

The line $y = mx$ divides quadrilateral $ABCD$ into two equal areas.

The value of m is

- (A) 1 (B) $\frac{2}{3}$ (C) $\frac{8}{17}$ (D) $\frac{8}{15}$ (E) $\frac{8}{25}$



21-25 题, 每题 5 分
Questions 21 to 25, 5 marks each

21. 小敏有三种方式从家里到距离 8km 的工作地点: 如果他驾车上班, 则费时 12 分钟; 如果他骑自行车上班, 费时 24 分钟; 如果他步行上班, 费时 1 小时 44 分钟。他想知道当自行车轮胎没有气时, 如何以最短的时间抵达工作地点。

他有三种策略:

- (i) 如果他靠近他家, 则他走路回家, 然后改成驾车上班。
- (ii) 如果他靠近工作地点, 则剩下的路程他改用步行的方式。
- (iii) 如果在中点附近, 则花费 20 分钟修理自行车的轮胎然后继续骑车上班。

他知道在前往工作地点的途中, 有两个位置必须改变策略。请问这两个位置的距离是什么?

Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

- (i) If he is close to home, walk back home and then drive his car.
- (ii) If he is close to work, just walk the rest of the way.
- (iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

He knows there are two locations along the route to work where the strategy should change. How far apart are they?

- (A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

22. 一枚半径为 1 cm 的硬币沿着一个正方形的内壁滚动但不能滑动或空转, 移动时恒与正方形的边相切。当这枚硬币回到出发点时, 它恰好转一整圈。请问这正方形的边长为多少 cm?



A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square.

When it returns to where it started, the coin has performed exactly one whole revolution.

In centimetres, what is the side length of the square?

- (A) π (B) 3.5 (C) $1 + \pi$ (D) 4 (E) $2 + \frac{\pi}{2}$

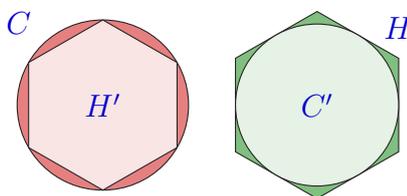
23. 有一列长度为 200 m 的客车以 80 km/h 的速度行驶，一列长度为 2 km 的货车以 20 km/h 的速度相向行驶。在同一条路径上，请问从两列火车相遇的点到两列火车相离的点之距离为何？

A passenger train 200 m long and travelling at 80 km/h passes a goods train 2 km long travelling in the opposite direction at 20 km/h. What is the distance, measured along one of the tracks, between the point at which the fronts of the trains pass each other and the point at which their back ends pass each other?

- (A) 1.28 km (B) 1.4 km (C) 1.56 km (D) 1.8 km (E) 1.88 km

24. 圆 C 与正六边形 H 之面积相等。一个正六边形 H' 内接于圆 C 、一个圆 C' 内切于正六边形 H 。

请问 H' 的面积与 C' 的面积之比是什么？



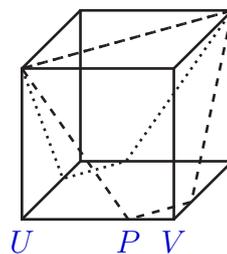
A circle C and a regular hexagon H have equal area. A regular hexagon H' is inscribed in C , and a circle C' is inscribed in H .

What is the ratio of the area of H' to the area of C' ?

- (A) 1 : 1 (B) 3 : π (C) 9 : π^2 (D) 3 : 4 (E) $3\sqrt{3} : 2\pi$

25. 通过一个边长为 1 的正立方体顶面之对角线的两个平面将此正立方体分割为等体积的三块。其中一个平面与棱边 \overline{UV} 交于点 P 。请问 PV 的长度是什么？

A cube of side length 1 is cut into three pieces of equal volume by two planes passing through the diagonal of the top face. One plane cuts the edge \overline{UV} at the point P . What is the length PV ?



- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\sqrt{3} - 1$ (E) $\frac{\sqrt{5} - 1}{2}$

问题 26-30 的答案为 000-999 之间的整数，
请将答案填在答案卡上对应的位置。

第 26 题占 6 分，第 27 题占 7 分，第 28 题占 8 分，
第 29 题占 9 分，第 30 题占 10 分。

For questions 26 to 30, shade the answer as an integer from 0 to 999
in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

26. 数 35 有一个性质，当我们把它的数码全部各加上 2，然后相乘，则可以得到 $5 \times 7 = 35$ ，与原来的数相等。
找出所有二位数中，当我们把它的数码全部加上 2，然后相乘，最后所得的结果与原数值相同。请问满足上述条件的所有二位数之和是什么？

The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is $5 \times 7 = 35$, equal to the original number.

Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.

27. 在一个数列中，如果有三个连续项的和是一个奇数，则称它们为一个奇和三元组。例如，我们将 1 到 6 的数写成以下的顺序

6 4 2 1 3 5

则其中恰有二个奇和三元组：(4, 2, 1) 与 (1, 3, 5)。

以某种顺序写下从 1 到 1000，请问最多可以得到多少个奇和三元组？

In a list of numbers, an *odd-sum triple* is a group of three numbers in a row that add to an odd number. For instance, if we write the numbers from 1 to 6 in this order,

6 4 2 1 3 5

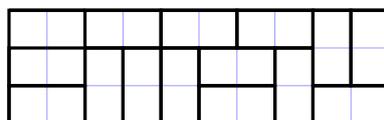
then there are exactly two odd-sum triples: (4, 2, 1) and (1, 3, 5).

What is the greatest number of odd-sum triples that can be made by writing the numbers from 1 to 1000 in some order?

28. 小德有一个立体，它共有四个三角形的表面。其中三个面互相垂直，而第四个面的边长为 11、20、21。请问这个立体的体积为多少？

Terry has a solid shape that has four triangular faces. Three of these faces are at right angles to each other, while the fourth face has side lengths 11, 20 and 21. What is the volume of the solid shape?

29. 下图所示为用 15 片 1×2 矩形瓷砖铺成一个 3×10 矩形的其中一种方法。



由于这个铺法不对称，我们将它的旋转、翻转之铺法视为不同的铺法。

要铺成这个 3×10 的矩形，请问总共有多少种不同的铺法？

The diagram shows one way in which a 3×10 rectangle can be tiled by 15 rectangles of size 1×2 .

Since this tiling has no symmetry, we count rotations and reflections of this tiling as different tilings.

How many different tilings of this 3×10 rectangle are possible?

30. 在正整数集合内定义函数 f ，其中 $f(1) = 2$ 、 $f(2) = 3$ 。并且若 n 为偶数，则 $f(f(f(n))) = n + 2$ ；若 n 为奇数，则 $f(f(f(n))) = n + 4$ 。请问 $f(777)$ 之值为何？

A function f , defined on the set of positive integers, has $f(1) = 2$ and $f(2) = 3$. Also $f(f(f(n))) = n + 2$ if n is even and $f(f(f(n))) = n + 4$ if n is odd. What is $f(777)$?

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E 组 2019 试卷 答案

- 1: B
- 2: B
- 3: D
- 4: B
- 5: B
- 6: C
- 7: E
- 8: E
- 9: B
- 10: B
- 11: C
- 12: A
- 13: D
- 14: A
- 15: A
- 16: D
- 17: A
- 18: E
- 19: E
- 20: C
- 21: C
- 22: E
- 23: C
- 24: C
- 25: E
- 26: 127
- 27: 997
- 28: 180
- 29: 571
- 30: 390

- A: 4
- B: 6
- C: 6
- D: 3
- E: 6