

THURSDAY 1 AUGUST 2019

NAME: \_\_\_\_\_

**TIME ALLOWED: 75 minutes**

### INSTRUCTIONS AND INFORMATION

#### General

- 1 Do not open the booklet until told to do so by your teacher.
- 2 NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
- 3 Diagrams are NOT drawn to scale. They are intended only as aids.
- 4 There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
- 5 This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
- 6 Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
- 7 When your teacher gives the signal, begin working on the problems.

#### The answer sheet

- 1 Use only lead pencil.
- 2 Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
- 3 Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

#### Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

**Reminder: You may sit this competition once, in one division only, or risk no score.**

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## Intermediate Division

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### Questions 1 to 10, 3 marks each

1. 算式  $20.19 - 19$  的值是

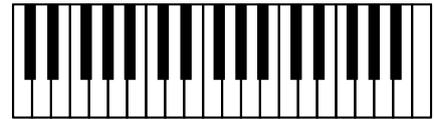
The value of  $20.19 - 19$  is

- (A) 39.19                      (B) 20.38                      (C) 20                      (D) 1.19                      (E) 1
- 

2. Sharyn 的钢琴课历时 40 分钟，她在下午 4.10 下课。请问她在什么时候开始上课？

Sharyn's piano lesson was 40 minutes long, and finished at 4.10 pm. When did it start?

- (A) 3.30 pm                      (B) 3.40 pm                      (C) 3.50 pm  
(D) 4.40 pm                      (E) 4.50 pm



3. 以下哪个数值最接近  $7 \times 1.8$ ？

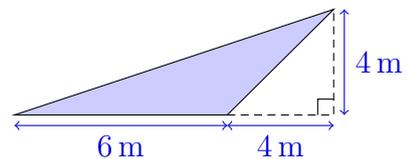
Which of the following is closest to  $7 \times 1.8$ ?

- (A) 10                      (B) 11                      (C) 12                      (D) 13                      (E) 14
- 

4. 请问右图阴影三角形的面积是多少？

What is the area of the shaded triangle?

- (A)  $8 \text{ m}^2$                       (B)  $12 \text{ m}^2$                       (C)  $14 \text{ m}^2$   
(D)  $20 \text{ m}^2$                       (E)  $24 \text{ m}^2$



5. 一个数的八分之五是 200。请问这个数是多少？

Five-eighths of a number is 200. What is the number?

- (A) 120                      (B) 320                      (C) 275                      (D) 75                      (E) 280
- 

6. 在这个  $3 \times 3$  的表格中每一行和每一列都必须包含数字 1、2 和 3。请问  $N+M$  的值是多少？

Every row and every column of this  $3 \times 3$  square must contain each of the numbers 1, 2 and 3.

What is the value of  $N + M$ ?

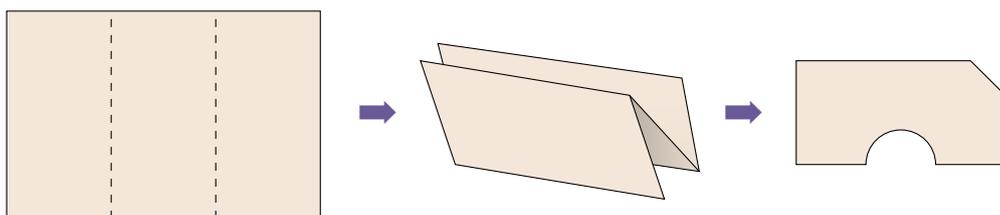
- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

1		
	2	$N$
		$M$

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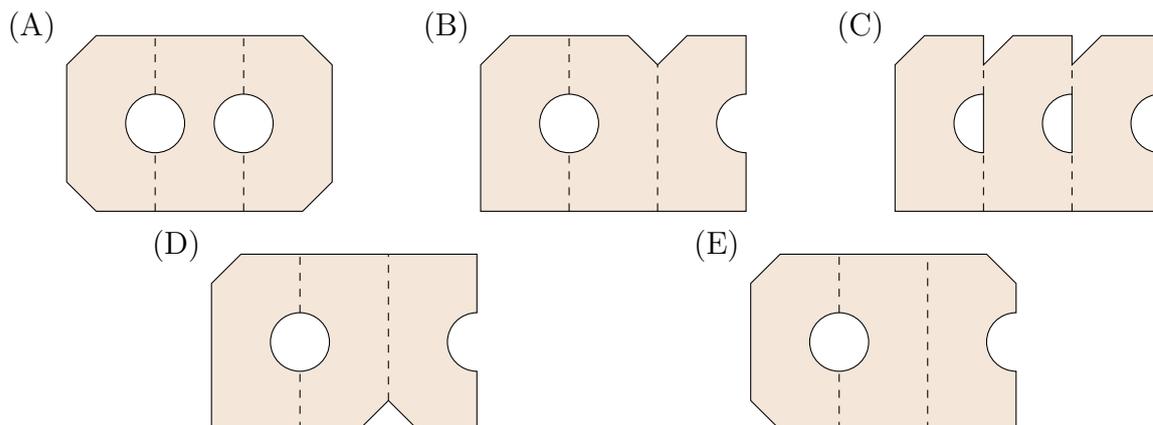
7. 将一张纸折成三等分，然后剪出一个半圆并直着剪一角，如下图所示

A piece of paper is folded in three, then a semi-circular cut and a straight cut are made, as shown in the diagram.



当纸张展开时，请问它是什么样子的？

When the paper is unfolded, what does it look like?



8. 将一个矩形切成两半，可以得到两个正方形。如果每个正方形的周长是 48，那么原始矩形的周长是多少？

When a rectangle is cut in half, two squares are formed. If each square has a perimeter of 48, what is the perimeter of the original rectangle?

- (A) 96                      (B) 72                      (C) 36                      (D) 24                      (E) 12

9. 考虑一个起伏的数列。每四项循环一次。最前 3 项的累计总和为 12。最前 7 项的累计总和为 28。请问下列哪一个选项内的值可能是这个数列的累计总和？

Consider the *undulating* number sequence

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \dots,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.

Which one of the following is also a running total of this sequence?

- (A) 61                      (B) 62                      (C) 67                      (D) 66                      (E) 65

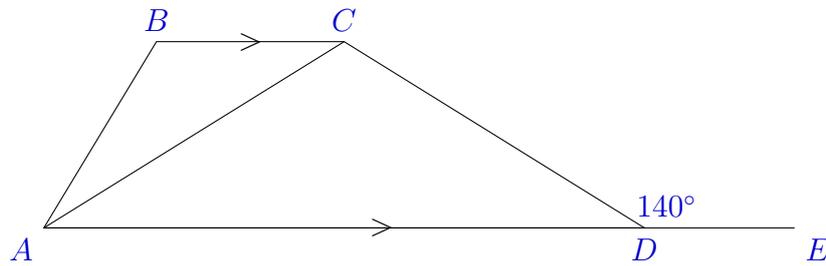
10. Sebastien 有一个由四位数字组成的个人识别码 (PIN)。它的前三位数码依序是 591。如果 Sebastien 的 PIN 可以被 3 整除，那么最后一位数码有多少种可能？

Sebastien has a Personal Identification Number (PIN) consisting of four digits. The first three digits in order are 591. If Sebastien's PIN is divisible by 3, then how many possibilities are there for the final digit?

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

11. 四边形ABCD中， $AD \parallel BC$ ， $AB = BC$  且  $AC = CD$ 。已知  $\angle CDE = 140^\circ$ 。如图所示。请问 $\angle ABC$ 为多少度？

A quadrilateral  $ABCD$  has  $AD \parallel BC$ ,  $AB = BC$  and  $AC = CD$ . The external  $\angle CDE = 140^\circ$ . What is the value, in degrees, of  $\angle ABC$ ?



- (A) 90                      (B) 100                      (C) 110                      (D) 120                      (E) 130

12. 在我的舞蹈课上，有14位学生比小柏高，有12位比小艾矮。有四位学生既比小艾矮，又比小柏高。请问我的舞蹈课共有多少学生？

In my dance class, 14 students are taller than Bob, and 12 are shorter than Alice. Four students are both shorter than Alice and taller than Bob. How many students are in my dance class?

- (A) 22                      (B) 24                      (C) 26                      (D) 28                      (E) 30

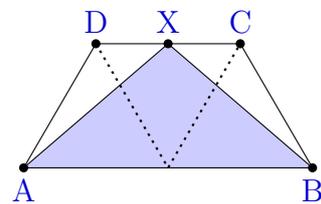
13. 三个等边三角形连接形成如图所示的四边形ABCD。X点位于CD的中间。请问阴影部分的面积占ABCD面积的几分之几？

Three equilateral triangles are joined to form the quadrilateral  $ABCD$  shown.

The point  $X$  is halfway along  $CD$ .

What fraction of the area of  $ABCD$  is shaded?

- (A)  $\frac{1}{2}$                       (B)  $\frac{3}{4}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{5}{6}$                       (E)  $\frac{3}{5}$



14. 在一个10年级的数学班里，有30名学生。每位学生都是15或16岁，且都是左撇子或右撇子。右撇子与左撇子的学生人数比例为4:1，而15岁学生的人数与16岁学生人数的比例为1:2，且15岁左撇子人数与16岁左撇子人数的比例为1:5。如果将本班学生的名牌放在一顶帽子内，从中随机抽取一个，请问抽出的学生是15岁且是右撇子的概率是多少？

In a year 10 Maths class, there are 30 students. Each student is either 15 or 16 years old, and either left- or right-handed.

The ratio of right-handed students to left-handed students is 4 : 1, the ratio of 15 year olds to 16 year olds is 1 : 2 and the ratio of left-handed 15 year olds to left-handed 16 year olds is 1 : 5.

If the names of the students in this class are placed in a hat and one is selected at random, what is the probability that the student selected is 15 years old and right-handed?

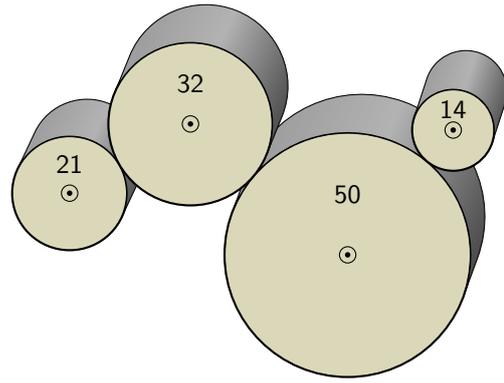
- (A)  $\frac{1}{30}$                       (B)  $\frac{1}{6}$                       (C)  $\frac{3}{10}$                       (D)  $\frac{1}{2}$                       (E)  $\frac{4}{5}$

15. 四个有固定轴心的圆形滚轴，并在不脱离位置的情况下每个滚轴的转动都能带动下一个滚轴转动。它们的直径分别为 21 厘米、32 厘米、50 厘米和 14 厘米。当 21 cm 的这个滚轮旋转一整圈 ( $360^\circ$ ) 时，请问 14 cm 滚轮旋转的角度为多少？

The set of four rollers shown has fixed axles and transfers rotation from each roller to the next without slipping. Their diameters are 21 cm, 32 cm, 50 cm and 14 cm respectively.

While the 21 cm roller makes a full rotation ( $360^\circ$ ), through which angle does the 14 cm roller rotate?

- (A)  $180^\circ$       (B)  $310^\circ$       (C)  $360^\circ$   
(D)  $540^\circ$       (E)  $620^\circ$



16. 在一盒苹果中， $\frac{3}{7}$  的苹果是红色的，其余的都是绿色的。若将盒子里又加入五个青苹果。则现在绿色的苹果占八分之五。请问现在盒子里总共有多少个苹果？

In a box of apples,  $\frac{3}{7}$  of the apples are red and the rest are green. Five more green apples are added to the box. Now  $\frac{5}{8}$  of the apples are green.

How many apples are there now in the box?

- (A) 32      (B) 33      (C) 38      (D) 40      (E) 48

17. Asha 从 1 到 5 中选择一个整数并公开宣布它。然后 Richy 从 1 到 5 中选择一个整数并公开宣布它。最后，Asha 从 1 到 5 中选择一个整数并公开宣布它。如果公布的三个数字之和是 7 的倍数，则 Asha 获胜；否则 Richy 赢了。请问 Asha 第一轮应该选哪一个数才能确保获胜？

Asha chooses a whole number from 1 to 5 and announces it. Then Richy chooses a whole number from 1 to 5 and announces it. Finally, Asha chooses a whole number from 1 to 5 and announces it.

If the sum of the three numbers announced is a multiple of 7, then Asha wins; otherwise, Richy wins.

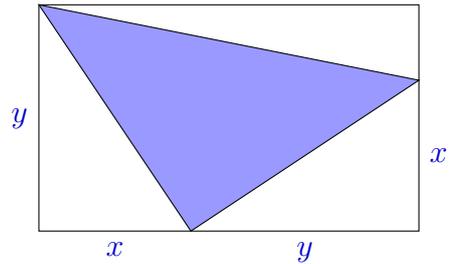
What number should Asha choose on her first turn to guarantee that she can win?

- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

## 18. 这个矩形内阴影三角形的面积是？

The area of the shaded triangle inside this rectangle is

- (A)  $\frac{1}{2}(x+y)^2$       (B)  $x(x+y)$       (C)  $y(x+y)$   
 (D)  $\frac{1}{2}(y^2 - x^2)$       (E)  $\frac{1}{2}(x^2 + y^2)$



## 19. 这三个立方体上的标记都完全相同，其中 6 个面上分别标有 A、M、C、D、E 和 F 6 个字母：

These three cubes are labelled in exactly the same way, with the 6 letters A, M, C, D, E and F on their 6 faces:



现在将立方体排成一排，使得从正面看起来像这样：

The cubes are now placed in a row so that the front looks like this:



当从这些正立方体的后方看过去时，请问我们将会看到：

When we look at the cubes from the opposite side, we will see

- (A) (B) (C)   
 (D) (E)

## 20. 五个数字排成一排。从第三个数字开始，每个数字都是前两个数字的平均值。已知第一个数字是 12，最后一个数字是 7。请问第三个数字是什么？

Five numbers are placed in a row. From the third number on, each number is the average of the previous two numbers. The first number is 12 and the last number is 7. What is the third number?

- (A) 4      (B) 5      (C) 6      (D) 7      (E) 8

## 21. 一座水池可以通过三根水管同时或分开注入水。如果只使用一根水管，则池子在 21 小时内充满。如果只使用第二根水管，水池则会在 24 小时后充满。如果三根管子一起使用，则 8 小时可将水池注满水。请问如果只使用第三根水管，需要多久才能将水池注满？

A pool can be filled through three pipes that can be used together or separately. If only the first pipe is used, the pool is filled in 21 hours. If only the second pipe is used, the pool is filled in 24 hours. If all three pipes are used, the pool is filled in 8 hours.

How long will it take to fill the pool using only the third pipe?

- (A) 12 hours      (B) 14 hours      (C) 27 hours      (D) 28 hours      (E) 30 hours

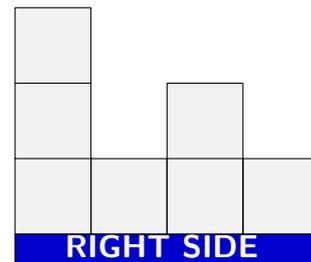
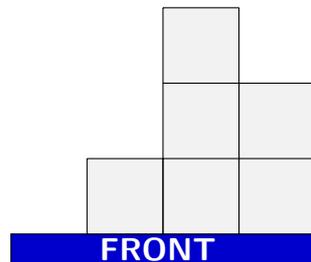
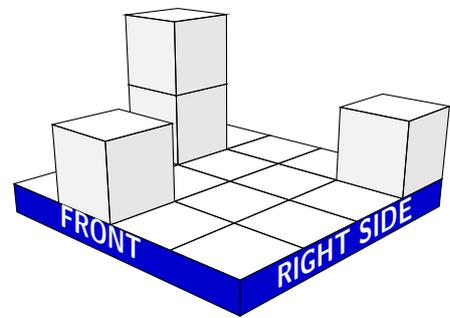
22. 在一块  $4\text{ cm} \times 4\text{ cm}$  的平台上放置有一些  $1\text{ cm}^3$  的正立方体，如图所示。现在将这个平台清空，然后放上若干个正立方体。它的前视图和右视图如下图所示。请问在这个平台上最多可以有多少个立方体？

A  $4\text{ cm} \times 4\text{ cm}$  board can have  $1\text{ cm}^3$  cubes placed on it as shown.

The board is cleared, then a number of these cubes are placed on the grid. The front and right side views are shown.

What is the maximum number of cubes there could be on the board?

- (A) 10      (B) 11      (C) 16      (D) 17      (E) 18



23. Manny 从家到工作的 8 公里路程有 3 种方式：开车需要 12 分钟，骑自行车需要 24 分钟，步行需要 1 小时 44 分钟。他想知道如果他骑自行车轮胎没有气时，如何以最短的时间赶到工作地点上班。

他有三个策略：

- (i) 如果他离家很近，则步行回家，然后改成开车上班。
  - (ii) 如果他离工作地点很近，则剩下的路程他改用步行的方式。
  - (iii) 如果在 midpoint 附近的距离，则花费 20 分钟修理轮胎，然后继续骑自行车上班。
- 他知道在前往工作地点的途中，有两个位置必须改变策略。请问这两个位置相距多远？

Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

- (i) If he is close to home, walk back home and then drive his car.
- (ii) If he is close to work, just walk the rest of the way.
- (iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

He knows there are two locations along the route to work where the strategy should change. How far apart are they?

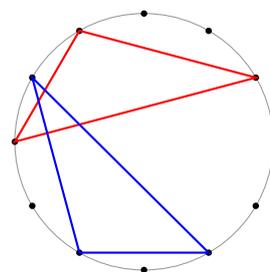
- (A) 2 km            (B) 3 km            (C) 4 km            (D) 5 km            (E) 6 km
- 

24. 现代音乐理论中，出现了以下问题。在一个圆周上有十二个等距离的点。连接其中三个点可以构造一个三角形。如果两个三角形经过旋转，但不可翻转，会完全吻合在一起，则视这两个三角形为相同的。例如，右图所示的两个三角形是相同的。请问总共可以构造多少个不同的三角形？

Out of modern musical theory comes the following question. Twelve points are equally spaced around a circle. Three points are to be joined to make a triangle.

We count two triangles as being the same only if they match perfectly after rotating, but not reflecting. For instance, the two triangles shown are the same.

How many different triangles can be made?



- (A) 10            (B) 14            (C) 19            (D) 20            (E) 22
- 

25. 一枚半径为 1 厘米的圆形硬币在正方形内部滚动而不滑动，移动时恒与正方形的边相切。当这枚硬币回到出发点时，它恰好转一整圈。以厘米为单位，请问正方形的边长是多少？

A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square.

When it returns to where it started, the coin has performed exactly one whole revolution.

In centimetres, what is the side length of the square?



- (A)  $\pi$             (B) 3.5            (C)  $1 + \pi$             (D) 4            (E)  $2 + \frac{\pi}{2}$
- 

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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26. 如果一个正整数至少有一个数码与它在这个数中的位数之值相同，则称这个数码为好数。例如，78247 是个好数，因为数码 4 出现在第 4 位。请问在三位数中有多少个好数？

A positive whole number is called *stable* if at least one of its digits has the same value as its position in the number. For example, 78247 is stable because a 4 appears in the 4<sup>th</sup> position. How many stable 3-digit numbers are there?

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- 27 当我们将一个正整数除以 15，其余数会是从 0 到 14 中的一个整数，当我们将一个正整数除以 27，其余数是从 0 到 26 中的一个整数。例如，这个正整数是 100，则余数分别是 10 和 19，它们不相等。请问从 1 到 1000 中，有多少个整数除以 15 与除以 27 所得的余数相同？
-

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When I divide an integer by 15, the remainder is an integer from 0 to 14. When I divide an integer by 27, the remainder is an integer from 0 to 26.

For instance, if the integer is 100 then the remainders are 10 and 19, which are different.

How many integers from 1 to 1000 leave the same remainders after division by 15 and after division by 27?

- 28 数35有一个性质，当我们把它的数码全部各加上2，然后相乘，则可以得到 $5 \times 7 = 35$ ，与原来的数相等。找出所有二位数中，当我们把它的数码全部加上2，然后相乘，最后所得的结果与原数相同。请问满足上序条件的所有二位数之和是多少？

The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is  $5 \times 7 = 35$ , equal to the original number.

Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.

- 
29. 有位暴君下令数码0不可以用来计算任何数，因为0代表没有任何东西，所以在计数中不允许出现任何数码0。因此在这个国家的初始几个计数数为1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, ... 其中第十个数是11。当你写下初始一千个该国允许的计数数字时，请问最后一个数的末三位数码是什么？

The Leader of Zip decrees that the digit 0, since it represents nothing, will no longer be used in any counting number. Only counting numbers without 0 digits are allowed. So the counting numbers in Zip begin 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, ... , where the tenth counting number is 11.

When you write out the first one thousand allowable counting numbers in Zip, what are the last three digits of the final number?

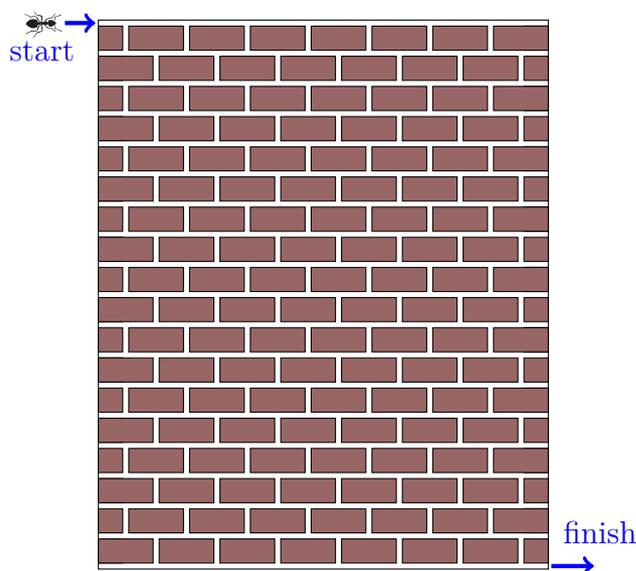
- 
30. 一只蚂蚁在这堵砖墙的左上角，需要爬行到达右下角。由于天气炎热，它避开了深色的砖块，只走在砖块之间、墙顶和墙底的较凉白色的砂浆上爬行。这道砖墙有18排砖，每排有7块整砖又半块的砖，它们以交替的方式排列。请问这只蚂蚁要爬行到对角的最快路径总共有多少种不同的方法？

Antony the ant is at the top-left corner of this brick wall and needs to get to the bottom-right corner.

Because it is a hot day, he avoids the dark bricks and only walks on the cooler white mortar between the bricks and at the top and bottom of the wall.

There are 18 rows of bricks, each with 7 whole bricks and one half-brick in an alternating pattern.

How many different ways are there for Antony to walk to the opposite corner as quickly as possible?



# 澳洲AMC 九十年级 答案

2019年

1. D

2. A

3. D

4. B

5. B

6. C

7. B

8. B

9. E

10. D

11. B

12. A

13. C

14. C

15. D

16. D

17. A

18. E

19. D

20. E

21. D

22. D

23. C

24. C

25. E

26. 252

27. 119

28. 127

29. 331

30. 103