AUSTRALIAN MATHS TRUST

2020 AMC

AUSTRALIAN MATHEMATICS COMPETITION

Senior Years 11–12
(Australian school years)

CHINESE VERSION

NAME _____

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

General

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

The answer sheet

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder: You may sit this competition once, in one division only, or risk no score.

1-10 题，每题 3 分
Questions 1 to 10, 3 marks each

1. 请问算式 $2020 \div 20$ 等于多少？

What is the value of $2020 \div 20$?

(A) 2000 (B) 2040 (C) 11 (D) 101 (E) 1001

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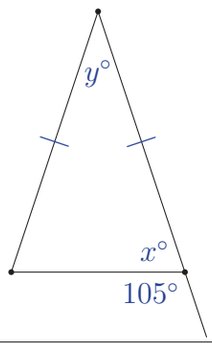
2. 在右图中，请问 x 与 y 之和等于多少？

In the diagram provided, find the sum of x and y.



(A) 30 (B) 75 (C) 95

(D) 105 (E) 180

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3. 请问根式 $\sqrt{7+18 \div (10-1^5)}$ 的值等于多少？

Evaluate $\sqrt{7+18 \div (10-1^5)}$.

(A) $\frac{5}{3}$ (B) 9 (C) 3 (D) 5 (E) $\frac{1}{27}$

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4. 小莎心里想着两个数，它们的和为 26、差为 14。请问小莎想的这两个数的乘积是多少？

Sebastien is thinking of two numbers whose sum is 26 and whose difference is 14. The product of Sebastien's two numbers is

(A) 80 (B) 96 (C) 105 (D) 120 (E) 132

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5. 已知 $\frac{7}{8}$ 的 $\frac{\star}{7}$ 的 $\frac{5}{6}$ 的 $\frac{4}{5}$ 等于 1，请问 $\star$ 的值为多少？

If $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{\star}{7}$ of $\frac{7}{8}$ is equal to 1, then the value of $\star$ is

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

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6. 将一个面积为 $10000\,\text{m}^2$ 正方形花园的长与宽各增加 10%，请问这个花园的面积会增加多少 m^2？

A square garden of area $10\,000\,\text{m}^2$ is to be enlarged by increasing both its length and width by 10%. The increase in area, in square metres, is

(A) 1000 (B) 2000 (C) 2100 (D) 2400 (E) 4000

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7. 已知 $f(x) = 2x^2 - 3x + c$ 且 $f(2) = 6$，请问 c 的值是多少？

Given that $f(x) = 2x^2 - 3x + c$ and $f(2) = 6$, then c is equal to

(A) 4 (B) 3 (C) 6 (D) 8 (E) 12

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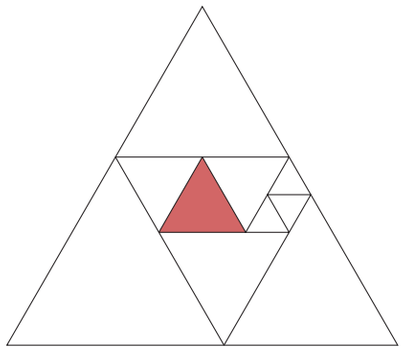
8. 将一个正三角形分割成若干个小正三角形，如图所示。涂上阴影的三角形边长为 2。
请问原来这个大正三角形的周长是多少？



An equilateral triangle is subdivided into a number of smaller equilateral triangles, as shown. The shaded triangle has side length 2. What is the perimeter of the large triangle?

(A) 24 (B) 27 (C) 30

(D) 33 (E) 36

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9. 若 $a \neq 0$，请问 $\dfrac{a^{x+y}}{a^x}$ 等于下列哪一项？

If $a \neq 0$, then $\dfrac{a^{x+y}}{a^x}$ is equivalent to

(A) a^y (B) $\dfrac{1}{a^y}$ (C) $-a^y$ (D) a^{1+y} (E) $1 + a^y$

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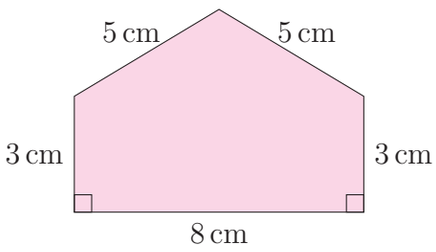
10. 请问右图所示五边形的面积是多少？

What is the area of the pentagon shown?

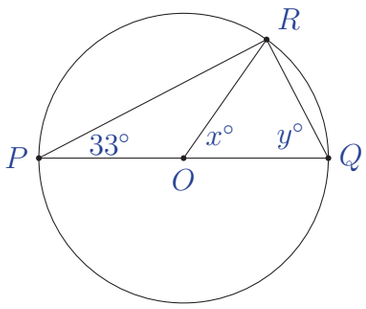


(A) $32\,\text{cm}^2$ (B) $36\,\text{cm}^2$ (C) $42\,\text{cm}^2$

(D) $56\,\text{cm}^2$ (E) $64\,\text{cm}^2$

11-20 题，每题 4 分
Questions 11 to 20, 4 marks each

11. 右图中，PQ 是圆的直径，OR 是半径，且 $\angle OPR = 33°$。请问 $x + y$ 等于多少？



In the diagram, PQ is a diameter of the circle, OR is a radius, and $\angle OPR = 33°$.
The value of $x + y$ is

(A) 99 (B) 113 (C) 115

(D) 123 (E) 137

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12. 右图全部都是由一些半圆构成的。八个最小半圆的直径都正好等于两个最大半圆直径的四分之一。请问这个大圆内的几分之几被涂成阴影？

This diagram is composed entirely of semicircles. The diameter of each of the eight smallest semicircles is exactly one-quarter of the diameter of the two biggest semicircles.
What fraction of the large circle is shaded?

(A) $\frac{9}{16}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$ (E) $\frac{5}{8}$

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13. 一整天的天气不是晴天就是雨天。
如果今天是晴天，则明天是晴天的机率为 $\frac{3}{4}$。
如果今天是雨天，则明天是晴天的机率为 $\frac{1}{3}$。
今天是星期五且是晴天。我打算星期日进行户外烧烤，请问星期日是晴天的机率是多少？

In Paradise, all days are either fine or wet.
If today is fine, the probability of tomorrow being fine is $\frac{3}{4}$.
If today is wet, the probability of tomorrow being fine is $\frac{1}{3}$.
Today is Friday and it is fine. I am having a BBQ on Sunday. What is the probability that it will be fine on Sunday?

(A) $\frac{25}{48}$ (B) $\frac{29}{48}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{31}{48}$

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14. 已知 x 与 y 都是整数且 $2^{x+1}+2^x=3^{y+2}-3^y$。请问 $x+y$ 的值等于多少？

Given that x and y are both integers and $2^{x+1}+2^x=3^{y+2}-3^y$, the value of $x+y$ is
(A) 0 (B) 1 (C) 4 (D) 7 (E) 9

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15. 一个袋子内恰好有 50 枚硬币。这些硬币的面额分别为 10 元、20 元或和50 元，且每种面额的硬币袋子中至少有一枚。已知袋子中硬币的总金额为 1000 元。请问袋子中总共有多少种不同的情况？

A bag contains exactly 50 coins. The coins are either worth 10 cents, 20 cents or 50 cents, and there is at least one of each. The total value of the coins is $10.
How many different ways can this occur?

(A) 2 (B) 4 (C) 8 (D) 12 (E) 16

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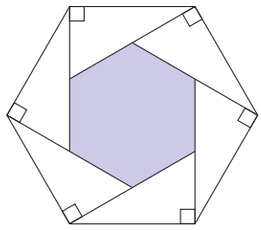
16. 一个正六边形内部有一部分被六个直角三角形覆盖，如图所示。
请问这个正六边形还有几分之几没有被覆盖？



A regular hexagon is partially covered by six right-angled triangles, as shown.
What fraction of the hexagon is not covered?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

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17. 在一个牧羊场，公羊的数量是母羊数量的 4 倍。在另一个牧场，母羊的数量是公羊的 5 倍。将两个牧羊场的羊群合并在一起，则公羊的数量与母羊的数量相等。请问这两群羊总数的最小可能值是多少？

In a paddock of sheep, there are 4 times as many male sheep as female sheep. In another paddock, there are 5 times as many females as males. When the two flocks of sheep are combined, there are equal numbers of males and females. What is the smallest possible total number of sheep?

(A) 20 (B) 26 (C) 30 (D) 38 (E) 42

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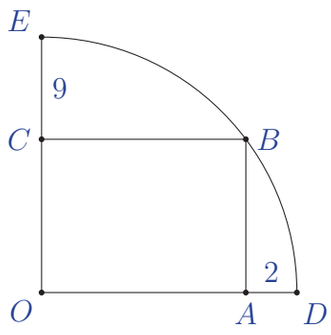
18. 在四分之一圆 ODE 内部画一个矩形 $OABC$，使得 $AD = 2$ 且 $CE = 9$。
请问这个圆的半径为多少？



The rectangle $OABC$ is drawn in the quadrant of a circle ODE, so that $AD = 2$ and $CE = 9$.
What is the radius of the circle?

(A) 11 (B) 13 (C) 15 (D) 17 (E) 20

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19. 请问函数 $f(x) = 2^{x^2-2x-3}$ 的最小值是多少？

The minimum value of the function $f(x) = 2^{x^2-2x-3}$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{16}$

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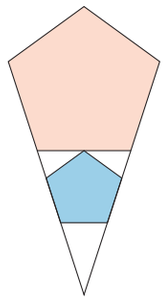
20. 将正五边形的两边延长构成一个三角形。然后在这个三角形的内部画一个小正五边形，如图所示。
请问大正五边形的面积是小正五边形面积的多少倍？



Two sides of a regular pentagon are extended to create a triangle. Inside this triangle, a smaller regular pentagon is drawn, as shown. In area, how many times bigger is the larger pentagon than the smaller pentagon?

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $\dfrac{\sqrt{5}+3}{2}$ (E) $\sqrt{5}$

21-25 题，每题 5 分
Questions 21 to 25, 5 marks each

21. 对于 $n \geq 1$，定义s_n为n个1连续组成的数，故 $s_1 = 1$、$s_2 = 11$、$s_3 = 111$，以此类推。
请问下列哪一项内的数可被 7 整除？

For $n \geq 1$, s_n is defined to be the number consisting of n consecutive ones, so $s_1 = 1$, $s_2 = 11$, $s_3 = 111$, and so on.
Which one of the following numbers is divisible by 7?

(A) s_{902} (B) s_{903} (C) s_{904} (D) s_{905} (E) s_{906}

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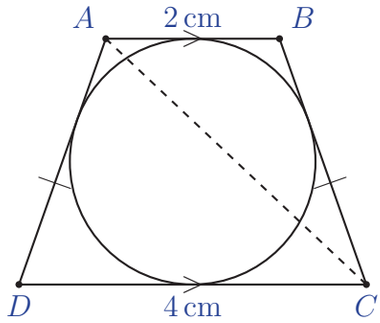
22. 一个圆内切于四边形 $ABCD$，如图所示。
已知边 AB 与 DC 平行且长度分别为 2 cm 与 4 cm，边 AD 与 BC 的长度相等。
请问 AC 的长度是多少 cm？



A circle is inscribed in the quadrilateral $ABCD$ so that it touches all four sides, as shown. Sides AB and DC are parallel with lengths 2 cm and 4 cm, respectively, and sides AD and BC have equal length.
What, in centimetres, is the length of AC?

(A) $\sqrt{17}$ (B) $2\sqrt{5}$ (C) $\sqrt{13}$ (D) 5 (E) $3\sqrt{2}$

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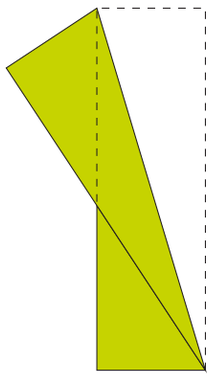
23. 一张矩形纸片的长是宽的三倍。将它沿着一条对角线折叠成为一个五边形，如图所示。
请问这个五边形的面积与原来矩形面积之比为几比几？



A rectangular sheet of paper that is three times as tall as it is wide is folded along one diagonal, making the pentagon shown. What is the ratio of the area of this pentagon to the area of the original rectangle?

(A) 13 : 18 (B) 3 : 4 (C) 7 : 12 (D) 2 : 3 (E) $\sqrt{10} : 4$

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24. 小亚计算以下算式的和，算式中最后一项是由 2020 个连续的数字 9 组成的：

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots9}_{\text{2019 个九}} + \underbrace{99\ldots9}_{\text{2020 个九}}$$

请问数字 1 在答案中重复出现了几次？

Alex writes down the value of the following sum, where the final term is the number consisting of 2020 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots9}_{\text{2019 nines}} + \underbrace{99\ldots9}_{\text{2020 nines}}$$

How many times does the digit 1 appear in the answer?

(A) 0 (B) 2016 (C) 2018 (D) 2020 (E) 2021

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25. 三个实数 a、b 与 c 满足

$$a+b+c=4 \quad \text{且} \quad \frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=5$$

请问算式 $\dfrac{c}{a+b}+\dfrac{a}{b+c}+\dfrac{b}{c+a}$ 的值等于多少？

Three real numbers a, b and c are such that

$$a+b+c=4 \quad \text{and} \quad \frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=5$$

Then, $\dfrac{c}{a+b}+\dfrac{a}{b+c}+\dfrac{b}{c+a}$ is equal to

(A) $\dfrac{3}{2}$ (B) $\dfrac{4}{5}$ (C) 2 (D) 20 (E) 17

问题 26-30 的答案为 000-999 之间的整数，
请将答案填在答案卡上对应的位置。

第 26 题占 6 分，第 27 题占 7 分，第 28 题占 8 分，
第 29 题占 9 分，第 30 题占 10 分。

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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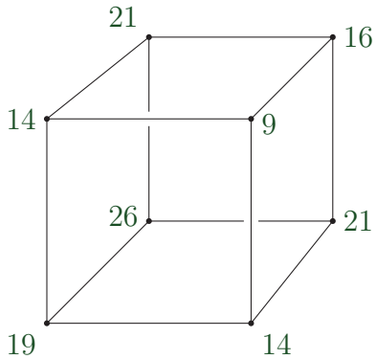
26. 在一个正立方体的每个表面都放置 1 到 10 之中的一个整数，且互不相同，然后在每个顶点上标记在此顶点交会的三个平面上的数之和。
右图只显示各顶点上的数。
在各面上的数之中，请问最小四个数的乘积是多少？



A different integer from 1 to 10 is placed on each of the faces of a cube. Each vertex is then assigned a number which is the sum of the numbers on the three faces which touch that vertex.
Only the vertex numbers are shown here.
What is the product of the 4 smallest face numbers?

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27. 多项式函数 P（x）的系数都是非负整数。已知 $P(2) = 40$ 且 $P(40) = 2\,688\,008$，请问 $P(3)$ 的值是多少？

The coefficients of a polynomial function $P(x)$ are all non-negative integers. Given that $P(2) = 40$ and $P(40) = 2\,688\,008$, what is the value of $P(3)$?

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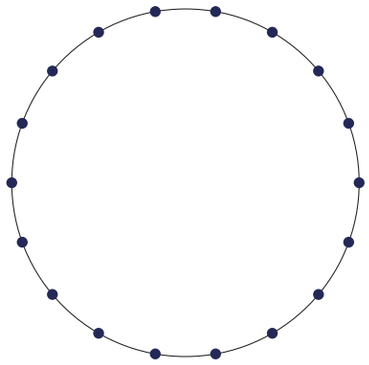
28. 这个圆上标记有 18 个等距的点。共有 816 种方法连结其中三个点而构成一个三角形。
请问这些三角形中有多少个三角形具有两个相差 30° 的角？

This circle has 18 equally spaced points marked. There are 816 ways of joining 3 of these points to form a triangle.
How many of these triangles have a pair of angles that differ by 30°?



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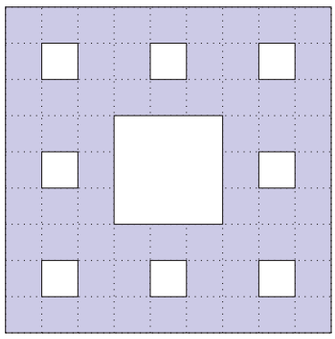
29. 从一个 $9 \times 9 \times 9$ 的正立方体开始。小史沿着每个面的九个正方形打通，使得最后的主视图、左视图与俯视图都与右图所示相同。
请问从原来的正方体到多孔立方体，表面积增加了多少？

Starting with a 9×9×9 cube, Steve mined out nine square tunnels through each face so that the resulting solid shape had front view, top view and side view all the same, as shown.

Going from the original cube to the perforated cube, how much did the surface area increase?



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30. 我每天开车到学校需要经过八个交通信号灯，每个信号灯是由绿灯 (G)、黄灯 (Y) 和红灯 (R) 组成的。由于同步化，我发现黄灯总是紧跟着绿灯，而红灯不会紧跟着红灯。
所以这些信号灯的排列可能是 GYRY，但不可能是 RRGG。
请问这八个红绿灯共有多少种可能的排列方式？

When I drive to school every day, I pass eight traffic lights, each either green, yellow, or red. I find that, because of synchronization, a green light is always followed immediately by a yellow, and a red light is never immediately followed by a red.
Thus a sequence of lights may start with GYRY, but not RRGG.
How many possible sequences of the eight lights are there?

E 组 2020 试卷 答案

1: D
2: D
3: C
4: D
5: D
6: C
7: A
8: B
9: A
10: B
11: D
12: E
13: E
14: C
15: D
16: B
17: D
18: D
19: E
20: A
21: E
22: A
23: A
24: B
25: E
26: 168
27: 143
28: 180
29: 570
30: 595

A: 5
B: 4
C: 3
D: 8
E: 5