

2021

# AUSTRALIAN MATHEMATICS COMPETITION



**AMT.**  
AUSTRALIAN  
MATHS TRUST

Senior  
Years 11–12

## Instructions and Information

### General

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 000 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

### The answer sheet

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

### Integrity of the competition

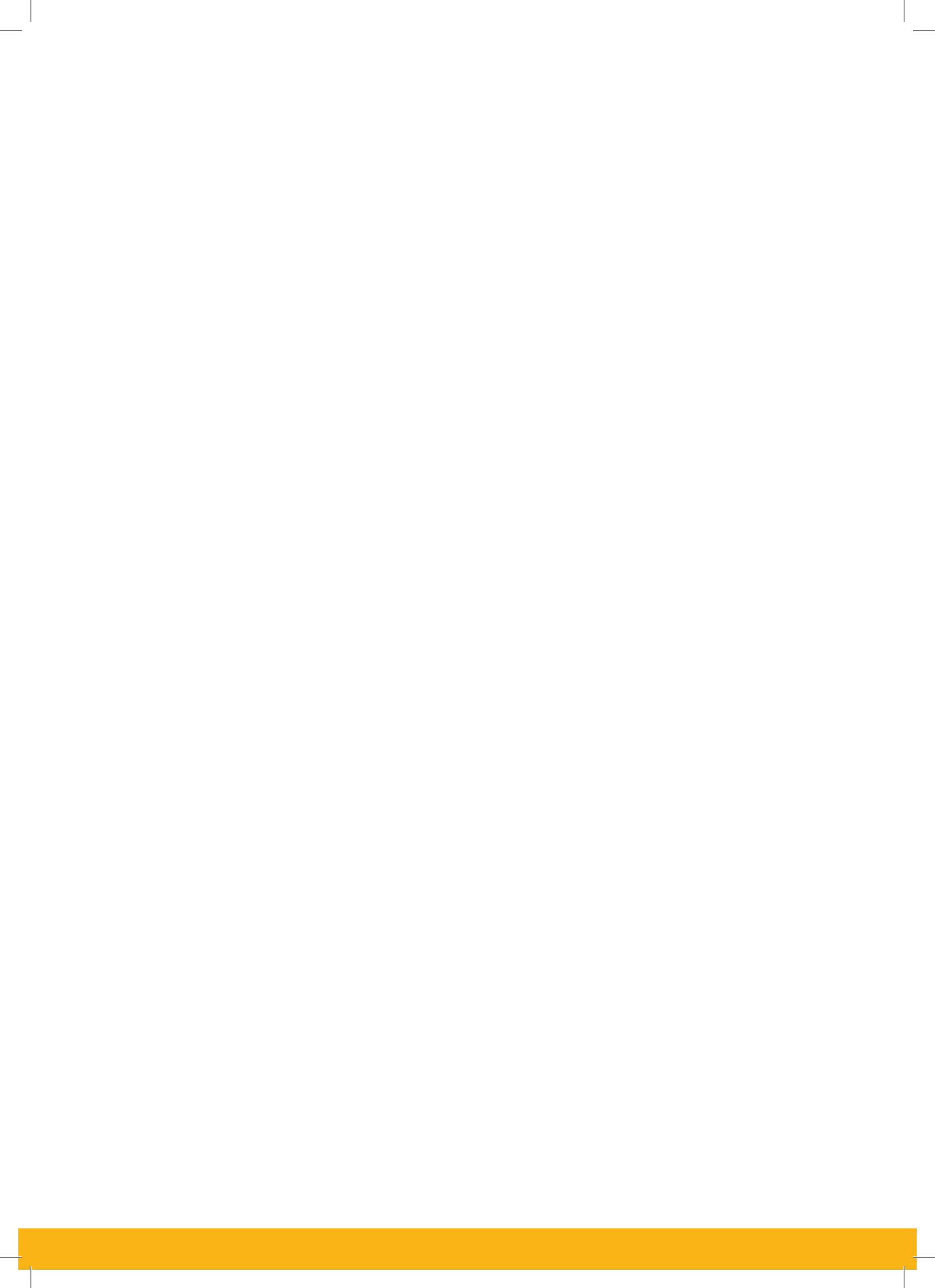
The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

### Reminder

You may sit this competition once, in one division only, or risk no score.

TIME ALLOWED

**75 minutes**



## Senior Division

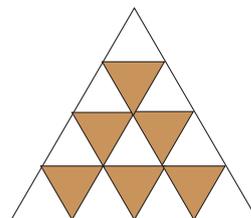
1-10 题, 每题 3 分

Questions 1 to 10, 3 marks each

1. 右图中每个小三角形的大小都相同。  
请问最大的三角形内有几分之几被涂上阴影？

Each small triangle is the same size.

What fraction of the largest triangle is shaded?



- (A)  $\frac{1}{2}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{4}$   
(D)  $\frac{2}{5}$       (E)  $\frac{3}{8}$

2. 2021 除以 7 所得的余数为

When 2021 is divided by 7 the remainder is

- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

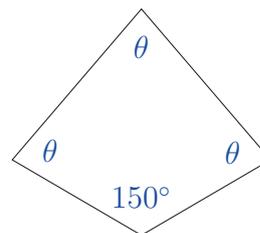
3. 请问算式  $12^1 - 12^{-1} - 12^0$  的值是多少？

What is  $12^1 - 12^{-1} - 12^0$ ?

- (A) 0      (B) 1      (C)  $10\frac{11}{12}$       (D) 11      (E)  $11\frac{1}{12}$

4. 在右图中, 请问标记  $\theta$  的角是多少度？

In this diagram, what is the size of the angles marked  $\theta$ ?



- (A)  $70^\circ$       (B)  $75^\circ$       (C)  $80^\circ$   
(D)  $85^\circ$       (E)  $90^\circ$

5. 算式  $\frac{(20 \times 21) + 21}{21}$  的值为

The value of  $\frac{(20 \times 21) + 21}{21}$  is

- (A) 20      (B) 21      (C) 22      (D) 41      (E) 42

6. 小亨骑电动滑板车行驶 1.5 公里用了 3 分 45 秒。  
请问小亨骑这段路程的平均速度是每小时多少公里？

Henry's electric scooter took him 1.5 km in 3 minutes and 45 seconds.  
What was the average speed of Henry's trip in kilometres per hour?

- (A) 20                      (B) 21                      (C) 24                      (D) 25                      (E) 30
- 

7.  $\frac{8^3 \times 3^6}{6^5} =$

- (A) 6                      (B) 48                      (C) 72                      (D) 128                      (E) 256
- 

8. 已知循环小数  $0.\dot{3}$  和  $0.\dot{6}$  的乘积等于循环小数  $0.\dot{x}$ 。请问  $x$  的值是多少？

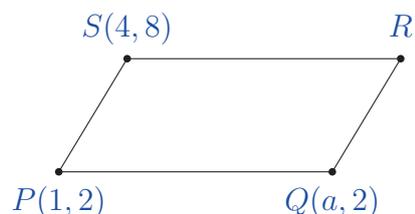
The product of recurring decimals  $0.\dot{3}$  and  $0.\dot{6}$  is the recurring decimal  $0.\dot{x}$ . What is the value of  $x$ ?

- (A) 1                      (B) 2                      (C) 5                      (D) 7                      (E) 9
- 

9. 右图是一个平行四边形，面积为 48 个平方单位。  
请问  $a$  的值是多少？

The parallelogram shown has an area of 48 square units. The value of  $a$  is

- (A) 7      (B) 8      (C) 9      (D) 10      (E) 11



10. 小莫想要粉刷他的长方形卧室的四面墙和天花板，但受到以下限制：共使用三种颜料：每个平面使用一种颜料；相邻的平面颜色不可以相同。  
他决定使用红色、白色和绿色颜料，请问他总共有多少种不同的方式来粉刷卧室？

Mervin is allowed to paint the four walls and the ceiling of his rectangular bedroom as he wishes, subject to the following constraints. He paints each surface in one of three colours. He cannot paint two adjacent surfaces the same colour. He decides to use red, white and green. How many different ways can he paint his room?和

- (A) 2                      (B) 3                      (C) 6                      (D) 12                      (E) 24
-

11-20 题, 每题 4 分

Questions 11 to 20, 4 marks each

11. 下面是一列分数, 当以最简形式写出后, 这些分数的分母都小于 6:

$$\frac{1}{5}, \boxed{\phantom{00}}, \frac{1}{3}, \frac{2}{5}, \boxed{\phantom{00}}, \boxed{\phantom{00}}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$$

已知这个数列是一个递增数列, 但其中有三个分数没被写出。请问这三个分数之和是多少?

Here is a list of fractions which, when written in simplest form, have a denominator less than 6:

$$\frac{1}{5}, \boxed{\phantom{00}}, \frac{1}{3}, \frac{2}{5}, \boxed{\phantom{00}}, \boxed{\phantom{00}}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$$

The list is in ascending order, but three fractions are omitted. The sum of these three fractions is

- (A) 1                      (B) 2                      (C)  $\frac{21}{20}$                       (D)  $\frac{27}{20}$                       (E)  $\frac{29}{20}$
- 

12. 一到秋天, 小恬家的草地就会随着天气迅速变化。

草地上的开花植物最初有 150 000 株, 但每周数量都会凋谢为之前的一半。同时, 菌类植物的数量最初有 20 株, 每周都会变为之前的三倍。

请问多久之后, 菌类植物的数量超过开花植物的数量? 请选出最接近的选项。

- (A) 3 个星期      (B) 5 个星期      (C) 8 个星期      (D) 11 个星期      (E) 13 个星期

In autumn, Tilly's meadow changes rapidly with the weather.

The number of flowering plants starts at 150 000 but they are dying off so each week the number halves. At the same time, the number of fungi starts at 20 and triples each week.

To the nearest week, how long will it be until the fungi outnumber the flowering plants?

- (A) 3 weeks      (B) 5 weeks      (C) 8 weeks      (D) 11 weeks      (E) 13 weeks
-

13. 在物理学中，有一条公式如下：

$$r = \frac{mV}{qB}$$

如果  $q$  的值变为之前的三倍， $m$  的值减半且  $r$  和  $B$  的值保持不变，那么  $V$  的值会

- (A) 增加为原来的 6 倍                      (B) 减少到原来的五分之一                      (C) 保持不变  
(D) 增加为原来的两倍                      (E) 增加为原来的 3 倍

A formula in physics is given as:

$$r = \frac{mV}{qB}$$

If  $q$  was trebled,  $m$  was halved and  $r$  and  $B$  remained the same, then  $V$  would

- (A) increase by a factor of 6                      (B) decrease by a factor of 5                      (C) stay the same  
(D) double                      (E) increase by a factor of 3

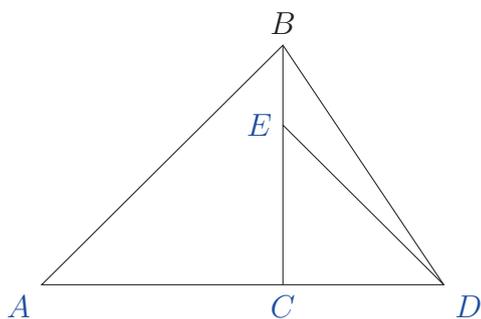
14. 在右图中，已知  $AD = 12$ ， $ABC$  与  $CDE$  都是等腰直角三角形。

如果三角形  $BDE$  的面积为 9，请问三角形  $ABD$  的面积是多少？

In this diagram,  $AD = 12$  and  $ABC$  and  $CDE$  are right-angled isosceles triangles.

The area of triangle  $BDE$  is 9.

What is the area of triangle  $ABD$ ?



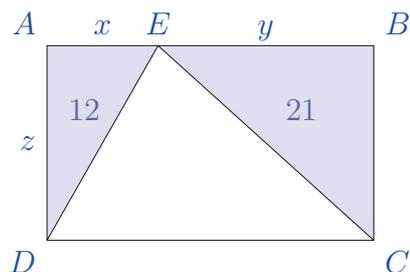
- (A) 36                      (B) 50                      (C) 54                      (D) 60                      (E) 72

15. 将数  $1^{40}$ 、 $2^{30}$ 、 $3^{20}$  与  $4^{10}$  递增排列，顺序为

The numbers  $1^{40}$ ,  $2^{30}$ ,  $3^{20}$  and  $4^{10}$ , in increasing order, are

- (A)  $1^{40}, 4^{10}, 2^{30}, 3^{20}$                       (B)  $1^{40}, 3^{20}, 4^{10}, 2^{30}$                       (C)  $4^{10}, 3^{20}, 2^{30}, 1^{40}$   
(D)  $1^{40}, 2^{30}, 3^{20}, 4^{10}$                       (E)  $1^{40}, 2^{30}, 4^{10}, 3^{20}$

16. 在矩形  $ABCD$  中, 标记有  $x$ 、 $y$  和  $z$  的线段长度都是正整数。  
已知三角形  $AED$  的面积为 12 个平方单位,  
三角形  $BCE$  的面积为 21 个平方单位。  
请问  $z$  有多少种可能值?

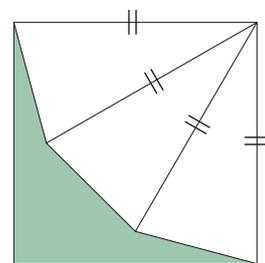


In the rectangle  $ABCD$ , the lengths marked  $x$ ,  $y$  and  $z$  are positive integers. Triangle  $AED$  has an area of 12 square units and triangle  $BCE$  has an area of 21 square units.

How many possible values are there for  $z$ ?

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

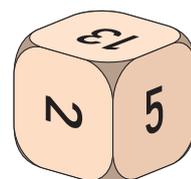
17. 如图所示, 将一个正方形分割成三个全等的等腰三角形和一个涂上阴影的五边形。请问涂上阴影的区域占正方形面积的几分之几?



A square is divided into three congruent isosceles triangles and a shaded pentagon, as shown. What fraction of the square's area is shaded?

- (A)  $\frac{1}{3}$                       (B)  $\frac{1}{4}$                       (C)  $\frac{\sqrt{2}}{6}$                       (D)  $\frac{\sqrt{3}}{8}$                       (E)  $2 - \sqrt{3}$

18. 一枚非标准骰子上标记有数字 2、3、5、8、13 和 21。将这枚骰子投掷两次, 并将得到的数字相加。请问相加后得到的结果也是骰子上标记的一个数字的概率是多少?



A non-standard dice has the numbers 2, 3, 5, 8, 13 and 21 on it. The dice is rolled twice and the numbers are added together. What is the probability that the resulting sum is also a value on the dice?

- (A)  $\frac{1}{9}$                       (B)  $\frac{5}{12}$                       (C)  $\frac{1}{3}$                       (D)  $\frac{1}{6}$                       (E)  $\frac{2}{9}$

19. 若  $a$  与  $b$  都是正整数, 则  $\sqrt{a^2 + \frac{1}{b^2}} \times \sqrt{b^2 + \frac{1}{a^2}}$  等于

If  $a$  and  $b$  are positive numbers, then  $\sqrt{a^2 + \frac{1}{b^2}} \times \sqrt{b^2 + \frac{1}{a^2}}$  is equal to

- (A)  $\frac{a}{b} + \frac{b}{a}$       (B)  $a^2b^2 + \frac{1}{a^2b^2}$       (C)  $ab + 2 + \frac{1}{ab}$       (D)  $a + b + \frac{1}{a} + \frac{1}{b}$       (E)  $ab + \frac{1}{ab}$

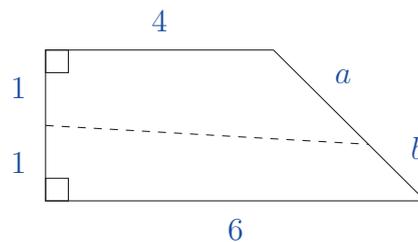
20. 如右图所示，将四边形沿着虚线切成面积相等的两部分。

请问  $a : b$  等于多少？

The quadrilateral shown is cut into two equal areas by the dashed line.

What is the ratio  $a : b$  ?

- (A) 2:1                      (B) 7:3                      (C) 5:2                      (D) 4:3                      (E) 3:2



21-25 题，每题 5 分

Questions 21 to 25, 5 marks each

21. 已知正整数  $x$  和  $y$  满足方程

$$x^2 + 2xy + 2y^2 + 2y = 1988$$

请问  $x + y$  的最大可能值是多少？

Positive integers  $x$  and  $y$  satisfy the equation

$$x^2 + 2xy + 2y^2 + 2y = 1988$$

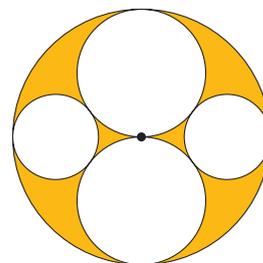
What is the largest possible value of  $x + y$ ?

- (A) 33                      (B) 38                      (C) 42                      (D) 46                      (E) 47

22. 请问右图大圆面积的几分之几被涂上了阴影？

What fraction of the area of the diagram is shaded?

- (A)  $\frac{1}{4}$                       (B)  $\frac{4}{9}$                       (C)  $\frac{5}{12}$   
(D)  $\frac{1}{2}$                       (E)  $\frac{5}{18}$



23. 小思正在用一张边长为 24 cm 的正方形纸巾折纸。他先沿着一条对角线对折后得到三角形  $ABC$ ，其中角  $A$  为直角。然后再将这个三角形折叠，使得点  $C$  落在  $AB$  上相交于点  $D$ 。设这条折线与  $BC$  交于点  $X$ 。小思继续折叠使得点  $B$  与点  $X$  重合，此时他注意到这条折线经过点  $D$ 。请问点  $A$  与点  $D$  之间的距离是多少 cm？

Sebastien is playing with a square paper serviette with side length 24 centimetres. He folds it in half along a diagonal to obtain a triangle  $ABC$  with a right angle at  $A$ . He then folds the triangle so that  $C$  ends up on line  $AB$  at some point  $D$ . Suppose that the fold created meets  $BC$  at the point  $X$ . Sebastien then folds  $B$  to meet  $X$  and notices that the fold created passes through the point  $D$ . The distance in centimetres between the points  $A$  and  $D$  is

- (A)  $6\sqrt{2}$                       (B)  $12(\sqrt{3} - 1)$                       (C) 10                      (D) 12                      (E)  $24(\sqrt{2} - 1)$

24. 已知分数  $\frac{a}{b}$  是正数且是最简形式，其中  $a$  和  $b$  都是正整数且没有公因数。  
 当将分数  $\frac{a}{b}$  的分子和分母同时加上正整数  $n$  后，得到的值是原来分数的两倍。  
 当将分数  $\frac{a}{b}$  的分子和分母同时减去  $n$  后，得到的值是原来分数的三倍。  
 请问  $n$  的值是多少？

The fraction  $\frac{a}{b}$  is positive and in lowest terms, so that  $a$  and  $b$  are positive with no common factors.

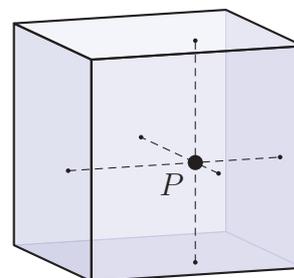
When I add the integer  $n$  to both the numerator and denominator of the fraction  $\frac{a}{b}$ , the result is double the original fraction.

When I subtract  $n$  from both the numerator and denominator, the result is triple the original value.

The value of  $n$  is

- (A) 13                      (B) 18                      (C) 21                      (D) 24                      (E) 28

25. 一个立方体内部有一个点  $P$ ，点  $P$  到立方体六个面六个面的垂直距离分别为 1 cm、2 cm、3 cm、4 cm、5 cm 和 6 cm。  
 请问这个立方体内部还有多少个其它的点具有这样的性质？



- (A) 5    (B) 11    (C) 23    (D) 47    (E) 无限多个

A cube has an internal point  $P$  such that the perpendicular distances from  $P$  to the six faces of the cube are 1 cm, 2 cm, 3 cm, 4 cm, 5 cm and 6 cm.

How many **other** internal points of the cube have this property?

- (A) 5                      (B) 11                      (C) 23                      (D) 47                      (E) infinitely many

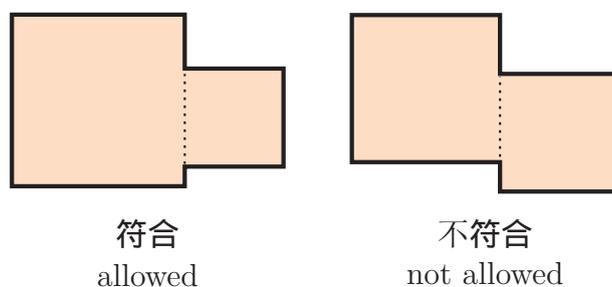
问题 26-30 的答案为 000-999 之间的整数，  
请将答案填在答案卡上对应的位置。

第 26 题占 6 分，第 27 题占 7 分，第 28 题占 8 分，  
第 29 题占 9 分，第 30 题占 10 分。

For questions 26 to 30, shade the answers as an integer from 000 to 999 in the space provided on the answer sheet.  
Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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26. 将一条 70 cm 长的绳圈摆放成由两个相邻正方形构成的图形，如下方左图所示。其中小正方形的边必须完全位于大正方形的边内，所以下方右图的例子不符合。



请问构成的图形面积的最小值是多少  $\text{cm}^2$ ?

A 70 cm long loop of string is to be arranged into a shape consisting of two adjacent squares, as shown on the left. The side of the smaller square must lie entirely within the side of the larger one, so the example on the right is not allowed.

What is the minimum area of the resulting shape, in square centimetres?

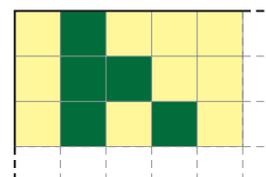
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27. 请问存在多少个数对  $(m, n)$ ，可以使得  $m$  和  $n$  是 2310 的两个不同因子，且  $n$  能整除  $m$ ？请注意：1 与 2310 都是 2310 的因子。

How many pairs  $(m, n)$  exist, where  $m$  and  $n$  are different divisors of 2310 and  $n$  divides  $m$ ? Both 1 and 2310 are considered divisors of 2310.

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28. 将一个高度为 20 个小方格、宽度为 21 个小方格的网格内所有小方格都涂成绿色或金色。右图显示了**这个网格**左上角部分的涂色情况。



网格涂色时须遵循以下规则：

- 位于最左侧一列的所有小方格都是金色的。
- 在最上面一行中，只有最左侧第二个小方格是绿色的。
- 对于**所有**这个方向的三个方格 , 金色小方格的数量均为奇数。

请问在这  $20 \times 21 = 420$  的网格中有多少个被涂成绿色？

A grid that measures 20 squares tall and 21 squares wide has each of its squares painted either green or gold. The diagram shows part of the grid, including the top-left corner.

The pattern follows these rules:

- All squares in the leftmost column are gold.
- Only the second-leftmost square in the top row is green.
- For every triplet of squares in this orientation , the number of gold squares is odd.

How many of the  $20 \times 21 = 420$  squares are painted green?

29. 小莎从一张  $1\text{ m} \times \sqrt{2}\text{ m}$  的长方形纸片中剪去一个最大的正方形，剩下一个长方形。然后对剩下的长方形重复这样的过程，剪成另一个正方形与一个较小的长方形。因为  $\sqrt{2} \approx 1.41421356$  是一个无理数，理论上她可以无限操作下去，剪出无穷多个正方形纸片。

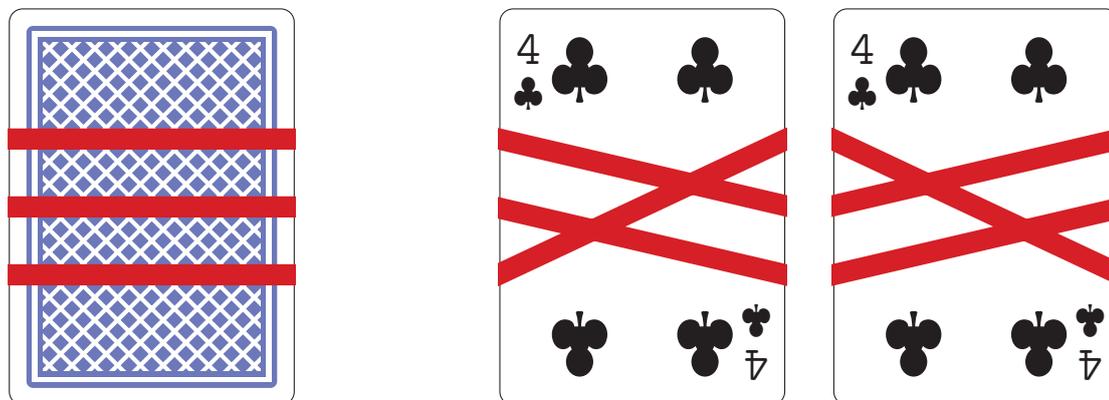
请问这些无穷多的正方形纸片的周长总和最接近多少厘米？

Starting with a paper rectangle measuring  $1 \times \sqrt{2}$  metres, Sadako makes a single cut to remove the largest square possible, leaving a rectangle. She repeats this process with the remaining rectangle, producing another square and a smaller rectangle.

Since  $\sqrt{2} \approx 1.41421356$  is irrational, she can in theory keep doing this forever, producing an infinite sequence of paper squares.

To the nearest centimetre, what would be the total perimeter of this infinite pile of squares?

30. 用一条橡皮筋将一叠扑克牌缠绕三圈，使得这副扑克牌的背面呈现三条水平线条，如下左图所示。不考虑橡皮筋自身重叠的不同方式，实际上这副扑克牌的正面有两种不同的交叉方式，如下右图所示。



如果扑克牌正面的一种交叉方式旋转  $180^\circ$  后和另一种交叉方式相同，则将这两种交叉方式视为相同。如果这条橡皮筋缠绕扑克牌后背面呈现七条水平线条，请问这副扑克牌的正面有多少种不同的交叉方式？

An elastic band is wound around a deck of playing cards three times so that three horizontal stripes are formed on the top of the deck, as shown on the left. Ignoring the different ways the rubber band could overlap itself, there are essentially two different patterns it could make on the under side of the deck, as shown on the right.

Treating two patterns as the same if one is a  $180^\circ$  rotation of the other, how many different patterns are possible on the under side of the deck if the rubber band is wound around to form seven horizontal stripes on top?

# Answer Key

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	B	E	C	D	E
2	C	E	B	C	D
3	E	E	B	A	C
4	D	A	B	D	A
5	C	A	C	B	B
6	D	D	E	B	C
7	E	A	B	E	B
8	D	E	A	B	B
9	B	D	C	B	C
10	A	D	E	D	C
11	E	E	B	C	D
12	E	D	A	D	B
13	E	D	D	C	A
14	B	A	D	A	C
15	B	B	D	C	A
16	A	C	B	E	D
17	C	C	C	A	B
18	B	C	E	C	E
19	A	C	A	E	E
20	C	A	C	E	E
21	C	E	B	E	C
22	B	B	B	C	E
23	D	A	C	D	E
24	A	B	C	B	C
25	E	D	A	E	D
26	198	666	356	71	245
27	200	72	84	36	211
28	385	45	250	55	99
29	602	17	40	93	966
30	17	117	232	792	384