2021

# AUSTRALIAN MATHEMATICS COMPETITION

**AMC.**

**AUSTRALIAN MATHS TRUST**

**Intermediate Years 9–10**

## Instructions and Information

TIME ALLOWED

**75 minutes**

### General

1. Do not open the booklet until told to do so by your teacher.

2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.

3. Diagrams are NOT drawn to scale. They are intended only as aids.

4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 000 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.

5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.

6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.

7. When your teacher gives the signal, begin working on the problems.

### The answer sheet

1. Use only lead pencil.

2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.

3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.
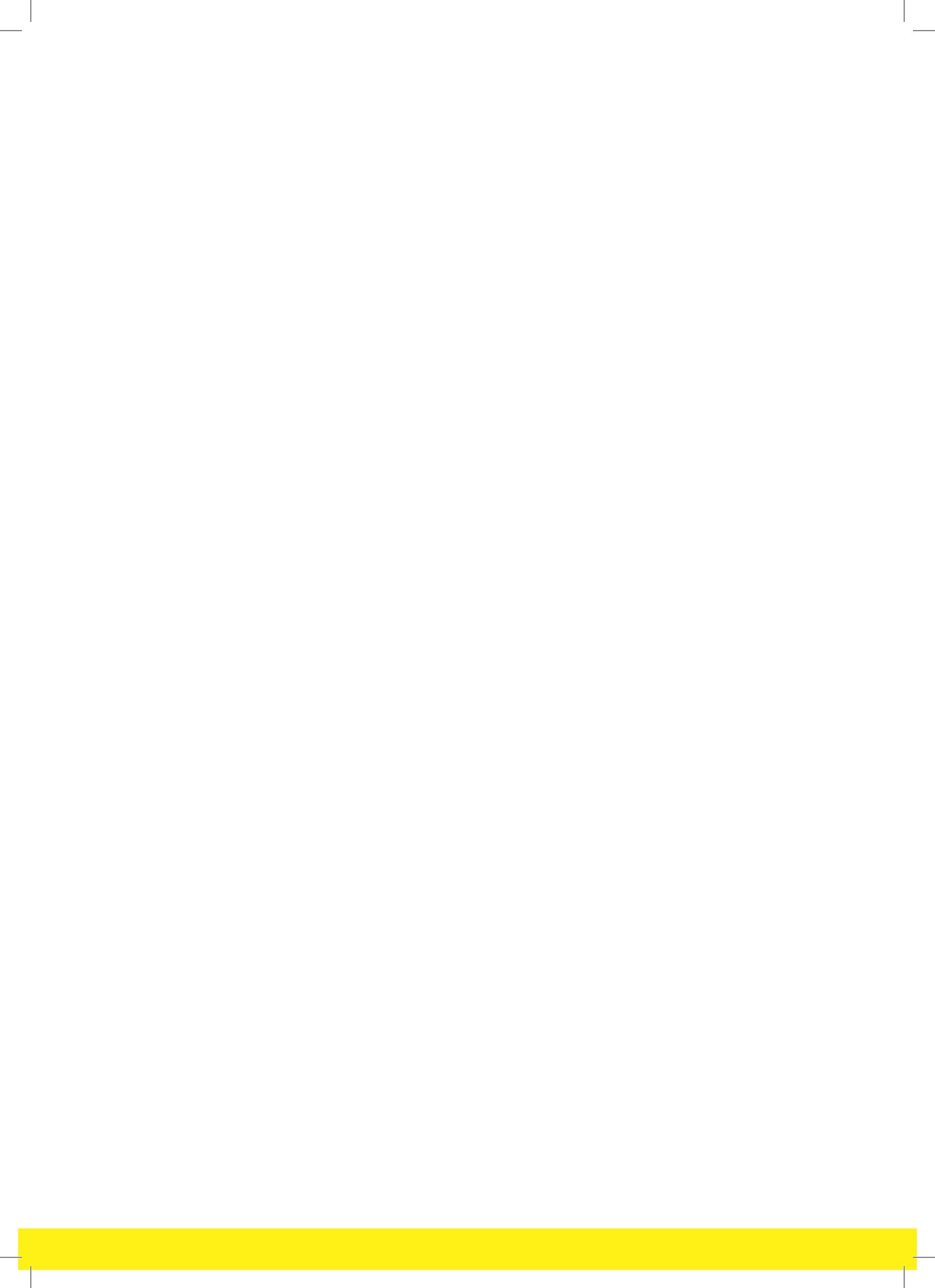
# Intermediate Division

**1-10 题，每题 3 分**
Questions1 to 10, 3 marks each

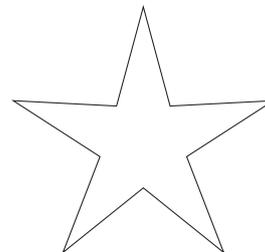**1.** 星形的每条边长度都是 2 cm。
请问它的周长是 ?
Each edge of this star is 2 cm long.
What is its perimeter?

(A) 5 cm      (B) 10 cm      (C) 15 cm
     (D) 20 cm      (E) 25 cm

**2.** 算式 $2000 - 200 + 20 - 2$ 的值为
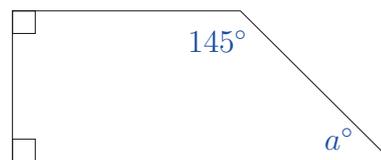The value of $2000 - 200 + 20 - 2$ is

(A) 1778      (B) 1782      (C) 1818      (D) 1822      (E) 1888

**3.** 请问右图 $a$ 的值是多少?
What is the value of $a$ in the diagram?

(A) 35      (B) 45      (C) 55
     (D) 65      (E) 75

**4.** 请问比 $\frac{1}{2}$ 50% 的数是 ?

What is 50% more than $\frac{1}{2}$?
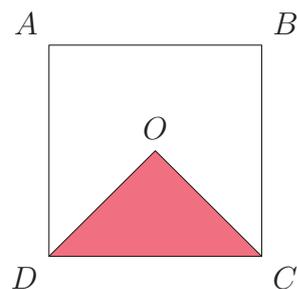
(A) $\frac{1}{4}$      (B) $\frac{5}{8}$      (C) $\frac{3}{2}$      (D) $\frac{3}{4}$      (E) 50.5

**5.** $\dfrac{1+3+5+7+9}{2+4+6+8+10} =$

(A) $\frac{1}{2}$      (B) $\frac{5}{6}$      (C) $\frac{11}{12}$      (D) $\frac{9}{10}$      (E) $\frac{63}{256}$

**6.** 正方形 $ABCD$ 的中心为点 $O$。
已知阴影区域的面积为 16　　　　　,
请问这个正方形的边长是多少　　单位？

Square $ABCD$ has centre $O$.
The shaded area is 16 square units.
What is the length of the side of the square?



(A) 4　　　　　(B) 8　　　　　(C) 16　　　　　(D) 32　　　　　(E) 64

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**7.** 数　上　　$10^2$　$10^4$　中　　点　　　　　　　?

On the number line, which number is halfway between $10^2$ and $10^4$?

(A) 500　　　　(B) 550　　　　(C) 1010　　　　(D) 2021　　　　(E) 5050

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**8.** 小金将三袋燕麦　与一袋含 20% 苜蓿、80% 燕麦的　混合　　　　　　　。
所有袋子的　都相同，请问这个混合饲料中苜蓿占百分之多少？

To feed a horse, Kim mixes three bags of oats with one bag containing 20% lucerne and 80% oats. If all the bags have the same volume, what percentage of the combined feed mixture is lucerne?

(A) 3　　　　　(B) 5　　　　　(C) 6　　　　　(D) 20　　　　　(E) 60

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**9.** 我有一块圆柱　的实心木块。　　顶　及底　都与侧曲面
相交成直角。　我　一个平面将　圆柱体　两个较小
的木块，请问下列哪一个选项的图形**不**可能是切

I have a solid block of wood in the shape of a cylinder. The top and bottom faces meet the curved side at right angles. Suppose that I slice the cylinder along a plane to create two smaller blocks of wood.

Which of the following could **not** be the shape of the resulting faces created by the slice?
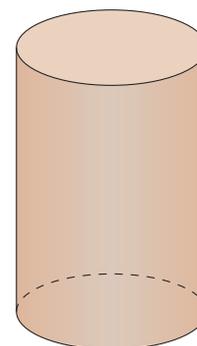


(A)　　　　　(B)　　　　　(C)　　　　　(D)　　　　　(E)
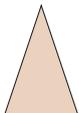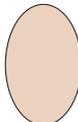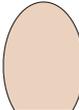
**10.** 小蒂在郊区 骑自行车 计时。骑了五圈之后，她的秒表 的时间为18 分 15 秒。

请问小蒂每骑一圈平均 多 ？

(A) 3 分 3 秒     (B) 3 分 15 秒     (C) 3 分 27 秒

      (D) 3 分 39 秒     (E) 3 分 51 秒

Diya timed herself cycling laps around her suburb. After five laps, her stopwatch indicated a time of 18 minutes and 15 seconds.

What was Diya's average time per lap?

(A) 3 minutes and 3 seconds     (B) 3 minutes and 15 seconds

(C) 3 minutes and 27 seconds     (D) 3 minutes and 39 seconds

      (E) 3 minutes and 51 seconds

---

### 11-20 题，每题 4 分
### Questions 11 to 20, 4 marks each

**11.** 有四个连续的奇数，其中最大的数比最小的数 两倍少 1。

请问下列哪一个选项 的数是这四个 数中最大 数？

I have four consecutive odd numbers. The largest is one less than twice the smallest. Which of the following is the largest of the four numbers?

(A) 9     (B) 11     (C) 13     (D) 15     (E) 21

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**12.** 光盘上未 压缩的音乐数据 每 $44\,100$ 个 ，其中每个 需要 4 个 的数据。请问下列哪一个选项内的数量最接近在光盘上存储 5 分钟音乐所需的 数？

(A) $100,0000$   (B) $500,0000$   (C) $1000,0000$   (D) $5000,0000$   (E) $10000,0000$
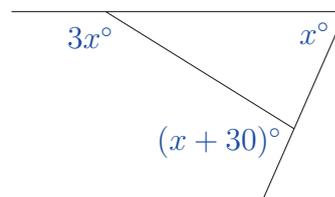
On a compact disc, uncompressed music data is stored as $44\,100$ samples for each second of music, where each sample requires 4 bytes of data. Which of the following is closest to the number of bytes required to store 5 minutes of music on the disc?

(A) 1 million     (B) 5 million     (C) 10 million     (D) 50 million     (E) 100 million

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**13.** 在右图 ，请问 $x$ 的值 ？

In the figure, the value of $x$ is
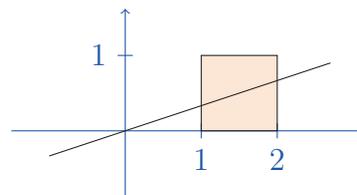
(A) 30     (B) 40     (C) 50

    (D) 60     (E) 70
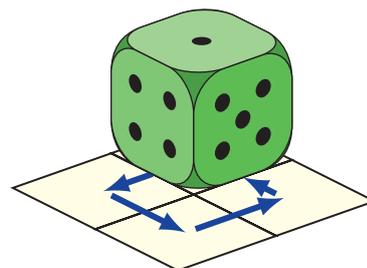
**14.** 在右图内，请问经过点 $(0,0)$ 且将正方形　平分的直线方程是什么？

What is the equation of the line passing through $(0,0)$ that bisects the square in the diagram?

(A) $y = \dfrac{x}{3}$       (B) $y = \dfrac{x}{2}$       (C) $y = \dfrac{x}{4}$

     (D) $y = 2x$       (E) $y = 3x$

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**15.** 将各面分别有 1 到 6 个点且相对两面上的点数之和　为 7 的一枚标准骰子放置在 $2 \times 2$ 的方格上，如图所示。

将这枚骰子翻越一　边依　滚遍四个小方格然后回到原来的小方格上请问　骰子　　朝上

A standard dice numbered 1 to 6 with opposite sides adding to 7 is placed on a 2 by 2 square as shown.

The dice is rolled over one edge onto each of the four base squares in turn and then back on to the original square, as indicated by the arrows.
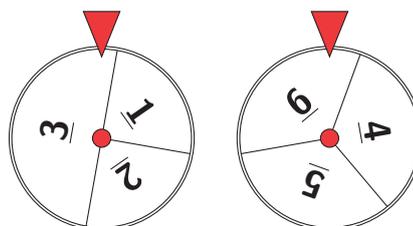
Which side of the dice is now facing upwards?

(A)    (B)    (C)    (D)    (E)

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**16.** 旋转　图所示的　　转盘，当它们停止后将箭头所指　的数　记录下来。
例如，　　　　　两个数是 3　　6。
请问　　　两个数　之和　偶数的　率是多少？

The two spinners shown are spun and the numbers that the arrows point to when they stop are recorded.

For example, the numbers here are 3 and 6.
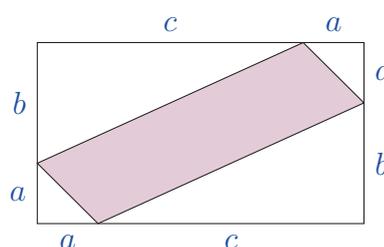
What is the probability that the sum of the two numbers is even?

(A) $\dfrac{1}{2}$      (B) $\dfrac{3}{8}$      (C) $\dfrac{3}{4}$      (D) $\dfrac{2}{3}$      (E) $\dfrac{5}{12}$

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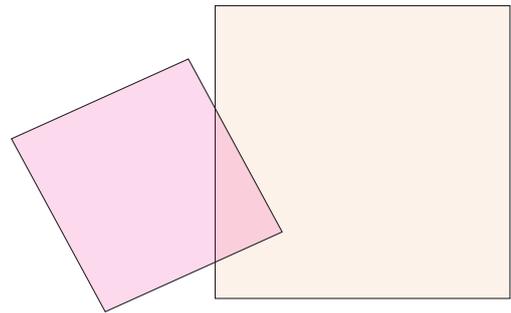**17.** 请问右图中阴影区域的面积　　　？

The area of the shaded region is given by

(A) $ab + ac$           (B) $a\sqrt{b^2 + c^2}$
(C) $bc + a^2 - ab - ac$     (D) $ab + ac - bc$
        (E) $ab + ac - a^2$

**18.** 已知 $k$ 和 $n$ 都是正整数，且 $\sqrt{10n+k}=k$，请问 $k$ 的最小可能值是 ?

If $k$ and $n$ are positive integers, and $\sqrt{10n+k}=k$, then the smallest possible value for $k$ is

(A) 3          (B) 4          (C) 5          (D) 6          (E) 10

---

**19.** 图所示方式画出两个正方形。已知小正方形覆盖住大正方形的 $\frac{1}{8}$，大正方形覆盖住小正方形的 $\frac{2}{9}$。
请问大正方形边长 小正方形边长之比是 ?

Two squares are drawn as shown.

The smaller square covers $\frac{1}{8}$ of the larger square and the larger square covers $\frac{2}{9}$ of the smaller square.

What is the ratio of the side length of the larger square to the side length of the smaller square?

(A) 3:2          (B) 7:3          (C) 7:4          (D) 5:3          (E) 4:3

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**20.** 如图所示 将六个全等的飞镖形嵌入一个正六边形内。每个飞镖形都有三个 30° 内角 一个 270° 内角 请问这个大正六边形内有几分之几被涂上阴影？
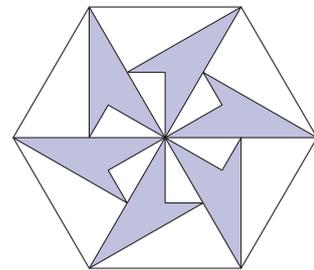
Six identical darts fit inside a regular hexagon as shown. Each dart has three interior angles of 30°, and one of 270°.

What fraction of the large hexagon is shaded?

(A) $\frac{1}{2}$          (B) $\frac{1}{3}$          (C) $\frac{2}{5}$          (D) $\frac{4}{9}$          (E) $\frac{3}{8}$

**21-25 题，每题 5 分**
Questions 21 to 25, 5 marks each

**21.** 我们想要在右图的每个空白小方格内填入一个数，使得所填入的每个数都等于所有与它直接相连的小方格内的数之平均值。

请问位于最上面一行中央的小方格内应该填入的数是什么？
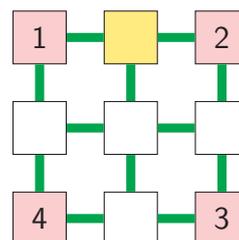


We want to place numbers into each of the blank squares in this diagram so that each of the numbers we place is the average of those in the squares directly connected to it.

What number should we put in the middle square of the top row?

(A) $\frac{5}{3}$ 
(B) $\frac{3}{2}$ 
(C) $\frac{10}{9}$ 
(D) $\frac{11}{9}$ 
(E) $\frac{11}{6}$

**22.** 为了设定微波炉上的计时器，小克 左到右依 表示时、分 秒。例如，输入"12345"， 将计时器设定为 1 小时 23 分 45 秒，输入"408"将其设定为 4 分 8 秒。

某 ，小克不小心漏输入最后一位数 ， 计时器比预期的时间提前了 4 分 42 秒。请问他漏输入的数 是 ?

To set the timer on his microwave oven, Rick enters the digits of the hours, minutes and seconds in order from left to right. For example, entering '12345' sets the timer to 1 hour 23 minutes 45 seconds, while entering '408' sets it to 4 minutes 8 seconds. One day, Rick accidentally missed the last digit and the timer finished 4 minutes and 42 seconds earlier than he was expecting. What was the missing digit?

(A) 3 
(B) 4 
(C) 5 
(D) 6 
(E) 7

**23.** 我用若干个单位立方体组成了一个大立方体。然后将大立方体的某几个面 颜色。当拆散这个大立方体 ，我发现有 288 个单位立方体没有任何一面被涂上颜色。请问这个大立方体有多少个面涂上 颜色？

I build a large cube from unit cubes. Then I completely paint a number of faces of the large cube. When I dismantle the large cube, I find that I have 288 unit cubes without any paint on them. How many faces of the large cube were painted?
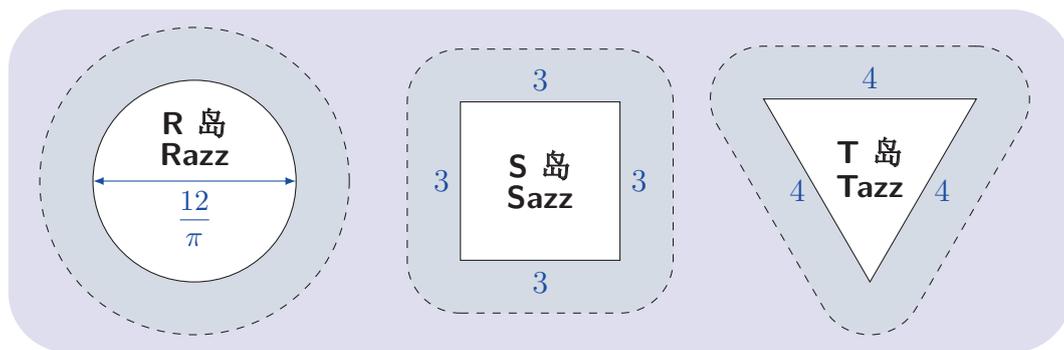
(A) 1 
(B) 2 
(C) 3 
(D) 4 
(E) 5

**24.** 乘式  $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{15^2}\right)$  的值等于

The product  $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{15^2}\right)$  is equal to

(A) $\frac{7}{13}$  (B) $\frac{8}{15}$  (C) $\frac{9}{16}$  (D) $\frac{10}{21}$  (E) $\frac{13}{24}$

**25.** 在浅　　建造三个人工岛 R 岛、S 岛　 T 岛，每个岛都有　 长为 12 km 的海岸线，如图所示。



每个岛屿　都　一个钓鱼区，钓鱼区　　　　　　　　　　　　　1 km 。
请问哪些岛屿的钓鱼区　　　 ?

(A) 仅有 R 岛  (B) 仅有 S 岛  (C) 仅有 T 岛

(D) S 岛与 T 岛  (E) 三个面积都一样大

Three artificial islands Razz, Sazz and Tazz were constructed in a shallow sea, each with a coastline of 12 km.

Around each island is a fishing zone, consisting of all points in the sea within 1 km of the island. Which islands have a fishing zone of the largest area?

(A) Razz only  (B) Sazz only  (C) Tazz only

(D) Sazz and Tazz  (E) All three have the same area

问题 **26-30** 的答案为 **000-999** 之间的整数，
请将答案填在答案卡上对应的位置。

第 **26** 题占 **6** 分，第 **27** 题占 **7** 分，第 **28** 题占 **8** 分，
第 **29** 题占 **9** 分，第 **30** 题占 **10** 分。

For questions 26 to 30, shade the answer as an integer from 000 to
999 in the space provided on the answer sheet.
Questions 26– 30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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**26.** 在澳式橄榄球比赛中，球队 " " 可得 6 分， " "，可得 1
在比赛中，小威喜欢用6 与 1 成的数列 记 球队 得分情况。恰好
的数列来记录最终得分为 7 分 ，即 6,1 与 1,6 与 1,1,1,1,1,1,1。
请问总共有多少种不同的数列可 记录最终得分为 20 分 ？

In Australian Rules football, a team scores six points for a 'goal' and one point for a
'behind'. During a game, Vladislav likes to record his team's score with a sequence
of sixes and ones. There are exactly three distinct sequences which give a final score
of 7 points, namely 6,1 and 1,6 and 1,1,1,1,1,1,1.
How many different sequences give a final score of 20 points?

---

**27.** 使得数

$$N = 100000 \times 100002 \times 100006 \times 100008 + n$$

为完全平方数 数 $n$ 是多少?

What is the smallest natural number $n$ such that the number

$$N = 100000 \times 100002 \times 100006 \times 100008 + n$$

is a perfect square?

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**28.** 我有 火柴棒，它们有四种颜色：红、黄、蓝、绿。 用它们来拼一些边长
都是一根火柴棒长的正方形。
如果拼出 的两个正方形经过旋转、翻转后 颜色 ，则将这两个正方形
视为相同 。
请问最多可以拼出多少种不同的正方形?

I have a large supply of matchsticks in four colours: red, yellow, blue and green. I use
them to make squares where each side is one matchstick long.
I count two squares as the same if one can be rotated and/or reflected to match the
shape and colour of the other.
How many different squares can be created?

---

**29.** 小贝将数 499 分别除以 $1, 2, 3, \ldots, 499$ 并　　　　记录　所得的余数。所以她所记录下来的数列前几项为：

$$0, 1, 1, 3, 4, 1, \ldots$$

　　$M$ 是这 499 个余数的总和。

小翔将数 500 分别除以 $1, 2, 3, \ldots, 500$ 并　　　　记录　所得的余数。所以他所记录下来的数列前几项为：

$$0, 0, 2, 0, 0, 2, \ldots$$

　　$N$ 是这 500 个余数的总和。

请问数 $M$　$N$　　多少？

Bluey divides the number 499 by each of the numbers $1, 2, 3, \ldots, 499$ and records the remainders in order. So her sequence begins:

$$0, 1, 1, 3, 4, 1, \ldots$$

Let $M$ be the sum of these 499 remainders.

Jean-Luc divides the number 500 by each of the numbers $1, 2, 3, \ldots, 500$ and records the remainders in order. So his sequence begins:
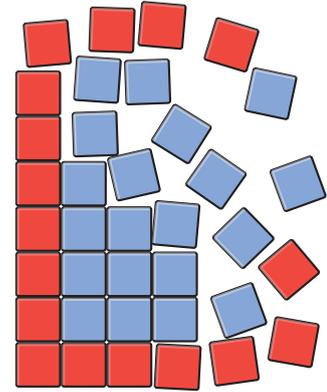
$$0, 0, 2, 0, 0, 2, \ldots$$

Let $N$ be the sum of these 500 remainders.

What is the difference between the numbers $M$ and $N$?

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**30.** 小泰有　　　　　　　正方形瓷砖，其中

　　　　　　　　　　　1

　　　　　　　　　　　　　　瓷砖
　　　　　　　　　　　　　　1

请问小泰总共有多少块蓝色瓷砖？

Tyler has a large number of square tiles, all the same size. He has four times as many blue tiles as red tiles. He builds a large rectangle using all the tiles, with the red tiles forming a boundary 1 tile wide around the blue tiles.

He then breaks up this rectangle and uses the tiles to make two smaller rectangles. Like the large rectangle, each of the smaller rectangles has four times as many blue tiles as red tiles, and the red tiles form a boundary 1 tile wide around the blue tiles.

How many blue tiles does Tyler have?

# Answer Key

| Question | Middle Primary | Upper Primary | Junior | Intermediate | Senior |
|---|---|---|---|---|---|
| 1 | B | E | C | D | E |
| 2 | C | E | B | C | D |
| 3 | E | E | B | A | C |
| 4 | D | A | B | D | A |
| 5 | C | A | C | B | B |
| 6 | D | D | E | B | C |
| 7 | E | A | B | E | B |
| 8 | D | E | A | B | B |
| 9 | B | D | C | B | C |
| 10 | A | D | E | D | C |
| 11 | E | E | B | C | D |
| 12 | E | D | A | D | B |
| 13 | E | D | D | C | A |
| 14 | B | A | D | A | C |
| 15 | B | B | D | C | A |
| 16 | A | C | B | E | D |
| 17 | C | C | C | A | B |
| 18 | B | C | E | C | E |
| 19 | A | C | A | E | E |
| 20 | C | A | C | E | E |
| 21 | C | E | B | E | C |
| 22 | B | B | B | C | E |
| 23 | D | A | C | D | E |
| 24 | A | B | C | B | C |
| 25 | E | D | A | E | D |
| 26 | 198 | 666 | 356 | 71 | 245 |
| 27 | 200 | 72 | 84 | 36 | 211 |
| 28 | 385 | 45 | 250 | 55 | 99 |
| 29 | 602 | 17 | 40 | 93 | 966 |
| 30 | 17 | 117 | 232 | 792 | 384 |