2000 AMC 12 Problems/Problem 1

The following problem is from both the 2000 AMC 12 #1 and 2000 AMC 10 #1, so both problems redirect to this page.

Problem

In the year 2001, the United States will host the International Mathematical Olympiad. Let $I, M,$ and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

$\mathrm{(A)}\ 23 \qquad \mathrm{(B)}\ 55 \qquad \mathrm{(C)}\ 99 \qquad \mathrm{(D)}\ 111 \qquad \mathrm{(E)}\ 671$

Solution

The sum is the highest if two factors are the lowest.

So, $1 \cdot 3 \cdot 667 = 2001$ and $1 + 3 + 667 = 671 \Longrightarrow \boxed{\text{(E)}}$.

See Also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Algebra Problems | Introductory Number Theory Problems

2000 AMC 12 Problems/Problem 2

The following problem is from both the 2000 AMC 12 #2 and 2000 AMC 10 #2, so both problems redirect to this page.

Problem

$2000(2000^{2000}) =$

$\text{(A) } 2000^{2001} \qquad \text{(B) } 4000^{2000} \qquad \text{(C) } 2000^{4000} \qquad \text{(D) } 4,000,000^{2000} \qquad \text{(E) } 2000^{4,000,000}$

Solution

$$2000(2000^{2000}) = (2000^1)(2000^{2000}) = 2000^{2001} \Rightarrow \boxed{A}$$

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2000_AMC_12_Problems/Problem_2&oldid=53551"

Category: Introductory Algebra Problems

2000 AMC 12 Problems/Problem 3

The following problem is from both the 2000 AMC 12 #3 and 2000 AMC 10 #3, so both problems redirect to this page.

Problem

Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

$\text{(A)}\ 40 \qquad \text{(B)}\ 50 \qquad \text{(C)}\ 55 \qquad \text{(D)}\ 60 \qquad \text{(E)}\ 75$

Solution

Since Jenny eats 20% of her jelly beans per day, $80\% = \frac{4}{5}$ of her jelly beans remain after one day.

Let x be the number of jelly beans in the jar originally.

$$\frac{4}{5} \cdot \frac{4}{5} \cdot x = 32$$

$$\frac{16}{25} \cdot x = 32$$

$$x = \frac{25}{16} \cdot 32 = 50 \Rightarrow \boxed{B}$$

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2000_AMC_12_Problems/Problem_3&oldid=53552"

Category: Introductory Algebra Problems

2000 AMC 10 Problems/Problem 4

Problem

Chandra pays an on-line service provider a fixed monthly fee plus an hourly charge for connect time. Her December bill was $\$12.48$, but in January her bill was $\$17.54$ because she used twice as much connect time as in December. What is the fixed monthly fee?

$\text{(A)}\ \$ 2.53 \qquad\text{(B)}\ \$ 5.06 \qquad\text{(C)}\ \$ 6.24 \qquad\text{(D)}\ \$ 7.42 \qquad\text{(E)}\ \$ 8.77$

Solution

Let x be the fixed fee, and y be the amount she pays for the minutes she used in the first month.

$x+y=12.48$

$x+2y=17.54$

$y=5.06$

$x=7.42$

We want the fixed fee, which is $\boxed{\text{D}}$

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 10 Problems/Problem 5

Problem

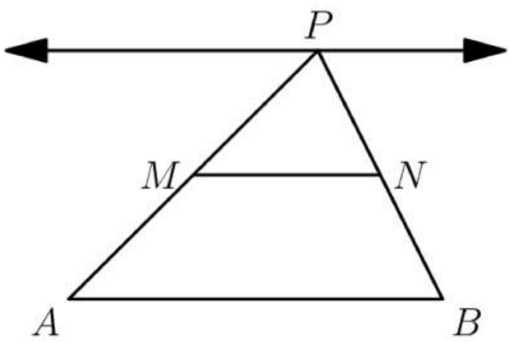
Points M and N are the midpoints of sides PA and PB of $\triangle PAB$. As P moves along a line that is parallel to side AB, how many of the four quantities listed below change?

(a) the length of the segment MN

(b) the perimeter of $\triangle PAB$

(c) the area of $\triangle PAB$

(d) the area of trapezoid $ABNM$



$\text{(A)}\ 0 \qquad \text{(B)}\ 1 \qquad \text{(C)}\ 2 \qquad \text{(D)}\ 3 \qquad \text{(E)}\ 4$

Solution

(a) Clearly AB does not change, and $MN = \frac{1}{2}AB$, so MN doesn't change either.

(b) Obviously, the perimeter changes.

(c) The area clearly doesn't change, as both the base AB and its corresponding height remain the same.

(d) The bases AB and MN do not change, and neither does the height, so the area of the trapezoid remains the same.

Only 1 quantity changes, so the correct answer is $\boxed{\text{B}}$.

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 4

The following problem is from both the 2000 AMC 12 #4 and 2000 AMC 10 #6, so both problems redirect to this page.

Problem

The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ starts with two 1s, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

$\text{(A)}\ 0 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 9$

Solution

Note that any digits other than the units digit will not affect the answer. So to make computation quicker, we can just look at the Fibonacci sequence in $\bmod{10}$:

$$1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, \ldots.$$

The last digit to appear in the units position of a number in the Fibonacci sequence is $6 \Longrightarrow \boxed{\text{C}}$.

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

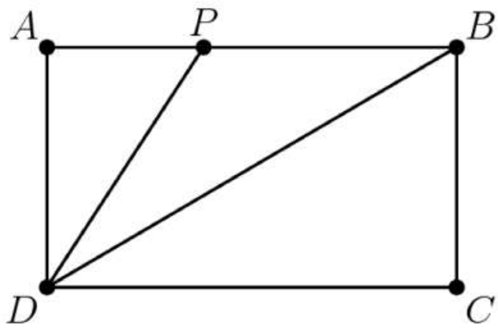
2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Combinatorics Problems

2000 AMC 10 Problems/Problem 7

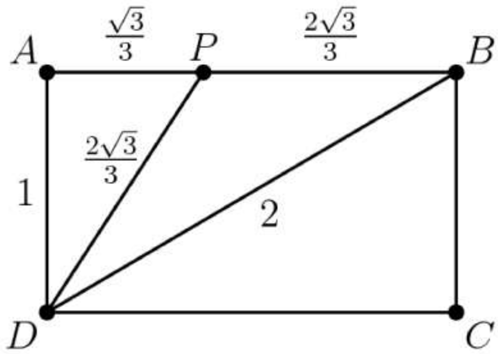
Problem

In rectangle $ABCD$, $AD=1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



$\text{(A)}\ 3+\frac{\sqrt{3}}{3} \qquad \text{(B)}\ 2+\frac{4\sqrt{3}}{3} \qquad \text{(C)}\ 2+2\sqrt{2} \qquad \text{(D)}\ \frac{3+3\sqrt{5}}{2} \qquad \text{(E)}\ 2+\frac{5\sqrt{3}}{3}$

Solution



$AD=1$.

Since $\angle ADC$ is trisected, $\angle ADP=\angle PDB=\angle BDC=30^\circ$.

Thus, $PD=\frac{2\sqrt{3}}{3}$

$DB=2$

$BP=\sqrt{3}-\frac{\sqrt{3}}{3}=\frac{2\sqrt{3}}{3}$.

Adding, $2+\frac{4\sqrt{3}}{3}$.

$\boxed{\text{B}}$

See Also

2000 AMC 10 Problems/Problem 8

Problem

At Olympic High School, $\frac{2}{5}$ of the freshmen and $\frac{4}{5}$ of the sophomores took the AMC-10. Given that the number of freshmen and sophomore contestants was the same, which of the following must be true?

(A) There are five times as many sophomores as freshmen.

(B) There are twice as many sophomores as freshmen.

(C) There are as many freshmen as sophomores.

(D) There are twice as many freshmen as sophomores.

(E) There are five times as many freshmen as sophomores.

Solution

Let f be the number of freshman and s be the number of sophomores.

$$\frac{2}{5}f = \frac{4}{5}s$$

$$2f = 4s$$

$$f = 2s$$

There are twice as many freshmen as sophomores. $\boxed{\text{D}}$

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 5

The following problem is from both the 2000 AMC 12 #5 and 2000 AMC 10 #9, so both problems redirect to this page.

Problem

If $|x-2|=p$, where $x<2$, then $x-p=$

$\text{(A)}\ -2 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 2-2p \qquad \text{(D)}\ 2p-2 \qquad \text{(E)}\ |2p-2|$

Solution

When $x<2, x-2$ is negative so $|x-2| = 2-x = p$ and $x = 2-p$.

Thus $x-p = (2-p)-p = 2-2p \Longrightarrow (C)$.

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2000_AMC_12_Problems/Problem_5&oldid=53554"

Category: Introductory Algebra Problems

2000 AMC 10 Problems/Problem 10

Problem

The sides of a triangle with positive area have lengths 4, 6, and x. The sides of a second triangle with positive area have lengths 4, 6, and y. What is the smallest positive number that is not a possible value of $|x-y|$?

$\text{(A)}\ 2 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 10$

Solution

From the triangle inequality, $2<x<10$ and $2<y<10$. The smallest positive number not possible is $10-2$, which is 8. $\boxed{\text{D}}$

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 6

The following problem is from both the 2000 AMC 12 #6 and 2000 AMC 10 #11, so both problems redirect to this page.

Contents

Problem

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

$\mathrm{(A) \ 21 } \qquad \mathrm{(B) \ 60 } \qquad \mathrm{(C) \ 119 } \qquad \mathrm{(D) \ 180 } \qquad \mathrm{(E) \ 231 }$

Solution 1

All prime numbers between 4 and 18 have an odd product and an even sum. Any odd number minus an even number is an odd number, so we can eliminate B and D. Since the highest two prime numbers we can pick are 13 and 17, the highest number we can make is $(13)(17)-(13+17) = 221 - 30 = 191$. Thus, we can eliminate E. Similarly, the two lowest prime numbers we can pick are 5 and 7, so the lowest number we can make is $(5)(7)-(5+7) = 23$. Therefore, A cannot be an answer. So, the answer must be $\mathrm{(C)}$.

Solution 2

Let the two primes be p and q. We wish to obtain the value of $pq-(p+q)$, or $pq-p-q$. Using Simon's Favorite Factoring Trick, we can rewrite this expression as $(1-p)(1-q)-1$ or $(p-1)(q-1)-1$. Noticing that $(13-1)(11-1)-1 = 120-1 = 119$, we see that the answer is $\mathrm{(C)}$.

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

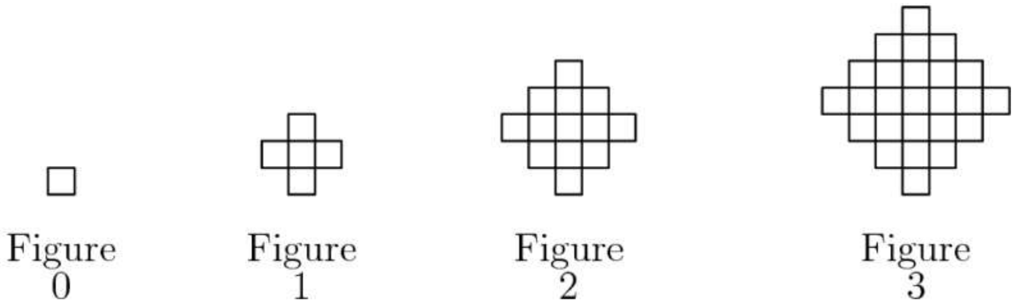
2000 AMC 12 Problems/Problem 8

The following problem is from both the 2000 AMC 12 #8 and 2000 AMC 10 #12, so both problems redirect to this page.

Contents

Problem

Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?



$\text{(A)}\ 10401 \qquad\text{(B)}\ 19801 \qquad\text{(C)}\ 20201 \qquad\text{(D)}\ 39801 \qquad\text{(E)}\ 40801$

Solution

Solution 1

We have a recursion:

$A_n = A_{n-1} + 4n.$

I.E. we add increasing multiples of 4 each time we go up a figure.

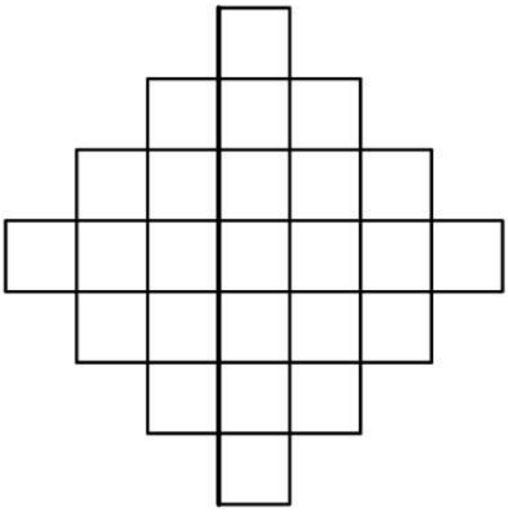
So, to go from Figure 0 to 100, we add

$4 \cdot 1 + 4 \cdot 2 + ... + 4 \cdot 99 + 4 \cdot 100 = 4 \cdot 5050 = 20200.$

We then add 20200 to the number of squares in Figure 0 to get 20201, which is choice $\boxed{\text{C}}$

Solution 2

We can divide up figure n to get the sum of the sum of the first $n+1$ odd numbers and the sum of the first n odd numbers. If you do not see this, here is the example for $n=3$:



The sum of the first n odd numbers is n^2, so for figure n, there are $(n+1)^2+n^2$ unit squares. We plug in $n=100$ to get 20201, which is choice $\boxed{\text{C}}$

Solution 3

Using the recursion from solution 1, we see that the first differences of $4, 8, 12, \ldots$ form an arithmetic progression, and consequently that the second differences are constant and all equal to 4. Thus, the original sequence can be generated from a quadratic function.

If $f(n)=an^2+bn+c$, and $f(0)=1$, $f(1)=5$, and $f(2)=13$, we get a system of three equations in three variables:

$f(0)=0$ gives $c=1$

$f(1)=5$ gives $a+b+c=5$

$f(2)=13$ gives $4a+2b+c=13$

Plugging in $c=1$ into the last two equations gives

$a+b=4$

$4a+2b=12$

Dividing the second equation by 2 gives the system:

$a+b=4$

$2a+b=6$

Subtracting the first equation from the second gives $a=2$, and hence $b=2$. Thus, our quadratic function is:

$f(n)=2n^2+2n+1$

Calculating the answer to our problem, $f(100)=20000+200+1=20201$, which is choice $\boxed{\text{C}}$

See Also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 Problems/Problem 13

Problem

There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?

$$\begin{array}{ccccc} \bullet & & & & \\ \bullet & \bullet & & & \\ \bullet & \bullet & \bullet & & \\ \bullet & \bullet & \bullet & \bullet & \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 5!\cdot 4!\cdot 3!\cdot 2!\cdot 1! \qquad \mathrm{(D)}\ \frac{15!}{5!\cdot 4!\cdot 3!\cdot 2!\cdot 1!} \qquad \mathrm{(E)}\ 15!$

Solution

In each column there must be one yellow peg. In particular, in the rightmost column, there is only one peg spot, therefore a yellow peg must go there.

In the second column from the right, there are two spaces for pegs. One of them is in the same row as the corner peg, so there is only one remaining choice left for the yellow peg in this column.

By similar logic, we can fill in the yellow pegs as shown:

$$\begin{array}{ccccc} Y & & & & \\ \bullet & Y & & & \\ \bullet & \bullet & Y & & \\ \bullet & \bullet & \bullet & Y & \\ \bullet & \bullet & \bullet & \bullet & Y \end{array}$$

After this we can proceed to fill in the whole pegboard, so there is only 1 arrangement of the pegs. The answer is

$$\begin{array}{ccccc} Y & & & & \\ R & Y & & & \\ G & R & Y & & \\ B & G & R & Y & \\ O & B & G & R & Y \end{array}$$

See Also

2000 AMC 12 Problems/Problem 9

The following problem is from both the 2000 AMC 12 #9 and 2000 AMC 10 #14, so both problems redirect to this page.

Contents

Problem

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were $71, 76, 80, 82,$ and 91. What was the last score Mrs. Walters entered?

$\text{(A)}\ 71 \qquad \text{(B)}\ 76 \qquad \text{(C)}\ 80 \qquad \text{(D)}\ 82 \qquad \text{(E)}\ 91$

Solution

Solution 1

The first number is divisible by 1.

The sum of the first two numbers is even.

The sum of the first three numbers is divisible by 3.

The sum of the first four numbers is divisible by 4.

The sum of the first five numbers is 400.

Since 400 is divisible by 4, the last score must also be divisible by 4. Therefore, the last score is either 76 or 80.

Case 1: 76 is the last number entered.

Since $400 \equiv 76 \equiv 1 \pmod{3}$, the fourth number must be divisible by 3, but none of the scores are divisible by 3.

Case 2: 80 is the last number entered.

Since $80 \equiv 2 \pmod{3}$, the fourth number must be $2 \pmod{3}$. That number is 71 and only 71. The next number must be 91, since the sum of the first two numbers is even. So the only arrangement of the scores $76, 82, 91, 71, 80 \Rightarrow \text{(C)}$

Solution 2

We know the first sum of the first three numbers must be divisible by 3, so we write out all 5 numbers $\pmod{3}$, which gives $2,1,2,1,1$, respectively. Clearly the only way to get a number divisible by 3 by adding three of these is by adding the three ones. So those must go first. Now we have an odd sum, and since the next average must be divisible by 4, 71 must be next. That leaves 80 for last, so the answer is C.

See also

2000 AMC 12 Problems/Problem 11

The following problem is from both the 2000 AMC 12 #11 and 2000 AMC 10 #15, so both problems redirect to this page.

Problem

Two non-zero real numbers, a and b, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

$\text{(A)}\ -2 \qquad \text{(B)}\ \frac{-1}{2} \qquad \text{(C)}\ \frac{1}{3} \qquad \text{(D)}\ \frac{1}{2} \qquad \text{(E)}\ 2$

Solution

$$\frac{a}{b} + \frac{b}{a} - ab = \frac{a^2 + b^2}{ab} - (a - b) = \frac{a^2 + b^2}{a - b} - \frac{(a-b)^2}{(a-b)} = \frac{2ab}{a - b} = 2 \Rightarrow \text{(E)}.$$

Alternatively, we could test simple values, like $(a,b) = \left(1, \frac{1}{2}\right)$, which would yield $\frac{a}{b} + \frac{b}{a} - ab = 2.$

Another way is to solve the equation for b, giving $b = \frac{a}{a+1}$; then substituting this into the expression and simplifying gives the answer of 2.

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

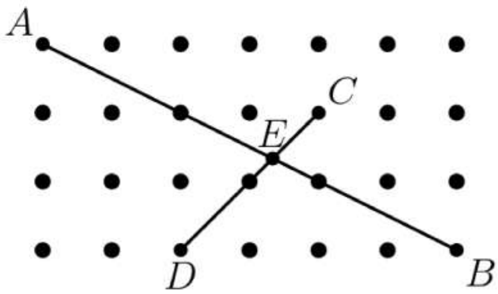
Category: Introductory Algebra Problems

2000 AMC 10 Problems/Problem 16

Contents

Problem

The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E. Find the length of segment AE.



$\mathrm{(A)}\ \frac{4\sqrt{5}}{3} \qquad \mathrm{(B)}\ \frac{5\sqrt{5}}{3} \qquad \mathrm{(C)}\ \frac{12\sqrt{5}}{7} \qquad \mathrm{(D)}\ 2\sqrt{5} \qquad \mathrm{(E)}\ \frac{5\sqrt{65}}{9}$

Solution

Solution 1

Let l_1 be the line containing A and B and let l_2 be the line containing C and D. If we set the bottom left point at $(0,0)$, then $A=(0,3)$, $B=(6,0)$, $C=(4,2)$, and $D=(2,0)$.

The line l_1 is given by the equation $y=m_1x+b_1$. The y-intercept is $A=(0,3)$, so $b_1=3$. We are given two points on l_1, hence we can compute the slope, m_1 to be $\frac{0-3}{6-0}=\frac{-1}{2}$, so l_1 is the line $y=\frac{-1}{2}x+3$

Similarly, l_2 is given by $y=m_2x+b_2$. The slope in this case is $\frac{2-0}{4-2}=1$, so $y=x+b_2$. Plugging in the point $(2,0)$ gives us $b_2=-2$, so l_2 is the line $y=x-2$.

At E, the intersection point, both of the equations must be true, so

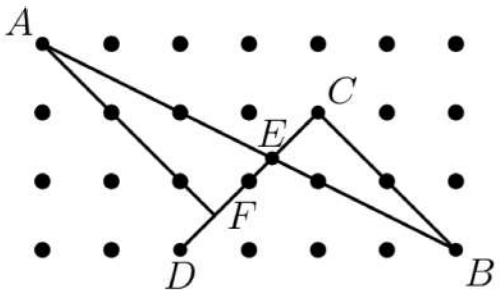
$$y=x-2, y=\frac{-1}{2}x+3 \Rightarrow x-2=\frac{-1}{2}x+3$$
$$\Rightarrow x=\frac{10}{3}$$
$$\Rightarrow y=\frac{4}{3}$$

We have the coordinates of A and E, so we can use the distance formula here:

$$\sqrt{\left(\frac{10}{3}-0\right)^2+\left(\frac{4}{3}-3\right)^2}=\frac{5\sqrt{5}}{3}$$

which is answer choice $\boxed{\text{B}}$

Solution 2



Draw the perpendiculars from A and B to CD, respectively. As it turns out, $BC \perp CD$. Let F be the point on CD for which $AF \perp CD$.

$m\angle AFE = m\angle BCE = 90^\circ$, and $m\angle AEF = m\angle CEB$, so by AA similarity,

$$\triangle AFE \sim \triangle BCE \Rightarrow \frac{AF}{AE}=\frac{BC}{BE}$$

By the Pythagorean Theorem, we have $AB=\sqrt{3^2+6^2}=3\sqrt{5}$, $AF=\sqrt{2.5^2+2.5^2}=2.5\sqrt{2}$, and $BC=\sqrt{2^2+2^2}=2\sqrt{2}$. Let $AE=x$, so $BE=3\sqrt{5}-x$, then

$$\frac{2.5\sqrt{2}}{x}=\frac{2\sqrt{2}}{3\sqrt{5}-x}$$

$$x=\frac{5\sqrt{5}}{3}$$

This is answer choice $\boxed{\text{B}}$

Also, you could extend CD to the end of the box and create two similar triangles. Then use ratios and find that the distance is 5/9 of the diagonal AB. Thus, the answer is B.

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 10 Problems/Problem 17

Problem

Boris has an incredible coin changing machine. When he puts in a quarter, it returns five nickels; when he puts in a nickel, it returns five pennies; and when he puts in a penny, it returns five quarters. Boris starts with just one penny. Which of the following amounts could Boris have after using the machine repeatedly?

$\text{(A)}\ \$3.63$

$\text{(B)}\ \$5.13$

$\text{(C)}\ \$6.30$

$\text{(D)}\ \$7.45$

$\text{(E)}\ \$9.07$

Solution

Consider what happens each time he puts a coin in. If he puts in a quarter, he gets five nickels back, so the amount of money he has doesn't change. Similarly, if he puts a nickel in the machine, he gets five pennies back and the money value doesn't change. However, if he puts a penny in, he gets five quarters back, increasing the amount of money he has by 124 cents.

This implies that the only possible values, in cents, he can have are the ones one more than a multiple of 124. Of the choices given, the only one is $\boxed{\text{D}}$

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 10 Problems/Problem 18

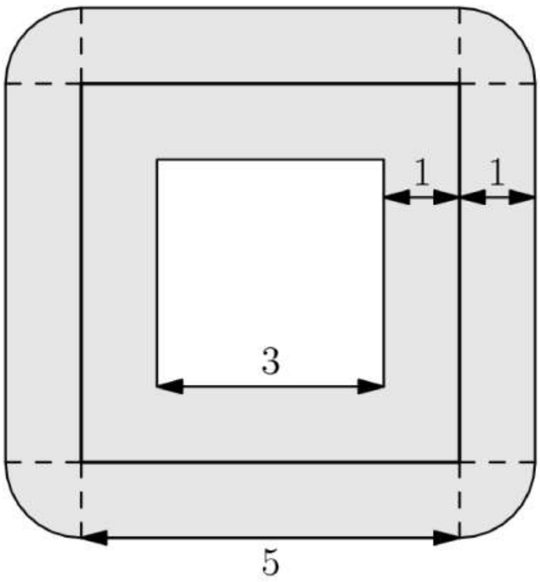
Problem

Charlyn walks completely around the boundary of a square whose sides are each 5 km long. From any point on her path she can see exactly 1 km horizontally in all directions. What is the area of the region consisting of all points Charlyn can see during her walk, expressed in square kilometers and rounded to the nearest whole number?

$(A) 24 \qquad (B)\ 27 \qquad (C)\ 39 \qquad (D)\ 40 \qquad (E)\ 42$

Solution

The area she sees looks at follows:



The part inside the walk has area $5 \cdot 5 - 3 \cdot 3 = 16$. The part outside the walk consists of four rectangles, and four arcs. Each of the rectangles has area $5 \cdot 1 = 5$. The four arcs together form a circle with radius 1.

Therefore the total area she can see is $16 + 4 \cdot 5 + \pi \cdot 1^2 = 36 + \pi \simeq 39.14$, which rounded to the nearest integer is 39. $\boxed{C}$

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 21

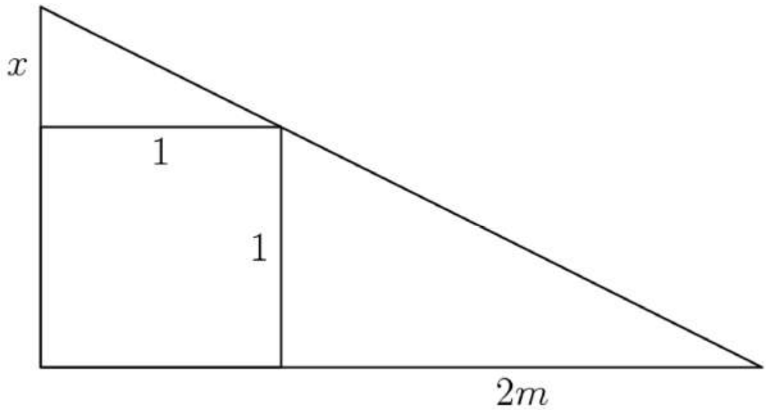
The following problem is from both the 2000 AMC 12 #21 and 2000 AMC 10 #19, so both problems redirect to this page.

Problem

Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

$\text{(A)}\ \frac{1}{2m+1} \qquad \text{(B)}\ m \qquad \text{(C)}\ 1-m \qquad \text{(D)}\ \frac{1}{4m} \qquad \text{(E)}\ \frac{1}{8m^2}$

Solution



WLOG, let a side of the square be 1. Simple angle chasing shows that the two right triangles are similar. Thus the ratio of the sides of the triangles are the same. Since $A = \frac{1}{2}bh = \frac{h}{2}$, the height of the triangle with area m is $2m$. Therefore $\frac{2m}{1} = \frac{1}{x}$ where x is the base of the other triangle. $x = \frac{1}{2m}$, and the area of that triangle is $\frac{1}{2} \cdot 1 \cdot \frac{1}{2m} = \frac{1}{4m}$ (D).

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 10 Problems/Problem 20

Problem

Let A, M, and C be nonnegative integers such that $A+M+C=10$. What is the maximum value of $A\cdot M\cdot C+A\cdot M+M\cdot C+C\cdot A$?

$\mathrm{(A)}\ 49 \qquad\mathrm{(B)}\ 59 \qquad\mathrm{(C)}\ 69 \qquad\mathrm{(D)}\ 79 \qquad\mathrm{(E)}\ 89$

Solution

The trick is to realize that the sum $AMC+AM+MC+CA$ is similar to the product $(A+1)(M+1)(C+1)$. If we multiply $(A+1)(M+1)(C+1)$, we get

$$(A+1)(M+1)(C+1) = AMC + AM + AC + MC + A + M + C + 1.$$

We know that $A+M+C=10$, therefore $(A+1)(M+1)(C+1) = (AMC + AM + MC + CA) + 11$ and

$$AMC + AM + MC + CA = (A+1)(M+1)(C+1) - 11.$$

Now consider the maximal value of this expression. Suppose that some two of A, M, and C differ by at least 2. Then this triple (A,M,C) is not optimal. (To see this, WLOG let $A\geq C+2$. We can then increase the value of $(A+1)(M+1)(C+1)$ by changing $A \to A-1$ and $C \to C+1$.)

Therefore the maximum is achieved when (A,M,C) is a rotation of $(3,3,4)$. The value of $(A+1)(M+1)(C+1)$ in this case is $4\cdot 4\cdot 5=80$, and thus the maximum of $AMC + AM + MC + CA$ is $80-11=\boxed{\mathrm{(C)}\ 69}$.

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 10 Problems/Problem 21

Problem

If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

$$\text{I. All alligators are creepy crawlers.}$$

$$\text{II. Some ferocious creatures are creepy crawlers.}$$

$$\text{III. Some alligators are not creepy crawlers.}$$

$$\text{(A) I only} \qquad \text{(B) II only} \qquad \text{(C) III only} \qquad \text{(D) II and III only} \qquad \text{(E) None must be true}$$

Solution

We interpret the problem statement as a query about three abstract concepts denoted as "alligators", "creepy crawlers" and "ferocious creatures". In answering the question, we may NOT refer to reality -- for example to the fact that alligators do exist.

To make more clear that we are not using anything outside the problem statement, let's rename the three concepts as A, C, and F.

We got the following information:

- If x is an A, then x is an F.
- There is some x that is a C and at the same time an A.

We CAN NOT conclude that the first statement is true. For example, the situation "Johnny and Freddy are As, but only Johnny is a C" meets both conditions, but the first statement is false.

We CAN conclude that the second statement is true. We know that there is some x that is a C and at the same time an A. Pick one such x and call it Bobby. Additionally, we know that if x is an A, then x is an F. Bobby is an A, therefore Bobby is an F. And this is enough to prove the second statement -- Bobby is an F that is also a C.

We CAN NOT conclude that the third statement is true. For example, consider the situation when A, C and F are equivalent (represent the same set of objects). In such case both conditions are satisfied, but the third statement is false.

Therefore the answer is $\boxed{\text{(B)} \quad \text{II only}}$.

See Also

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 13

The following problem is from both the 2000 AMC 12 #13 and 2000 AMC 10 #22, so both problems redirect to this page.

Contents

Problem

One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

$\text{(A)}\ 3 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 6 \qquad \text{(E)}\ 7$

Solution

Solution 1:

Let c be the total amount of coffee, m of milk, and p the number of people in the family. Then each person drinks the same total amount of coffee and milk (8 ounces), so

$$\left(\frac{c}{6}+\frac{m}{4}\right)p=c+m$$

Regrouping, we get $2c(6-p)=3m(p-4)$. Since both c, m are positive, it follows that $6-p$ and $p-4$ are also positive, which is only possible when $p=5\ \text{(C)}$.

Solution 2 (less rigorous):

One could notice that (since there are only two components to the mixture) Angela must have more than her "fair share" of milk and less then her "fair share" of coffee in order to ensure that everyone has 8 ounces. The "fair share" is $1/p$. So,

$$\frac{1}{6}<\frac{1}{p}<\frac{1}{4}$$

Which requires that p be $p=5\ \text{(C)}$, since p is a whole number.

Solution 3:

Again, let c, m, and p be the total amount of coffee, total amount of milk, and number of people in the family, respectively. Then the total amount that is drunk is $8p$, and also $c+m$. Thus, $c+m=8p$, so $m=8p-c$ and $c=8p-m$.

We also know that the amount Angela drank, which is $\frac{c}{6}+\frac{m}{4}$, is equal to 8 ounces, thus $\frac{c}{6}+\frac{m}{4}=8.$ Rearranging gives

$$24p-c=96.$$

Now notice that $c>0$ (by the problem statement). In addition, $m>0$, so $c=8p-m<8p$. Therefore, $0<c<8p$, and so $24p>24p-c>16p$. We know that $24p-c=96$, so

$$24p>96>16p.$$

From the leftmost inequality, we get $p > 4$, and from the rightmost inequality, we get $p < 6$. The only possible value of p is $p = 5\ (\mathrm{C})$.

Sidenote

If we now solve for c and m, we find that $m = 16$ and $c = 24$. Thus in total the family drank 16 ounces of milk and 24 ounces of coffee. Angela drank exactly 4 ounces of milk and 4 ounces of coffee.

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Number Theory Problems

2000 AMC 12 Problems/Problem 14

The following problem is from both the 2000 AMC 12 #14 and 2000 AMC 10 #23, so both problems redirect to this page.

Problem

When the mean, median, and mode of the list

$$10, 2, 5, 2, 4, 2, x$$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

$\text{(A)}\ 3 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 9 \qquad \text{(D)}\ 17 \qquad \text{(E)}\ 20$

Solution

- The mean is $\frac{10+2+5+2+4+2+x}{7} = \frac{25+x}{7}$.
- Arranged in increasing order, the list is $2, 2, 2, 4, 5, 10$, so the median is either $2, 4$ or x depending upon the value of x.
- The mode is 2, since it appears three times.

We apply casework upon the median:

- If the median is 2 $(x \le 2)$, then the arithmetic progression must be non-constant, which is fine.
- If the median is 4 $(x \ge 4)$, then the mean can either be $0, 3, 6$ to form an arithmetic progression. Solving for x yields $-25, -4, 17$ respectively, of which only 17 works.
- If the median is x $(2 \le x \le 4)$, then the mean can either be $1, 5/2, 4$ to form an arithmetic progression. Solving for x yields $-14, -7.5, 3$ respectively, of which only 3 works.

The answer is $3 + 17 = 20\ \text{(E)}$.

SOLUTION DOESNT MAKE SENSE 2,2,4,5,10 IS NON CONSTANT

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 15

The following problem is from both the 2000 AMC 12 #15 and 2000 AMC 10 #24, so both problems redirect to this page.

Problem

Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

$\text{(A)}\ -1/3 \qquad \text{(B)}\ -1/9 \qquad \text{(C)}\ 0 \qquad \text{(D)}\ 5/9 \qquad \text{(E)}\ 5/3$

Solution

Let $y = \frac{x}{3}$; then $f(y) = (3y)^2 + 3y + 1 = 9y^2 + 3y + 1$. Thus $f(3z) - 7 = 81z^2 + 9z - 6 = 3(9z-2)(3z+1) = 0$, and $z = -\frac{1}{3}, \frac{2}{9}$. These sum up to $\boxed{-\frac{1}{9} \text{(B)}}$. (We can also use Vieta's formulas to find the sum more quickly.)

Alternative solution: Set $f(\frac{x}{3}) = x^2 + x + 1 = 7$ to get $x^2 + x - 6 = 0$. From either finding the roots or using Vieta's formulas, we find the sum of these roots to be -1. Each root of this equation is 9 times greater than a corresponding root of $f(3z) = 7$ (because $\frac{x}{3} = 3z$ gives $x = 9z$), thus the sum of the roots in the equation $f(3z) = 7$ is $-\frac{1}{9}$.

Alternate Solution 2:

Since we have $f(x/3)$, $f(3z)$ occurs at $x = 9z$. Thus, $f(9z/3) = f(3z) = (9z)^2 + 9z + 1$. We set this equal to 7:

$81z^2 + 9z + 1 = 7 \Longrightarrow 81z^2 + 9z - 6 = 0$. For any quadratic $ax^2 + bx + c = 0$, the sum of the roots is $-\frac{b}{a}$. Thus, the sum of the roots of this equation is $-\frac{9}{81} = \boxed{-\frac{1}{9}} \Longrightarrow \boxed{\text{(B)}}$

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2000 AMC 12 Problems/Problem 18

The following problem is from both the 2000 AMC 12 #18 and 2000 AMC 10 #25, so both problems redirect to this page.

Problem

In year N, the 300^{th} day of the year is a Tuesday. In year $N+1$, the 200^{th} day is also a Tuesday. On what day of the week did the 100^{th} day of year $N-1$ occur?

$\text{(A)}\ \text{Thursday} \qquad \text{(B)}\ \text{Friday}\qquad \text{(C)}\ \text{Saturday}\qquad \text{(D)}\ \text{Sunday}\qquad \text{(E)}\ \text{Monday}$

Solution

There are either $65+200=265$ or $66+200=266$ days between the first two dates depending upon whether or not year $N+1$ is a leap year. Since 7 divides into 266, then it is possible for both dates to be Tuesday; hence year $N+1$ is a leap year and N is not a leap year. There are $265+300=565$ days between the date in years $N, N-1$, which leaves a remainder of 5 upon division by 7. Since we are subtracting days, we count 5 days before Tuesday, which gives us Thursday (A).

See also

2000 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2000))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2000 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2000))	
Preceded by Problem 24	Followed by Last Problem
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Number Theory Problems