2001 AMC 10 Problems/Problem 1

Problem

The median of the list $n, n+3, n+4, n+5, n+6, n+8, n+10, n+12, n+15$ is 10. What is the mean?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Solution

The median of the list is 10, and there are 9 numbers in the list, so the median must be the 5th number from the left, which is $n+6$.

We substitute the median for 10 and the equation becomes $n+6=10$.

Subtract both sides by 6 and we get $n=4$.

$$n+n+3+n+4+n+5+n+6+n+8+n+10+n+12+n+15=9n+63.$$

The mean of those numbers is $\frac{9n+63}{9}$ which is $n+7$.

Substitute n for 4 and $4+7=\boxed{\textbf{(E)}\ 11}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by First Question	Followed by Problem 2
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 2

Problem

A number x is 2 more than the product of its reciprocal and its additive inverse. In which interval does the number lie?

$\textbf{(A)}\ \ -4\le x\le -2\qquad\textbf{(B)}\ \ -2 < x\le 0\qquad\textbf{(C)}\ 0 < x \le 2\qquad \textbf{(D)}\ \ 2 < x\le 4\qquad\textbf{(E)}\ \ 4 < x\le 6$

Solution

We can write our equation as $x = \left(\frac{1}{x}\right) \cdot (-x) + 2 = -1 + 2 = 1$. Therefore, $\boxed{\textbf{(C)}\ 0 < x \le 2}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 1	Followed by Problem 3
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All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2001 AMC 12 Problems/Problem 1

The following problem is from both the 2001 AMC 12 #1 and 2001 AMC 10 #3, so both problems redirect to this page.

Problem

The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

$\text{(A)}\ 2S + 3 \qquad \text{(B)}\ 3S + 2 \qquad \text{(C)}\ 3S + 6 \qquad \text{(D)}\ 2S + 6 \qquad \text{(E)}\ 2S + 12$

Solution

Suppose the two numbers are a and b, with $a+b=S$. Then the desired sum is $2(a+3)+2(b+3)=2(a+b)+12=2S+12$, which is answer $\boxed{\text{(E)}}$.

See also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
Preceded by First question	Followed by Problem 2
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2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 2	Followed by Problem 4
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2001 AMC 10 Problems/Problem 4

Contents

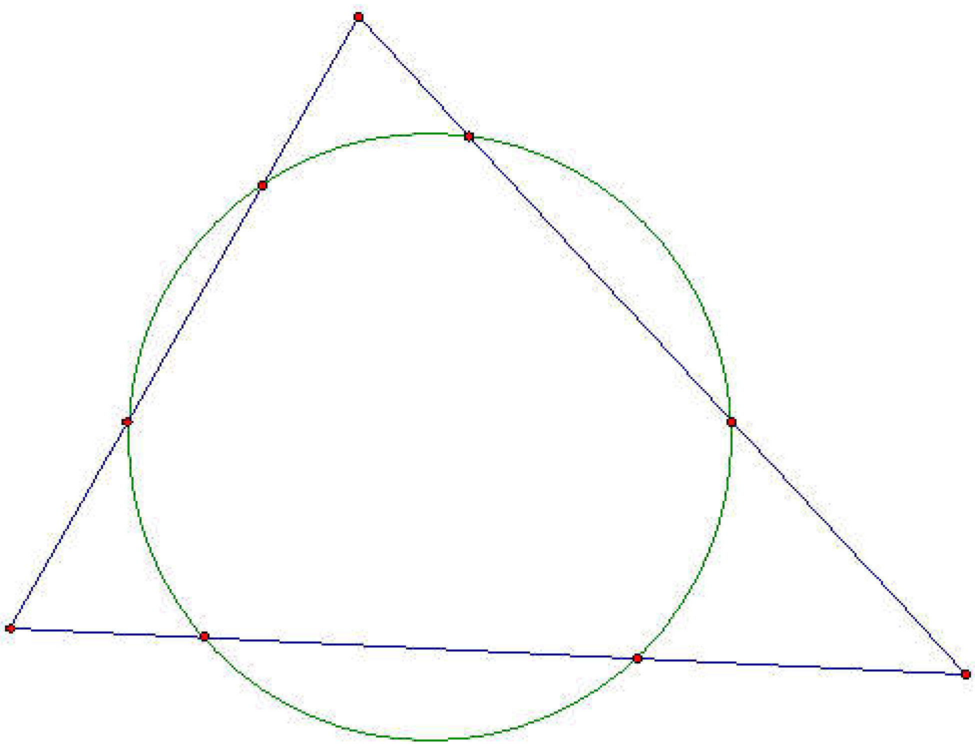
Problem

What is the maximum number of possible points of intersection of a circle and a triangle?

$\textbf{(A) }2\qquad\textbf{(B) }3\qquad\textbf{(C) }4\qquad\textbf{(D) }5\qquad\textbf{(E) }6$

Solution

Solution 1



We can draw a circle and a triangle, such that each side is tangent to the circle. This means that each side would intersect the circle at one point.

You would then have 3 points, but what if the circle was bigger? Then, each side would intersect the circle at 2 points.

Therefore, $2 \times 3 = \boxed{\textbf{(E) }6}$.

Solution 2

We know that the maximum amount of points that a circle and a line segment can intersect is 2. Therefore, because there are 3 line segments in a triangle, the maximum amount of points of intersection is $\boxed{\textbf{(E) }6}$.

See Also

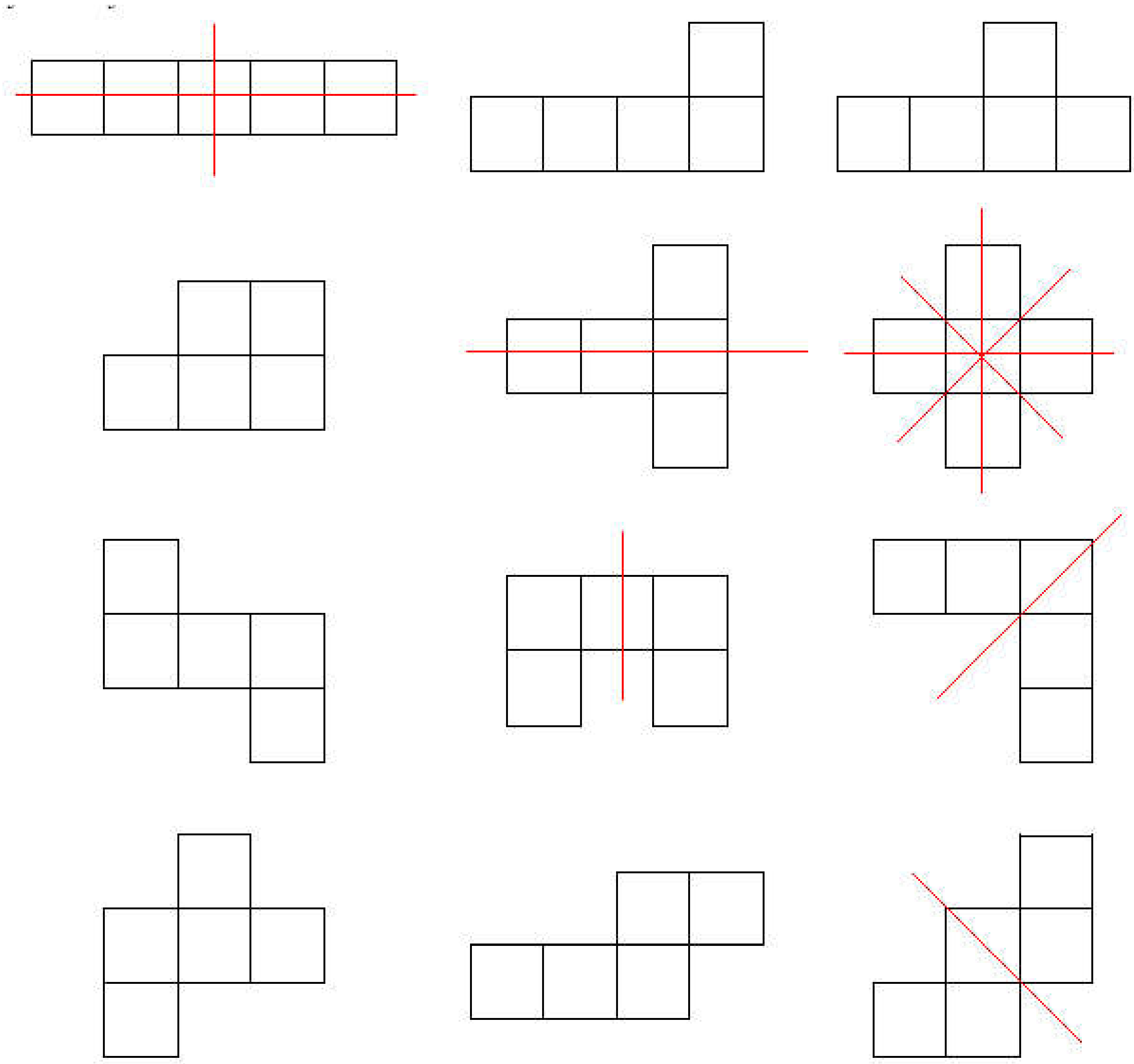
2001 AMC 10 Problems/Problem 5

Problem

How many of the twelve pentominoes pictured below at least one line of symmetry?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution



The ones with lines over the shapes have at least one line of symmetry. Counting the number of shapes that have line(s) on them, we find $\boxed{\textbf{(D)}\ 6}$ pentominoes.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 4	Followed by Problem 6
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 6

The following problem is from both the 2001 AMC 12 #2 and 2001 AMC 10 #6, so both problems redirect to this page.

Problem

Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N?

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Solution

Denote a and b as the tens and units digit of N, respectively. Then $N = 10a + b$. It follows that $10a + b = ab + a + b$, which implies that $9a = ab$. Since $a \neq 0$, $b = 9$. So the units digit of N is $\boxed{\textbf{(E)}\ 9}$.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
Preceded by Problem 1	Followed by Problem 3
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2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 7

Problem

When the decimal point of a certain positive decimal number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

$\textbf{(A)}\ 0.0002 \qquad \textbf{(B)}\ 0.002 \qquad \textbf{(C)}\ 0.02 \qquad \textbf{(D)}\ 0.2 \qquad \textbf{(E)}\ 2$

Solution

If x is the number, then moving the decimal point four places to the right is the same as multiplying x by 10000. This gives us the equation

$$10000x = 4 \cdot \frac{1}{x}$$

This is equivalent to

$$x^2 = \frac{4}{10000}$$

Since x is positive, it follows that $x = \frac{2}{100} = \boxed{\textbf{(C)}\ 0.02}$

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 6	Followed by Problem 8
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 8

Problem

Wanda, Darren, Beatrice, and Chi are tutors in the school math lab. Their schedule is as follows: Darren works every third school day, Wanda works every fourth school day, Beatrice works every sixth school day, and Chi works every seventh school day. Today they are all working in the math lab. In how many school days from today will they next be together tutoring in the lab?

$\textbf{(A)}\ 42 \qquad\textbf{(B)}\ 84 \qquad\textbf{(C)}\ 126 \qquad\textbf{(D)}\ 168 \qquad\textbf{(E)}\ 252$

Solution

We need to find the least common multiple of the four numbers given. That is, the next time they will be together. First, find the least common multiple of 3 and 4.

$3 \times 4 = 12.$

Find the least common multiple of 12 and 6.

Since 12 is a multiple of 6, the least common multiple is 12.

Lastly, the least common multiple of 12 and 7 is $12 \times\ 7 = \boxed{\textbf{(B)}\ 84}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 7	Followed by Problem 9
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All AMC 10 Problems and Solutions	

2001 AMC 12 Problems/Problem 3

The following problem is from both the 2001 AMC 12 #3 and 2001 AMC 10 #9, so both problems redirect to this page.

Problem

The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p+2)\%$ of any amount above $\$28000$. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

$\text{(A)}\,\textdollar 28000 \qquad \text{(B)}\,\textdollar 32000 \qquad \text{(C)}\,\textdollar 35000 \qquad \text{(D)}\,\textdollar 42000 \qquad \text{(E)}\,\textdollar 56000$

Solution

Let the income amount be denoted by A.

We know that $\frac{A(p+.25)}{100}=\frac{28000p}{100}+\frac{(p+2)(A-28000)}{100}$.

We can now try to solve for A:

$(p+.25)A=28000p+Ap+2A-28000p-56000$

$.25A=2A-56000$

$A=32000$

So the answer is $\boxed{B}$

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
Preceded by Problem 2	Followed by Problem 4
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2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 10

Contents

Problem

If x, y, and z are positive with $xy = 24$, $xz = 48$, and $yz = 72$, then $x + y + z$ is

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 19 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 22 \qquad \textbf{(E)}\ 24$

Solution 1

The first two equations in the problem are $xy = 24$ and $xz = 48$. Since $xyz \neq 0$, we have $\frac{xy}{xz} = \frac{24}{48} \implies 2y = z$. We can substitute $z = 2y$ into the third equation $yz = 72$ to obtain $2y^2 = 72 \implies y = 6$ and $2y = z = 12$. We replace y into the first equation to obtain $x = 4$.

Since we know every variable's value, we can substitute them in to find $x + y + z = 4 + 6 + 12 = \boxed{\textbf{(D)}\ 22}$.

Solution 2

These equations are symmetric, and furthermore, they use multiplication. This makes us think to multiply them all. This gives $(xyz)^2 = (xy)(yz)(xz) = (24)(48)(72) = (24 \times 12)^2 \implies xyz = 288$. We divide $xyz = 288$ by each of the given equations, which yields $x = 4$, $y = 6$, and $z = 12$. The desired sum is $4 + 6 + 12 = \boxed{22}$, so the answer is $\boxed{\textbf{(D)}}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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2001 AMC 10 Problems/Problem 11

Contents

Problem

Consider the dark square in an array of unit squares, part of which is shown. The first ring of squares around this center square contains 8 unit squares. The second ring contains 16 unit squares. If we continue this process, the number of unit squares in the 100^{th} ring is

$\textbf{(A)}\ 396 \qquad \textbf{(B)}\ 404 \qquad \textbf{(C)}\ 800 \qquad \textbf{(D)}\ 10{,}000 \qquad \textbf{(E)}\ 10{,}404$

Solutions

Solution 1

We can partition the n^{th} ring into 4 rectangles: two containing $2n+1$ unit squares and two containing $2n-1$ unit squares.

There are $2(2n+1)+2(2n-1) = 4n+2+4n-2 = 8n$ unit squares in the n^{th} ring.

Thus, the 100^{th} ring has $8 \times 100 = \boxed{\textbf{(C)} 800}$ unit squares.

Solution 2 (Alternate Solution)

We can make the n^{th} ring by removing a square of side length $2n-1$ from a square of side length $2n+1$.

This ring contains $(2n+1)^2-(2n-1)^2 = (4n^2+4n+1)-(4n^2-4n+1) = 8n$ unit squares.

Thus, the 100^{th} ring has $8 \times 100 = \boxed{\textbf{(C)}\ 800}$ unit squares.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 10	Followed by Problem 12
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Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2001_AMC_10_Problems/Problem_11&oldid=54189"

2001 AMC 10 Problems/Problem 12

Contents

Problem

Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 21 \qquad \textbf{(D)}\ 28 \qquad \textbf{(E)}\ 42$

Solution

Whenever n is the product of three consecutive integers, n is divisible by $3!$, meaning it is divisible by 6.

It also mentions that it is divisible by 7, so the number is definitely divisible by all the factors of 42.

In our answer choices, the one that is not a factor of 42 is $\boxed{\textbf{(D)}\ 28}$.

Solution 2

We can look for counterexamples. For example, letting $n = 13 \cdot 14 \cdot 15$, we see that n is not divisible by 28, so (D) is our answer.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
Preceded by Problem 11	Followed by Problem 13
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All AMC 10 Problems and Solutions	

2001 AMC 12 Problems/Problem 6

The following problem is from both the 2001 AMC 12 #6 and 2001 AMC 10 #13, so both problems redirect to this page.

Problem

A telephone number has the form ABC-DEF-GHIJ, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Solution

The last four digits GHIJ are either 9753 or 7531, and the other odd digit (1 or 9) must be A, B, or C. Since $A + B + C = 9$, that digit must be 1. Thus the sum of the two even digits in ABC is 8. DEF must be 864, 642, or 420, which respectively leave the pairs 2 and 0, 8 and 0, or 8 and 6, as the two even digits in ABC. Only 8 and 0 has sum 8, so ABC is 810, and the required first digit is $\boxed{\text{(E)} 8}$.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
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2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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2001 AMC 12 Problems/Problem 7

The following problem is from both the 2001 AMC 12 #7 and 2001 AMC 10A #14, so both problems redirect to this page.

Problem

A charity sells 140 benefit tickets for a total of 2001. Some tickets sell for full price (a whole dollar amount), and the rest sells for half price. How much money is raised by the full-price tickets?

$\text{(A) }\$782 \qquad \text{(B) }\$986 \qquad \text{(C) }\$1158 \qquad \text{(D) }\$1219 \qquad \text{(E) }\ \$1449$

Solution

Let's multiply ticket costs by 2, then the half price becomes an integer, and the charity sold 140 tickets worth a total of 4002 dollars.

Let h be the number of half price tickets, we then have $140-h$ full price tickets. The cost of $140-h$ full price tickets is equal to the cost of $280-2h$ half price tickets.

Hence we know that $h+(280-2h)=280-h$ half price tickets cost 4002 dollars. Then a single half price ticket costs $\frac{4002}{280-h}$ dollars, and this must be an integer. Thus $280-h$ must be a divisor of 4002. Keeping in mind that $0\leq h\leq 140$, we are looking for a divisor between 140 and 280, inclusive.

The prime factorization of 4002 is $4002=2\cdot 3\cdot 23\cdot 29$. We can easily find out that the only divisor of 4002 within the given range is $2\cdot 3\cdot 29=174$.

This gives us $280-h=174$, hence there were $h=106$ half price tickets and $140-h=34$ full price tickets.

In our modified setting (with prices multiplied by 2) the price of a half price ticket is $\frac{4002}{174}=23$. In the original setting this is the price of a full price ticket. Hence $23\cdot 34=\boxed{(\text{A}) 782}$ dollars are raised by the full price tickets.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
Preceded by Problem 6	Followed by Problem 8
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2001 AMC 10 Problems/Problem 15

Contents

Problem

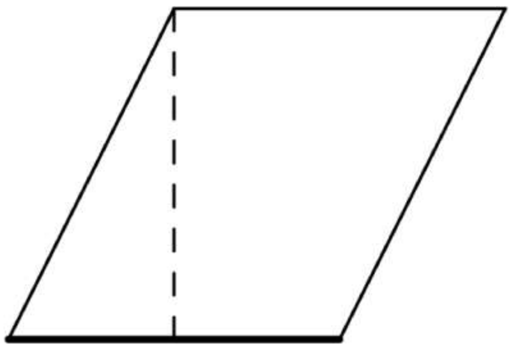
A street has parallel curbs 40 feet apart. A crosswalk bounded by two parallel stripes crosses the street at an angle. The length of the curb between the stripes is 15 feet and each stripe is 50 feet long. Find the distance, in feet, between the stripes.

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 25$

Solutions

Solution 1

Drawing the problem out, we see we get a parallelogram with a height of 40 and a base of 15, giving an area of .



If we look at it the other way, we see the distance between the stripes is the height and the base is 50.

The area is still the same, so the distance between the stripes is .

Solution 2

Alternatively, we could use similar triangles--the $30-40-50$ triangle (created by the length of the bordering stripe and the difference between the two curbs) is similar to the $x-y-15$ triangle, where we are trying to find y (the shortest distance between the two stripes). Therefore, y would have to be $\boxed{\textbf{(C)}\ 12}$.

2001 AMC 12 Problems/Problem 4

The following problem is from both the 2001 AMC 12 #4 and 2001 AMC 10 #16, so both problems redirect to this page.

Problem

The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is 5. What is their sum?

$\text{(A)}\ 5\qquad \text{(B)}\ 20\qquad \text{(C)}\ 25\qquad \text{(D)}\ 30\qquad \text{(E)}\ 36$

Solution

Let m be the mean of the three numbers. Then the least of the numbers is $m-10$ and the greatest is $m+15$. The middle of the three numbers is the median, 5. So $\frac{1}{3}[(m-10)+5+(m+15)]=m$, which implies that $m=10$. Hence, the sum of the three numbers is $3(10)=\boxed{\text{(D)} 30}$.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
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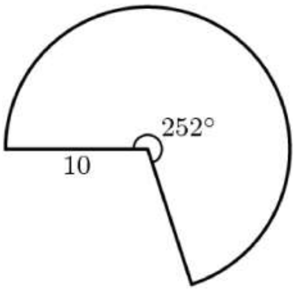
All AMC 10 Problems and Solutions	

2001 AMC 12 Problems/Problem 8

The following problem is from both the 2001 AMC 12 #8 and 2001 AMC 10 #17, so both problems redirect to this page.

Problem

Which of the cones listed below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?



$\textbf{(A)}$ A cone with slant height of 10 and radius 6

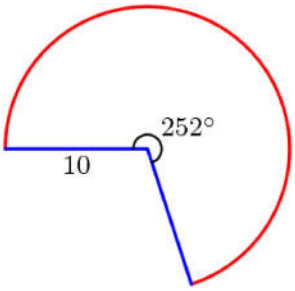
$\textbf{(B)}$ A cone with height of 10 and radius 6

$\textbf{(C)}$ A cone with slant height of 10 and radius 7

$\textbf{(D)}$ A cone with height of 10 and radius 7

$\textbf{(E)}$ A cone with slant height of 10 and radius 8

Solution



The blue lines will be joined together to form a single blue line on the surface of the cone, hence $\boxed{10}$ will be the $\boxed{\text{slant height}}$ of the cone.

The red line will form the circumference of the base. We can compute its length and use it to determine the radius.

The length of the red line is $\dfrac{252}{360}\cdot 2\pi \cdot 10 = 14\pi$. This is the circumference of a circle with radius $\boxed{7}$.

Therefore the correct answer is $\boxed{\text{C}}$.

See Also

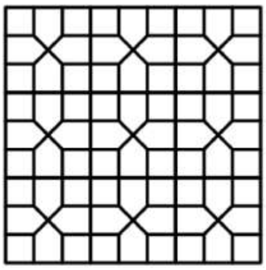
2001 AMC 12 Problems/Problem 10

The following problem is from both the 2001 AMC 12 #10 and 2001 AMC 10 #18, so both problems redirect to this page.

Problem

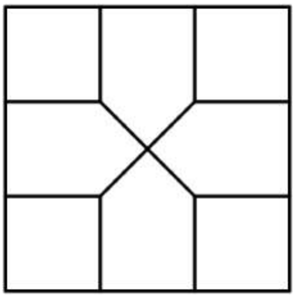
The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to

$\text{(A)}\ 50 \qquad \text{(B)}\ 52 \qquad \text{(C)}\ 54 \qquad \text{(D)}\ 56 \qquad \text{(E)}\ 58$



Solution

Consider any single tile:



If the side of the small square is a, then the area of the tile is $9a^2$, with $4a^2$ covered by squares and $5a^2$ by pentagons. Hence exactly $5/9$ of any tile are covered by pentagons, and therefore pentagons cover $5/9$ of the plane. When expressed as a percentage, this is $55.\bar{5}\%$, and the closest integer to this value is $\boxed{(D) 56}$.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
Preceded by Problem 9	Followed by Problem 11
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2001 AMC 10 Problems/Problem 19

Problem

Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Solution

Let the donuts be represented by Os. We wish to find all possible combinations of glazed, chocolate, and powdered donuts that give us 4 in all. The four donuts we want can be represented as $OOOO$. Notice that we can add two "dividers" to divide the group of donuts into three different kinds; the first will be glazed, second will be chocolate, and the third will be powdered. For example, $O|OO|O$ represents one glazed, two chocolate, and one powdered. We have six objects in all, and we wish to turn two into dividers, which can be done in $\binom{6}{2}=15$ ways. Our answer is hence $\boxed{\textbf{(D)}\ 15}$. Notice that this can be generalized to get the balls and urn identity.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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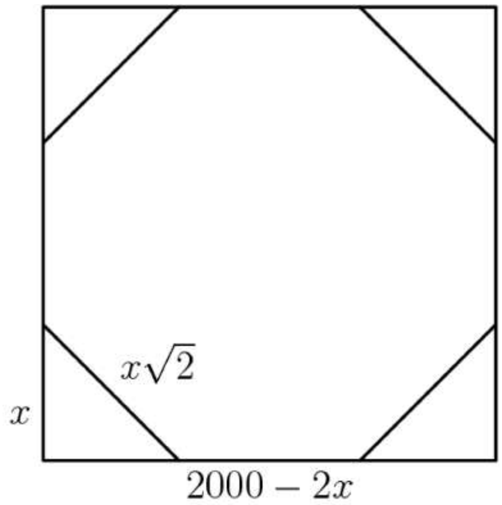
2001 AMC 10 Problems/Problem 20

Problem

A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

$\textbf{(A)}\frac{1}{3}(2000) \qquad \textbf{(B)}2000(\sqrt{2}-1) \qquad \textbf{(C)}2000(2-\sqrt{2}) \qquad \textbf{(D)}1000 \qquad \textbf{(E)}1000\sqrt{2}$

Solution



$$2000-2x=x\sqrt{2}$$
$$2000=x(2+\sqrt{2})$$
$$x=\frac{2000}{2+\sqrt{2}}=\frac{2000(2-\sqrt{2})}{2}=1000(2-\sqrt{2})$$

.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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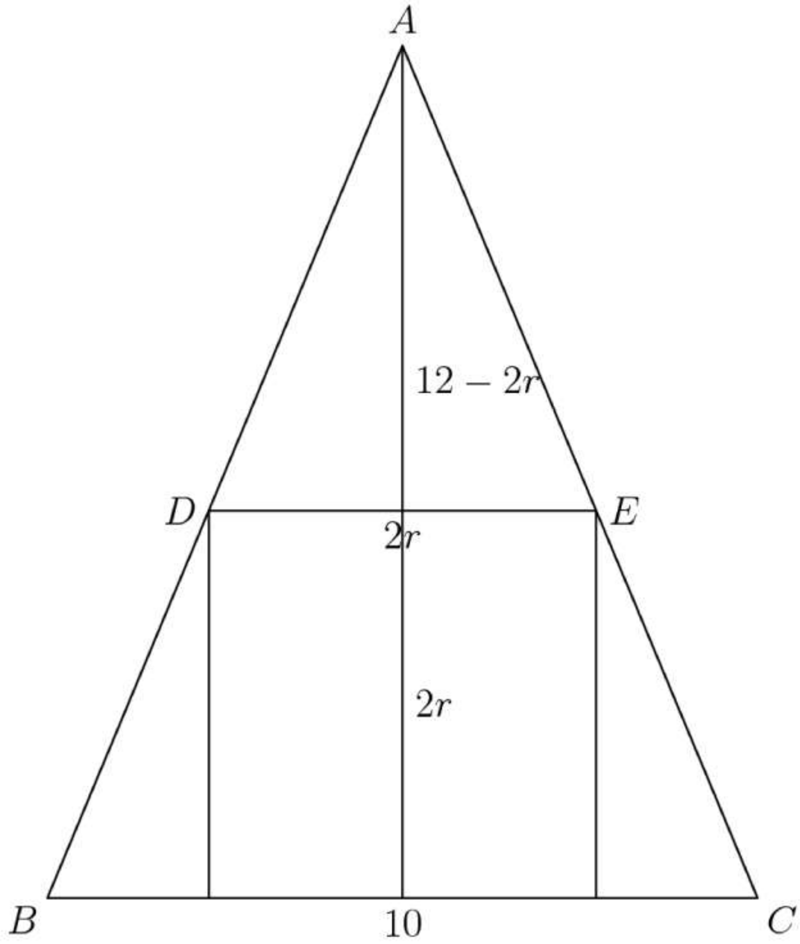
2001 AMC 10 Problems/Problem 21

Problem

A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

$\textbf{(A)}\ \frac{8}{3} \qquad \textbf{(B)}\ \frac{30}{11} \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ \frac{25}{8} \qquad \textbf{(E)}\ \frac{7}{2}$

Solution



Let the diameter of the cylinder be $2r$. Examining the cross section of the cone and cylinder, we find two similar triangles. Hence, $\frac{12-2r}{12}=\frac{2r}{10}$ which we solve to find $r=\frac{30}{11}$. Our answer is $\boxed{\textbf{(B)}\ \frac{30}{11}}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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All AMC 10 Problems and Solutions	

2001 AMC 10 Problems/Problem 22

Contents

Problem

In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by v, w, x, y, and z. Find $y+z$.

$\textbf{(A)}\ 43 \qquad \textbf{(B)}\ 44 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 46 \qquad \textbf{(E)}\ 47$

v	24	w
18	x	y
25	z	21

Solutions

Solution 1

We know that $y+z=2v$, so we could find one variable rather than two.

$v+24+w=43+v$

$24+w=43$

$w=19$

v	24	19
18	x	y
25	z	21

$44+x=24+x+z\ z=20$

v	24	19
18	x	y
25	20	21

The sum per row is $25+21+20=66$.

Thus $66-18-25=66-43=v=23$.

Since we needed $2v$ and we know $v=23$, $23\times 2=\boxed{\textbf{(D)}\ 46}$.

Solution 2

$v+24+w=43+v$

$24+w=43$

$w=19$

v	24	19
18	x	y
25	z	21

$44+x=24+x+z\ z=20$

v	24	19
18	x	y
25	20	21

The magic sum is determined by the bottom row. $25+20+21=66$.

Solving for y:

$y=66-19-21=66-40=26$.

To find our answer, we need to find $y+z$. $y+z=20+26=\boxed{\textbf{(D)}\ 46}$.

See Also

2001 AMC 12 Problems/Problem 11

The following problem is from both the 2001 AMC 12 #11 and 2001 AMC 10 #23, so both problems redirect to this page.

Contents

Problem

A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

$\text{(A) }\frac{3}{10} \qquad \text{(B) }\frac{2}{5} \qquad \text{(C) }\frac{1}{2} \qquad \text{(D) }\frac{3}{5} \qquad \text{(E) }\frac{7}{10}$

Solution

Imagine that we draw all the chips in random order, i.e., we do not stop when the last chip of a color is drawn. To draw out all the white chips first, the last chip left must be red, and all previous chips can be drawn in any order. Since there are 3 red chips, the probability that the last chip of the five is red (and so also the probability that the last chip drawn is white) is $\boxed{\text{(D)} \frac{3}{5}}$.

Solution 2

We wish to arrange the letters: W,W, R, R, R such that R appears last. The probability of this occurring is simply

$$\frac{\binom{4}{2,2}}{\binom{5}{3,2}} = \boxed{\text{(D)} \frac{3}{5}}$$

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
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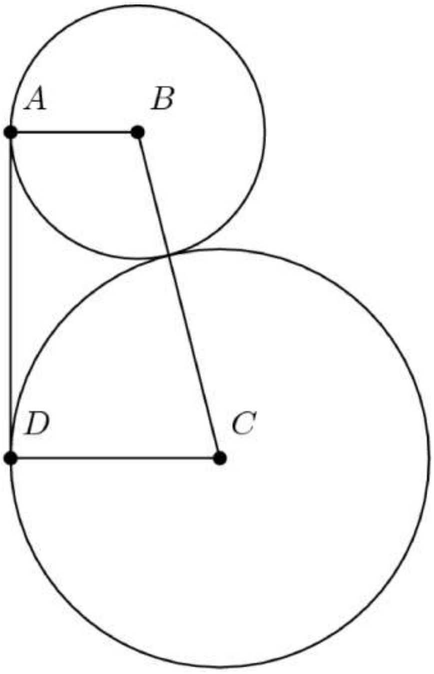
2001 AMC 10 Problems/Problem 24

Problem

In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$, with $AB+CD=BC$, $AB<CD$, and $AD=7$. What is $AB\cdot CD$?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 12.25 \qquad \textbf{(C)}\ 12.5 \qquad \textbf{(D)}\ 12.75 \qquad \textbf{(E)}\ 13$

Solution



If $AB=x$ and $CD=y$, then $BC=x+y$. By the Pythagorean theorem, we have $(x+y)^2=(y-x)^2+49$. Solving the equation, we get $4xy=49 \implies xy=\boxed{\textbf{(B)}\ 12.25}$.

See Also

2001 AMC 10 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2001))	
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Category: Introductory Geometry Problems

2001 AMC 12 Problems/Problem 12

The following problem is from both the 2001 AMC 12 #12 and 2001 AMC 10 #25, so both problems redirect to this page.

Problem

How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

$\text{(A) } 768 \qquad \text{(B) } 801 \qquad \text{(C) } 934 \qquad \text{(D) } 1067 \qquad \text{(E) } 1167$

Solution

Out of the numbers 1 to 12 four are divisible by 3 and three by 4, counting 12 twice. Hence 6 out of these 12 numbers are multiples of 3 or 4.

The same is obviously true for the numbers $12k+1$ to $12k+12$ for any positive integer k.

Hence out of the numbers 1 to $60=5\cdot 12$ there are $5\cdot 6=30$ numbers that are divisible by 3 or 4. Out of these 30, the numbers 15, 20, 30, 40, 45 and 60 are divisible by 5. Therefore in the set $\{1,\dots,60\}$ there are precisely $30-6=24$ numbers that satisfy all criteria from the problem statement.

Again, the same is obviously true for the set $\{60k+1,\dots,60k+60\}$ for any positive integer k.

We have $1980/60=33$, hence there are $24\cdot 33=792$ good numbers among the numbers 1 to 1980. At this point we already know that the only answer that is still possible is $\boxed{\text{(B)}}$, as we only have 20 numbers left.

By examining the remaining 20 by hand we can easily find out that exactly 9 of them match all the criteria, giving us $792+9=\boxed{801}$ good numbers.

See Also

2001 AMC 12 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2001))	
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