2002 AMC 10A Problems/Problem 1

Problem

The ratio $\frac{10^{2000}+10^{2002}}{10^{2001}+10^{2001}}$ is closest to which of the following numbers?

$\text{(A)}\ 0.1 \qquad \text{(B)}\ 0.2 \qquad \text{(C)}\ 1 \qquad \text{(D)}\ 5 \qquad \text{(E)}\ 10$

Solution

We factor $\frac{10^{2000}+10^{2002}}{10^{2001}+10^{2001}}$ as $\frac{10^{2000}(1+100)}{10^{2001}(1+1)}=\frac{101}{20}$. As $\frac{101}{20}=5.05$, our answer is $\boxed{\text{(D)}\ 5}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Number Theory Problems

2002 AMC 10A Problems/Problem 2

Problem

Given that a, b, and c are non-zero real numbers, define $(a,b,c)=\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$, find $(2,12,9)$.

$\text{(A)}\ 4 \qquad \text{(B)}\ 5 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 8$

Solution

$(2,12,9)=\frac{2}{12}+\frac{12}{9}+\frac{9}{2}=\frac{1}{6}+\frac{4}{3}+\frac{9}{2}=\frac{1}{6}+\frac{8}{6}+\frac{27}{6}=\frac{36}{6}=6$. Our answer is then $\boxed{\text{(C)}\ 6}$.

Alternate solution for the lazy: Without computing the answer exactly, we see that $2/12=\text{a little}$, $12/9=\text{more than }1$, and $9/2=4.5$. The sum is $4.5+(\text{more than }1)+(\text{a little})=(\text{more than }5.5)+(\text{a little})$, and as all the options are integers, the correct one is obviously 6.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Algebra Problems

2002 AMC 12A Problems/Problem 3

The following problem is from both the 2002 AMC 12A #3 and 2002 AMC 10A #3, so both problems redirect to this page.

Problem

According to the standard convention for exponentiation,

$$2^{2^{2^2}} = 2^{(2^{(2^2)})} = 2^{16} = 65536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 2 \qquad \mathrm{(D)}\ 3 \qquad \mathrm{(E)}\ 4$

Solution

The best way to solve this problem is by simple brute force.

It is convenient to drop the usual way how exponentiation is denoted, and to write the formula as $2\uparrow 2\uparrow 2\uparrow 2$, where $\uparrow$ denotes exponentiation. We are now examining all ways to add parentheses to this expression. There are 5 ways to do so:

1. $2\uparrow (2\uparrow (2\uparrow 2))$
2. $2\uparrow ((2\uparrow 2)\uparrow 2)$
3. $((2\uparrow 2)\uparrow 2)\uparrow 2$
4. $(2\uparrow (2\uparrow 2))\uparrow 2$
5. $(2\uparrow 2)\uparrow (2\uparrow 2)$

We can note that $2\uparrow (2\uparrow 2) = (2\uparrow 2)\uparrow 2 = 16$. Therefore options 1 and 2 are equal, and options 3 and 4 are equal. Option 1 is the one given in the problem statement. Thus we only need to evaluate options 3 and 5.

$((2\uparrow 2)\uparrow 2)\uparrow 2 = 16\uparrow 2 = 256$

$(2\uparrow 2)\uparrow (2\uparrow 2) = 4\uparrow 4 = 256$

Thus the only other result is 256, and our answer is $\boxed{\mathrm{(B)}\ 1}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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Preceded by Problem 2	Followed by Problem 4
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2002 AMC 12A Problems/Problem 6

The following problem is from both the 2002 AMC 12A #6 and 2002 AMC 10A #4, so both problems redirect to this page.

Contents

Problem

For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

$\mathrm{(A) \ } 4\qquad \mathrm{(B) \ } 6\qquad \mathrm{(C) \ } 9\qquad \mathrm{(D) \ } 12\qquad \mathrm{(E) \ }$ infinitely many

Solution

Solution 1

For any m we can pick $n = 1$, we get $m \cdot 1 \le m + 1$, therefore the answer is $\boxed{\text{(E) infinitely many}}$.

Solution 2

Another solution, slightly similar to this first one would be using Simon's Favorite Factoring Trick.

$$(m-1)(n-1) \le 1$$

Let $n = 1$, then

$$0 \le 1$$

This means that there are infinitely many numbers m that can satisfy the inequality. So the answer is $\boxed{\text{(E) infinitely many}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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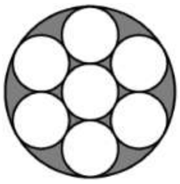
2002 AMC 12A Problems/Problem 5

The following problem is from both the 2002 AMC 12A #5 and 2002 AMC 10A #5, so both problems redirect to this page.

Problem

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



$\text{(A)}\ \pi \qquad \text{(B)}\ 1.5\pi \qquad \text{(C)}\ 2\pi \qquad \text{(D)}\ 3\pi \qquad \text{(E)}\ 3.5\pi$

Solution

The outer circle has radius $1+1+1=3$, and thus area 9π. The little circles have area π each; since there are 7, their total area is 7π. Thus, our answer is $9\pi-7\pi=\boxed{2\pi\Rightarrow \text{(C)}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 2

The following problem is from both the 2002 AMC 12A #2 and 2002 AMC 10A #6, so both problems redirect to this page.

Problem

Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

$\text{(A) } 15 \qquad \text{(B) } 34 \qquad \text{(C) } 43 \qquad \text{(D) } 51 \qquad \text{(E) } 138$

Solution

We work backwards; the number that Cindy started with is $3(43)+9=138$. Now, the correct result is $\frac{138-3}{9}=\frac{135}{9}=15$. Our answer is $\boxed{\text{(A) } 15}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 7

The following problem is from both the 2002 AMC 12A #7 and 2002 AMC 10A #7, so both problems redirect to this page.

Problem

A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

$\text{(A)}\ 4/9 \qquad \text{(B)}\ 2/3 \qquad \text{(C)}\ 5/6 \qquad \text{(D)}\ 3/2 \qquad \text{(E)}\ 9/4$

Solution

Let r_1 and r_2 be the radii of circles A and B, respectively.

It is well known that in a circle with radius r, a subtended arc opposite an angle of θ degrees has length $\frac{\theta}{360} \cdot 2\pi r$.

Using that here, the arc of circle A has length $\frac{45}{360}\cdot2\pi r_1=\frac{r_1\pi}{4}$. The arc of circle B has length $\frac{30}{360}\cdot2\pi r_2=\frac{r_2\pi}{6}$. We know that they are equal, so $\frac{r_1\pi}{4}=\frac{r_2\pi}{6}$, so we multiply through and simplify to get $\frac{r_1}{r_2}=\frac{2}{3}$. As all circles are similar to one another, the ratio of the areas is just the square of the ratios of the radii, so our answer is $\boxed{\text{(A)}\ 4/9}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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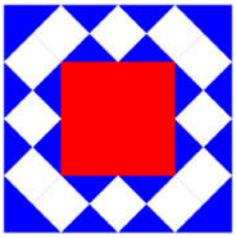
Category: Introductory Geometry Problems

2002 AMC 12A Problems/Problem 8

The following problem is from both the 2002 AMC 12A #8 and 2002 AMC 10A #8, so both problems redirect to this page.

Problem

Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



$\text{(A)}\ B = W \qquad \text{(B)}\ W = R \qquad \text{(C)}\ B = R \qquad \text{(D)}\ 3B = 2R \qquad \text{(E)}\ 2R = W$

Solution

The blue that's touching the center red square makes up 8 triangles, or 4 squares. Each of the corners is 2 squares and each of the edges is 1, totaling 12 squares. There are 12 white squares, thus we have $\boxed{B = W \Rightarrow \text{(A)}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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Category: Introductory Geometry Problems

2002 AMC 10A Problems/Problem 9

Problem

There are 3 numbers A, B, and C, such that $1001C - 2002A = 4004$, and $1001B + 3003A = 5005$. What is the average of A, B, and C?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}$ More than 1

Solution

Notice that we don't need to find what A, B, and C actually are, just their average. In other words, if we can find A+B+C, we will be done.

Adding up the equations gives $1001(A+B+C)=1001(9)$ so $A+B+C=9$ and the average is $\frac{9}{3}=3$. Our answer is $\boxed{\textbf{(B)}\ 3}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Algebra Problems

2002 AMC 12A Problems/Problem 1

The following problem is from both the 2002 AMC 12A #1 and 2002 AMC 10A #10, so both problems redirect to this page.

Contents

Problem

Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$

$\mathrm{(A) \ } \frac{7}{2}\qquad \mathrm{(B) \ } 4\qquad \mathrm{(C) \ } 5\qquad \mathrm{(D) \ } 7\qquad \mathrm{(E) \ } 13$

Solution

Solution 1

We expand to get $2x^2-8x+3x-12+2x^2-12x+3x-18=0$ which is $4x^2-14x-30=0$ after combining like terms. Using the quadratic part of Vieta's Formulas, we find the sum of the roots is $\frac{14}{4}=\boxed{\mathrm{(A)}\ 7/2}$.

Solution 2

Combine terms to get $(2x+3)\cdot\Big((x-4)+(x-6)\Big)=(2x+3)(2x-10)=0$, hence the roots are $-\frac{3}{2}$ and 5, thus our answer is $-\frac{3}{2}+5=\boxed{\mathrm{(A)}\ 7/2}$.

See also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 9

The following problem is from both the 2002 AMC 12A #9 and 2002 AMC 10A #11, so both problems redirect to this page.

Problem

Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB, 12 of the files take up 0.7 MB, and the rest take up 0.4 MB. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

$\text{(A)}\ 12 \qquad \text{(B)}\ 13 \qquad \text{(C)}\ 14 \qquad \text{(D)}\ 15 \qquad \text{(E)} 16$

Solution

A 0.8 MB file can either be on its own disk, or share it with a 0.4 MB. Clearly it is better to pick the second possibility. Thus we will have 3 disks, each with one 0.8 MB file and one 0.4 MB file.

We are left with 12 files of 0.7 MB each, and 12 files of 0.4 MB each. Their total size is $12\cdot(0.7+0.4)=13.2$ MB. The total capacity of 9 disks is $9\cdot 1.44=12.96$ MB, hence we need at least 10 more disks. And we can easily verify that 10 disks are indeed enough: six of them will carry two 0.7 MB files each, and four will carry three 0.4 MB files each.

Thus our answer is $3+10=\boxed{\text{(B)}\ 13}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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Category: Intermediate Algebra Problems

2002 AMC 12A Problems/Problem 11

The following problem is from both the 2002 AMC 12A #11 and 2002 AMC 10A #12, so both problems redirect to this page.

Contents

Problem

Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

$\text{(A)}\ 45 \qquad \text{(B)}\ 48 \qquad \text{(C)}\ 50 \qquad \text{(D)}\ 55 \qquad \text{(E)} 58$

Solution

Solution 1

Let the time he needs to get there in be t and the distance he travels be d. From the given equations, we know that $d = \left(t + \frac{1}{20}\right) 40$ and $d = \left(t - \frac{1}{20}\right) 60$. Setting the two equal, we have $40t + 2 = 60t - 3$ and we find $t = \frac{1}{4}$ of an hour. Substituting t back in, we find $d = 12$. From $d = rt$, we find that r, and our answer, is $\boxed{\text{(B)}\ 48}$.

Solution 2

Since either time he arrives at is 3 minutes from the desired time, the answer is merely the harmonic mean of 40 and 60. The harmonic mean of a and b is $\frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b}$. In this case, a and b are 40 and 60, so our answer is $\frac{4800}{100} = 48$, so $\boxed{\text{(B)}\ 48}$.

Solution 3

A more general form of the argument in Solution 2, with proof:

Let d be the distance to work, and let s be the correct average speed. Then the time needed to get to work is $t = \frac{d}{s}$.

We know that $t + \frac{3}{60} = \frac{d}{40}$ and $t - \frac{3}{60} = \frac{d}{60}$. Summing these two equations, we get: $2t = \frac{d}{40} + \frac{d}{60}$.

Substituting $t = \frac{d}{s}$ and dividing both sides by d, we get $\frac{2}{s} = \frac{1}{40} + \frac{1}{60}$, hence $s = \boxed{48}$.

(Note that this approach would work even if the time by which he is late was different from the time by which he is early in the other case - we would simply take a weighted sum in step two, and hence obtain a weighted harmonic mean in step three.)

2002 AMC 10A Problems/Problem 13

Contents

Problem

Give a triangle with side lengths 15, 20, and 25, find the triangle's smallest height.

$\text{(A)}\ 6 \qquad \text{(B)}\ 12 \qquad \text{(C)}\ 12.5 \qquad \text{(D)}\ 13 \qquad \text{(E)}\ 15$

Solution

Solution 1

This is a Pythagorean triple (a 3-4-5 actually) with legs 15 and 20. The area is then $\frac{(15)(20)}{2} = 150$. Now, consider an altitude drawn to any side. Since the area remains constant, the altitude and side to which it is drawn are inversely proportional. To get the smallest altitude, it must be drawn to the hypotenuse. Let the length be x; we have $\frac{(x)(25)}{2} = \frac{(15)(20)}{2}$, so $25x = 300$ and x is 12. Our answer is then $\boxed{\text{(B)}\ 12}$.

Solution 2

By Heron's formula, the area is 150, hence the shortest altitude's length is $2 \cdot \frac{150}{25} = \boxed{12 \Rightarrow \text{(B)}}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12A Problems/Problem 12

The following problem is from both the 2002 AMC 12A #12 and 2002 AMC 10A #14, so both problems redirect to this page.

Problem

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2 \qquad \text{(D) } 4 \qquad \text{(E) more than } 4$

Solution

Consider a general quadratic with the coefficient of x^2 being 1 and the roots being r and s. It can be factored as $(x - r)(x - s)$ which is just $x^2 - (r + s)x + rs$. Thus, the sum of the roots is the negative of the coefficient of x and the product is the constant term. (In general, this leads to Vieta's Formulas).

We now have that the sum of the two roots is 63 while the product is k. Since both roots are primes, one must be 2, otherwise the sum would be even. That means the other root is 61 and the product must be 122. Hence, our answer is $\boxed{\text{(B) } 1}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 10A Problems/Problem 15

Problem

Using the digits 1, 2, 3, 4, 5, 6, 7, and 9, form 4 two-digit prime numbers, using each digit only once. What is the sum of the 4 prime numbers?

$\text{(A)}\ 150 \qquad \text{(B)}\ 160 \qquad \text{(C)}\ 170 \qquad \text{(D)}\ 180 \qquad \text{(E)}\ 190$

Solution

Only odd numbers can finish a two-digit prime number, and a two-digit number ending in 5 is divisible by 5 and thus composite, hence our answer is $20+40+50+60+1+3+7+9=190 \Rightarrow \boxed{(E)=190}$.

(Note that we did not need to actually construct the primes. If we had to, one way to match the tens and ones digits to form four primes is 23, 41, 59, and 67.)

(you could also guess the numbers, if you have a lot of spare time)

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 10A Problems/Problem 16

Contents

Problem

Let $a+1=b+2=c+3=d+4=a+b+c+d+5$. What is $a+b+c+d$?

$\text{(A)}\ -5 \qquad \text{(B)}\ -10/3 \qquad \text{(C)}\ -7/3 \qquad \text{(D)}\ 5/3 \qquad \text{(E)}\ 5$

Solution 1

Let $x=a+1=b+2=c+3=d+4=a+b+c+d+5$. Since one of the sums involves a, b, c, and d, it makes sense to consider 4x. We have $4x=(a+1)+(b+2)+(c+3)+(d+4)=a+b+c+d+10=4(a+b+c+d)+20$. Rearranging, we have $3(a+b+c+d)=-10$, so $a+b+c+d=\frac{-10}{3}$. Thus, our answer is $\boxed{\text{(B)}\ -10/3}$.

Solution 2

Take $a+1=a+b+c+d+5$. Now we can clearly see: $-4=b+c+d$. Continuing this same method with $b+2, c+3$, and $d+4$ we get: $-4=b+c+d$, $-3=a+c+d$, $-2=a+b+d$, and $-1=a+b+c$, Adding, we see $-10=3a+3b+3c+3d$. Therefore, $a+b+c+d=\frac{-10}{3}$.

See Also

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Category: Introductory Algebra Problems

2002 AMC 12A Problems/Problem 10

The following problem is from both the 2002 AMC 12A #10 and 2002 AMC 10A #17, so both problems redirect to this page.

Problem

Sarah places four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then pours half the coffee from the first cup to the second and, after stirring thoroughly, pours half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

$(\text{A})\ \frac{1}{4}\qquad(\text{B})\ \frac{1}{3}\qquad(\text{C})\ \frac{3}{8}\qquad(\text{D})\ \frac{2}{5}\qquad(\text{E})\ \frac{1}{2}$

Solution

We will simulate the process in steps.

In the beginning, we have:

- 4 ounces of coffee in cup 1
- 4 ounces of cream in cup 2

In the first step we pour $4/2=2$ ounces of coffee from cup 1 to cup 2, getting:

- 2 ounces of coffee in cup 1
- 2 ounces of coffee and 4 ounces of cream in cup 2

In the second step we pour $2/2=1$ ounce of coffee and $4/2=2$ ounces of cream from cup 2 to cup 1, getting:

- $2+1=3$ ounces of coffee and $0+2=2$ ounces of cream in cup 1
- the rest in cup 2

Hence at the end we have $3+2=5$ ounces of liquid in cup 1, and out of these 2 ounces is cream. Thus the answer is $\boxed{(\text{D})\ \frac{2}{5}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 10A Problems/Problem 18

Problem

A 3x3x3 cube is made of 27 normal dice. Each die's opposite sides sum to 7. What is the smallest possible sum of all of the values visible on the 6 faces of the large cube?

$\text{(A)}\ 60 \qquad \text{(B)}\ 72 \qquad \text{(C)}\ 84 \qquad \text{(D)}\ 90 \qquad \text{(E)} 96$

Solution

In a 3x3x3 cube, there are 8 cubes with three faces showing, 12 with two faces showing and 6 with one face showing. The smallest sum with three faces showing is 1+2+3=6, with two faces showing is 1+2=3, and with one face showing is 1. Hence, the smallest possible sum is $8(6)+12(3)+6(1)=48+36+6=90$. Our answer is thus $\boxed{\text{(D)}\ 90}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2002 AMC 10A Problems/Problem 19

Problem

Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside of the doghouse that Spot can reach?

$\text{(A)}\ 2\pi/3 \qquad \text{(B)}\ 2\pi \qquad \text{(C)}\ 5\pi/2 \qquad \text{(D)}\ 8\pi/3 \qquad \text{(E)}\ 3\pi$

Solution

Part of what Spot can reach is $\frac{240}{360}=\frac{2}{3}$ of a circle with radius 2, which gives him $\frac{8\pi}{3}$. He can also reach two $\frac{60}{360}$ parts of a unit circle, which combines to give $\frac{\pi}{3}$. The total area is then 3π, which gives $\boxed{\text{(E)}}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Geometry Problems

2002 AMC 10A Problems/Problem 20

Contents

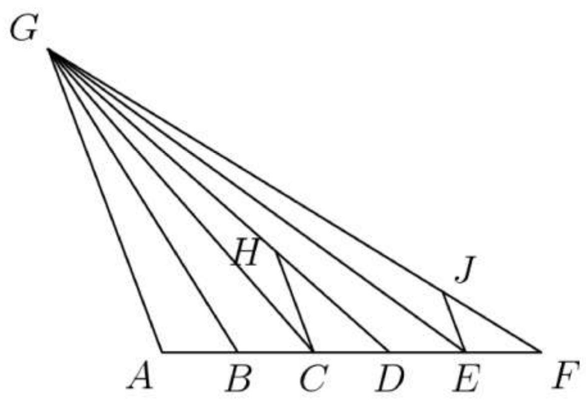
Problem

Points A, B, C, D, E and F lie, in that order, on $\overline{AF}$, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on $\overline{GD}$, and point J lies on $\overline{GF}$. The line segments $\overline{HC}, \overline{JE}$, and $\overline{AG}$ are parallel. Find HC/JE.

$\text{(A)}\ 5/4 \qquad \text{(B)}\ 4/3 \qquad \text{(C)}\ 3/2 \qquad \text{(D)}\ 5/3 \qquad \text{(E)}\ 2$

Solution 1

First we can draw an image.



Since $\overline{AG}$ and $\overline{CH}$ are parallel, triangles $\triangle GAD$ and $\triangle HCD$ are similar. Hence, $\frac{CH}{AG} = \frac{CD}{AD} = \frac{1}{3}$.

Since $\overline{AG}$ and $\overline{JE}$ are parallel, triangles $\triangle GAF$ and $\triangle JEF$ are similar. Hence, $\frac{EJ}{AG} = \frac{EF}{AF} = \frac{1}{5}$. Therefore, $\frac{CH}{EJ} = \left(\frac{CH}{AG}\right) \div \left(\frac{EJ}{AG}\right) = \left(\frac{1}{3}\right) \div \left(\frac{1}{5}\right) = \boxed{\frac{5}{3}}$. The answer is $\boxed{(D) 5/3}$.

Solution 2

As angle F is clearly congruent to itself, we get from AA similarity, $\triangle AGF \sim \triangle EJF$; hence $\frac{AG}{JE} = 5$. Similarly, $\frac{AG}{HC} = 3$. Thus, $\frac{HC}{JE} = \left(\frac{AG}{JE}\right)\left(\frac{HC}{AG}\right) = \boxed{\frac{5}{3} \Rightarrow \text{(D)}}$.

See Also

2002 AMC 12A Problems/Problem 15

The following problem is from both the 2002 AMC 12A #15 and 2002 AMC 10A #21, so both problems redirect to this page.

Contents

Problem

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

$\text{(A)}\ 11 \qquad \text{(B)}\ 12 \qquad \text{(C)}\ 13 \qquad \text{(D)}\ 14 \qquad \text{(E)}\ 15$

Solution

As the unique mode is 8, there are at least two 8s.

As the range is 8 and one of the numbers is 8, the largest one can be at most 16.

If the largest one is 16, then the smallest one is 8, and thus the mean is strictly larger than 8, which is a contradiction.

If the largest one is 15, then the smallest one is 7. This means that we already know four of the values: 8, 8, 7, 15. Since the mean of all the numbers is 8, their sum must be 64. Thus the sum of the missing four numbers is $64-8-8-7-15=26$. But if 7 is the smallest number, then the sum of the missing numbers must be at least $4\cdot 7=28$, which is again a contradiction.

If the largest number is 14, we can easily find the solution $(6,6,6,8,8,8,8,14)$. Hence, our answer is $\boxed{\text{(D)}\ 14}$.

Note

The solution for 14 is, in fact, unique. As the median must be 8, this means that both the 4^{th} and the 5^{th} number, when ordered by size, must be 8s. This gives the partial solution $(6,a,b,8,8,c,d,14)$. For the mean to be 8 each missing variable must be replaced by the smallest allowed value.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 10A Problems/Problem 22

Contents

Problem

A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

$\text{(A)}\ 10 \qquad \text{(B)}\ 11 \qquad \text{(C)}\ 18 \qquad \text{(D)}\ 19 \qquad \text{(E)}\ 20$

Solution

Solution 1

The pattern is quite simple to see after listing a couple of terms.

#	Removed	Left
1	10	90
2	9	81
3	9	72
4	8	64
5	8	56
6	7	49
7	7	42
8	6	36
9	6	30
10	5	25
11	5	20
12	4	16
13	4	12
14	3	9
15	3	6
16	2	4
17	2	2
$\boxed{18}$	1	1

Solution 2

Given n^2 tiles, a step removes n tiles, leaving $n^2 - n$ tiles behind. Now, $(n-1)^2 = n^2 - n + (1-n) < n^2 - n < n^2$, so in the next step $n-1$ tiles are removed. This gives $(n^2 - n) - (n-1) = n^2 - 2n + 1 = (n-1)^2$, another perfect square.

Thus each two steps we cycle down a perfect square, and in $(10-1)\times 2 = 18$ steps, we are left with 1 tile, hence our answer is $\boxed{\text{(C)}\ 18}$.

See also

2002 AMC 10A Problems/Problem 23

Problem 23

Points A, B, C and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.

$\text{(A)}\ 15/2 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 17/2 \qquad \text{(D)}\ 9 \qquad \text{(E)}\ 19/2$

Solution

First, we draw an altitude to BC from E.Let it intersect at M. As triangle BEC is isosceles, we immediately get MB=MC=6, so the altitude is 8. Now, let $AB = CD = x$. Using the Pythagorean Theorem on triangle EMA, we find $AE = \sqrt{x^2 + 12x + 100}$. From symmetry, $DE = \sqrt{x^2 + 12x + 100}$ as well. Now, we use the fact that the perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$.

We have $2\sqrt{x^2 + 12x + 100} + 2x + 12 = 2(32)$ so $\sqrt{x^2 + 12x + 100} = 26 - x$. Squaring both sides, we have $x^2 + 12x + 100 = 676 - 52x + x^2$ which nicely rearranges into $64x = 576 \rightarrow x = 9$. Hence, AB is 9 so our answer is $\boxed{\text{(D)}}$.

See Also

2002 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12A Problems/Problem 16

The following problem is from both the 2002 AMC 12A #16 and 2002 AMC 10A #24, so both problems redirect to this page.

Contents

Problem

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, ..., 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

$\text{(A)}\ 2/5 \qquad \text{(B)}\ 9/20 \qquad \text{(C)}\ 1/2 \qquad \text{(D)}\ 11/20 \qquad \text{(E)}\ 24/25$

Solution

Solution 1

This is not too bad using casework.

Tina gets a sum of 3: This happens in only one way $(1,2)$ and Sergio can choose a number from 4 to 10, inclusive. There are 7 ways that Sergio gets a desirable number here.

Tina gets a sum of 4: This once again happens in only one way $(1,3)$. Sergio can choose a number from 5 to 10, so 6 ways here.

Tina gets a sum of 5: This can happen in two ways $(1,4)$ and $(2,3)$. Sergio can choose a number from 6 to 10, so $2\cdot5=10$ ways here.

Tina gets a sum of 6: Two ways here $(1,5)$ and $(2,4)$. Sergio can choose a number from 7 to 10, so $2*4=8$ here.

Tina gets a sum of 7: Two ways here $(2,5)$ and $(3,4)$. Sergio can choose from 8 to 10, so $2\cdot3=6$ ways here.

Tina gets a sum of 8: Only one way possible $(3,5)$. Sergio chooses 9 or 10, so 2 ways here.

Tina gets a sum of 9: Only one way $(4,5)$. Sergio must choose 10, so 1 way.

In all, there are $7+6+10+8+6+2+1=40$ ways. Tina chooses two distinct numbers in $\binom{5}{2}=10$ ways while Sergio chooses a number in 10 ways, so there are $10\cdot10=100$ ways in all. Since , our answer is $\boxed{(\text{A})\frac{2}{5}}$.

Solution 2

We want to find the average of the smallest possible chance of Sergio winning and the largest possible chance of Sergio winning. This is because the probability decreases linearly. The largest possibility of Sergio winning if Tina chooses a 1 and a 2. The chances of Sergio winning is then $\frac{7}{10}$. The smallest possibility of Sergio winning is if Tina chooses a 4 and a 5. The chances of Sergio winning then is $\frac{1}{10}$. The average of $\frac{7}{10}$ and $\frac{1}{10}$ is $\boxed{(\mathrm{A})\frac{2}{5}}$.

Solution 3

The expected value of a number randomly selected form the set $\{1,2,3,4,5\}$ is 3. Therefore, Tina's expected sum is $3+3=6$. The probability that Sergio selects a number larger than 6 from his set is $\boxed{\frac{2}{5}}$. This works because of symmetry.

See Also

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Categories: Introductory Probability Problems | Introductory Combinatorics Problems

2002 AMC 10A Problems/Problem 25

Contents

Problem

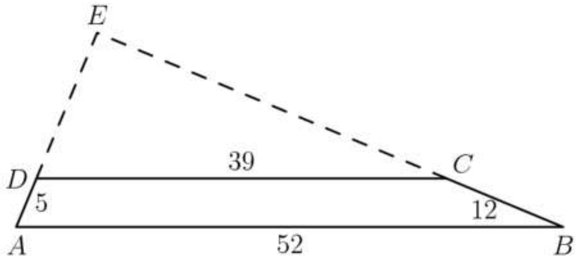
In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. The area of $ABCD$ is

$\text{(A)}\ 182 \qquad \text{(B)}\ 195 \qquad \text{(C)}\ 210 \qquad \text{(D)}\ 234 \qquad \text{(E)}\ 260$

Solution

Solution 1

It shouldn't be hard to use trigonometry to bash this and find the height, but there is a much easier way. Extend $\overline{AD}$ and $\overline{BC}$ to meet at point E:



Since $\overline{AB} || \overline{CD}$ we have $\triangle AEB \sim \triangle DEC$, with the ratio of proportionality being $\frac{39}{52} = \frac{3}{4}$. Thus

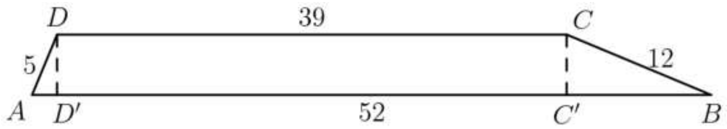
$$\frac{CE}{CE+12} = \frac{3}{4} \Longrightarrow CE = 36$$
$$\frac{DE}{DE+5} = \frac{3}{4} \Longrightarrow DE = 15$$

So the sides of $\triangle CDE$ are $15, 36, 39$, which we recognize to be a $5-12-13$ right triangle. Therefore (we could simplify some of the calculation using that the ratio of areas is equal to the ratio of the sides squared),

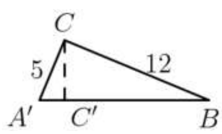
$$[ABCD] = [ABE] - [CDE] = \frac{1}{2} \cdot 20 \cdot 48 - \frac{1}{2} \cdot 15 \cdot 36 = \boxed{\text{(C)}\ 210}$$

Solution 2

Draw altitudes from points C and D:



Translate the triangle ADD' so that DD' coincides with CC'. We get the following triangle:



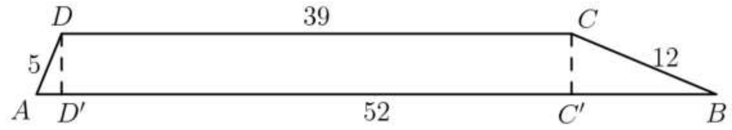
The length of $A'B$ in this triangle is equal to the length of the original AB, minus the length of CD. Thus $A'B = 52 - 39 = 13$.

Therefore $A'BC$ is a well-known $(5, 12, 13)$ right triangle. Its area is $[A'BC] = \frac{A'C \cdot BC}{2} = \frac{5 \cdot 12}{2} = 30$, and therefore its altitude CC' is $\frac{[A'BC]}{A'B} = \frac{60}{13}$.

Now the area of the original trapezoid is $\frac{(AB + CD) \cdot CC'}{2} = \frac{91 \cdot 60}{13 \cdot 2} = 7 \cdot 30 = \boxed{\text{(C)}\ 210}$

Solution 3

Draw altitudes from points C and D:



Call the length of AD' to be y, the length of BC' to be z, and the height of the trapezoid to be x. By the Pythagorean Theorem, we have:

$$z^2 + x^2 = 144$$

$$y^2 + x^2 = 25$$

Subtracting these two equation yields:

$$z^2 - y^2 = 119 \implies (z+y)(z-y) = 119$$

We also have: $z + y = 52 - 39 = 13$.

We can substitute the value of $z + y$ into the equation we just obtained, so we now have:

$$(13)(z-y) = 119 \implies z - y = \frac{119}{13}$$

.

We can add the $z + y$ and the $z - y$ equation to find the value of z, which simplifies down to be $\frac{144}{13}$. Finally, we can plug in z and use the Pythagorean theorem to find the height of the trapezoid.

$$\frac{12^4}{13^2} + x^2 = 12^2 \implies x^2 = \frac{(12^2)(13^2)}{13^2} - \frac{12^4}{13^2} \implies x^2 = \frac{(12 \cdot 13)^2 - (144)^2}{13^2} \implies x^2 = \frac{(156+144)(156-144)}{13^2} \implies x = \sqrt{\frac{3600}{169}} =$$

Now that we have the height of the trapezoid, we can multiply this by the median to find our answer.

The median of the trapezoid is $\frac{39+52}{2} = \frac{91}{2}$, and multiplying this and the height of the trapezoid gets us:

$$\frac{60 \cdot 91}{13 \cdot 2} = \boxed{\text{(C) } 210}$$

See also

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