2002 AMC 10B Problems/Problem 1

Problem

The ratio $\dfrac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ is:

$\mathrm{(A) \ } 1/6\qquad \mathrm{(B) \ } 1/3\qquad \mathrm{(C) \ } 1/2\qquad \mathrm{(D) \ } 2/3\qquad \mathrm{(E) \ } 3/2$

Solution

$\dfrac{2^{2001} \cdot 3^{2003}}{6^{2002}} = \dfrac{6^{2001} \cdot 3^{2}}{6^{2002}} = \dfrac{9}{6} = \dfrac{3}{2}$ or $\mathrm{(E)}$

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Number Theory Problems

2002 AMC 10B Problems/Problem 2

Problem

For the nonzero numbers a, b, and c, define

$$(a,b,c)=\frac{abc}{a+b+c}$$

Find $(2,4,6)$.

$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 4 \qquad (\mathrm{D})\ 6 \qquad (\mathrm{E})\ 24$

Solution

$$\frac{2\cdot4\cdot6}{2+4+6}=\frac{48}{12}=4\Longrightarrow(\mathrm{C})$$

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 1	Followed by Problem 3
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Category: Introductory Algebra Problems

2002 AMC 12B Problems/Problem 1

The following problem is from both the 2002 AMC 12B #1 and 2002 AMC 10B #3, so both problems redirect to this page.

Problem

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

$\mathrm{(A)}\ 0 \qquad\mathrm{(B)}\ 2 \qquad\mathrm{(C)}\ 4 \qquad\mathrm{(D)}\ 6 \qquad\mathrm{(E)}\ 8$

Solution

We wish to find $\frac{9+99+\cdots +999999999}{9}$, or $\frac{9(1+11+111+\cdots +111111111)}{9} = 123456789$. This does not have the digit 0, so the answer is $\boxed{\mathrm{(A)}\ 0}$

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 2	Followed by Problem 4
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Category: Introductory Algebra Problems

2002 AMC 12B Problems/Problem 2

The following problem is from both the 2002 AMC 12B #2 and 2002 AMC 10B #4, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution
- 3 Comment
- 4 See also

Problem

What is the value of $(3x-2)(4x+1)-(3x-2)4x+1$ when $x=4$?

$\mathrm{(A) \ } 0\qquad \mathrm{(B) \ } 1\qquad \mathrm{(C) \ } 10\qquad \mathrm{(D) \ } 11\qquad \mathrm{(E) \ } 12$

Solution

By the distributive property,

$$(3x-2)[(4x+1)-4x]+1 = 3x-2+1 = 3x-1 = 3(4)-1 = \boxed{\mathrm{(D)\ } 11}$$

Comment

It would be nicer if the organizers chose a more difficult substitution, say $x=4.7$, to penalize those solutions that do not take advantage of the distributive property.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 3	Followed by Problem 5
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2002 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
Preceded by Problem 1	Followed by Problem 3
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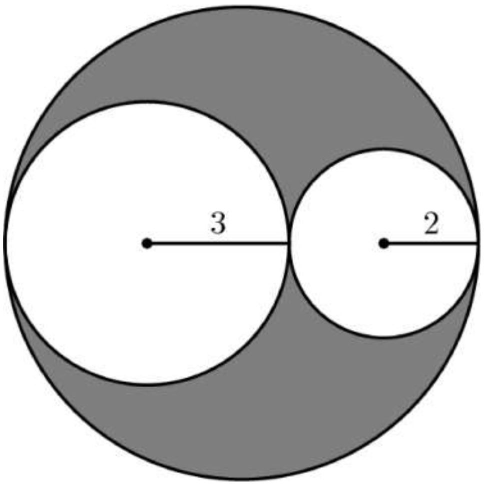
All AMC 12 Problems and Solutions	

2002 AMC 10B Problems/Problem 5

Problem

Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.



$\mathrm{(A) \ } 3\pi\qquad \mathrm{(B) \ } 4\pi\qquad \mathrm{(C) \ } 6\pi\qquad \mathrm{(D) \ } 9\pi\qquad \mathrm{(E) \ } 12\pi$

Solution

A line going through the centers of the two smaller circles also go through the diameter. The length of this line within the circle is $3+3+2+2=10.$ Because this is the length of the larger circle's diameter, the length of its radius is $5.$

The area of the large circle is 25π, and the area of the two smaller circles is $9\pi+4\pi=13\pi.$ To find the area of the shaded region, subtract the area of the two smaller circles from the area of the large circle.

$\longrightarrow 25\pi-13\pi=\boxed{\mathrm{(E) \ } 12\pi}$

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12B Problems/Problem 3

The following problem is from both the 2002 AMC 12B #3 and 2002 AMC 10B #6, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 See also

Problem

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

$\mathrm{(A)}\ \text{none} \qquad \mathrm{(B)}\ \text{one} \qquad \mathrm{(C)}\ \text{two} \qquad \mathrm{(D)}\ \text{more than two, but finitely many} \qquad \mathrm{(E)}\ \text{infinitely many}$

Solution 1

Factoring, we get $n^2 - 3n + 2 = (n-2)(n-1)$. Either $n-1$ or $n-2$ is odd, and the other is even. Their product must yield an even number. The only prime that is even is 2, which is when n is 3. The answer is $\boxed{\mathrm{(B)}\ \text{one}}$.

Solution 2

Considering parity, we see that $n^2 - 3n + 2$ is always even. The only even prime is 2, and so $n^2 - 3n = 0$ whence $n = 3 \Rightarrow \boxed{\mathrm{(B)}\ \text{one}}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 5	Followed by Problem 7
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Preceded by Problem 2	Followed by Problem 4
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Category: Introductory Number Theory Problems

2002 AMC 12B Problems/Problem 4

The following problem is from both the 2002 AMC 12B #4 and 2002 AMC 10B #7, so both problems redirect to this page.

Problem

Let n be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is not true:

$\mathrm{(A)}\ 2\ \text{divides}\ n \qquad \mathrm{(B)}\ 3\ \text{divides}\ n \qquad \mathrm{(C)}\ \ 6\ \text{divides}\ n \qquad \mathrm{(D)}\ 7\ \text{divides}\ n \qquad \mathrm{(E)}\ n > 84$

Solution

Since $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} = \frac{41}{42}$,

$$0 < \lim_{n \rightarrow \infty} \left(\frac{41}{42} + \frac{1}{n}\right) < \frac{41}{42} + \frac{1}{n} < \frac{41}{42} + \frac{1}{1} < 2$$

From which it follows that $\frac{41}{42} + \frac{1}{n} = 1$ and $n = 42$. The only answer choice that is not true is $\boxed{\mathrm{(E)}\ n > 84}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 6	Followed by Problem 8
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2002 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
Preceded by Problem 3	Followed by Problem 5
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Category: Introductory Algebra Problems

2002 AMC 12B Problems/Problem 8

The following problem is from both the 2002 AMC 12B #8 and 2002 AMC 10B #8, so both problems redirect to this page.

Problem

Suppose July of year N has five Mondays. Which of the following must occurs five times in the August of year N? (Note: Both months have 31 days.)

$\textbf{(A)}\ \text{Monday} \qquad \textbf{(B)}\ \text{Tuesday} \qquad \textbf{(C)}\ \text{Wednesday} \qquad \textbf{(D)}\ \text{Thursday} \qquad \textbf{(E)}\ \text{Friday}$

Solution

If there are five Mondays, there are only three possibilities for their dates: $(1,8,15,22,29)$, $(2,9,16,23,30)$, and $(3,10,17,24,31)$.

In the first case August starts on a Thursday, and there are five Thursdays, Fridays, and Saturdays in August.

In the second case August starts on a Wednesday, and there are five Wednesdays, Thursdays, and Fridays in August.

In the third case August starts on a Tuesday, and there are five Tuesdays, Wednesdays, and Thursdays in August.

The only day of the week that is guaranteed to appear five times is therefore $\boxed{\textbf{(D)}\ \text{Thursday}}$.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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Category: Introductory Algebra Problems

2002 AMC 10B Problems/Problem 9

Problem

Using the letters A, M, O, S, and U, we can form five-letter "words". If these "words" are arranged in alphabetical order, then the "word" $USAMO$ occupies position

$\mathrm{(A) \ } 112\qquad \mathrm{(B) \ } 113\qquad \mathrm{(C) \ } 114\qquad \mathrm{(D) \ } 115\qquad \mathrm{(E) \ } 116$

Solution

There are $4!\cdot 4$ "words" beginning with each of the first four letters alphabetically. From there, there are $3!\cdot 3$ with U as the first letter and each of the first three letters alphabetically. After that, the next "word" is $USAMO$, hence our answer is $4\cdot 4!+3\cdot 3!+1=\boxed{115\Rightarrow\text{(D)}}$.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 8	Followed by Problem 10
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Category: Introductory Combinatorics Problems

2002 AMC 12B Problems/Problem 6

The following problem is from both the 2002 AMC 12B #6 and 2002 AMC 10B #10, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See also

Problem

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. Then the pair (a, b) is

$\mathrm{(A)}\ (-2,1) \qquad\mathrm{(B)}\ (-1,2) \qquad\mathrm{(C)}\ (1,-2) \qquad\mathrm{(D)}\ (2,-1) \qquad\mathrm{(E)}\ (4,4)$

Solution

Solution 1

Since $(x-a)(x-b) = x^2 - (a+b)x + ab = x^2 + ax + b = 0$, it follows by comparing coefficients that $-a - b = a$ and that $ab = b$. Since b is nonzero, $a = 1$, and $-1 - b = 1 \Longrightarrow b = -2$. Thus $(a,b) = \boxed{\mathrm{(C)}\ (1,-2)}$.

Solution 2

Another method is to use Vieta's formulas. The sum of the solutions to this polynomial is equal to the opposite of the x coefficient, since the leading coefficient is 1; in other words, $a + b = -a$ and the product of the solutions is equal to the constant term (i.e, $a * b = b$). Since b is nonzero, it follows that $a = 1$ and therefore (from the first equation), $b = -2a = -2$. Hence, $(a,b) = \boxed{\mathrm{(C)}\ (1,-2)}$

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 9	Followed by Problem 11
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2002 AMC 12B Problems/Problem 7

The following problem is from both the 2002 AMC 12B #7 and 2002 AMC 10B #11, so both problems redirect to this page.

Problem

The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

$\mathrm{(A)}\ 50 \qquad\mathrm{(B)}\ 77 \qquad\mathrm{(C)}\ 110 \qquad\mathrm{(D)}\ 149 \qquad\mathrm{(E)}\ 194$

Solution

Let the three consecutive positive integers be $a-1$, a, and $a+1$. So, $a(a-1)(a+1)=24a \Rightarrow (a-1)(a+1)=24$. $24=4\times 6$, so $a=5$. Hence, the sum of the squares is $4^2+5^2+6^2=\boxed{\mathrm{(B)}\ 77}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 10	Followed by Problem 12
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Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2002_AMC_12B_Problems/Problem_7&oldid=62514"

Category: Introductory Algebra Problems

2002 AMC 10B Problems/Problem 12

Problem

For which of the following values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x?

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

Solution

The domain over which we solve the equation is $\mathbb{R} - \{2,6\}$.

We can now cross-multiply to get rid of the fractions, we get $(x-1)(x-6) = (x-k)(x-2)$.

Simplifying that, we get $7x-6 = (k+2)x + 2k$. Clearly for $k=5$ we get the equation $-6=10$ which is never true. The answer is $\boxed{\text{(E) } 5}$

For other k, one can solve for x: $x(5-k) = 6-2k$, hence $x = \frac{6-2k}{5-k}$. We can easily verify that for none of the other four possible values of k is this equal to 2 or 6, hence there is a solution for x in each of the other cases.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 11	Followed by Problem 13
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2002 AMC 10B Problems/Problem 13

Problem

Find the value(s) of x such that $8xy - 12y + 2x - 3 = 0$ is true for all values of y.

$\textbf{(A) } \frac23 \qquad \textbf{(B) } \frac32 \text{ or } -\frac14 \qquad \textbf{(C) } -\frac23 \text{ or } -\frac14 \qquad \textbf{(D) } \frac32 \qquad \textbf{(E) } -\frac32 \text{ or } -\frac14$

Solution

We have $8xy - 12y + 2x - 3 = 4y(2x-3) + (2x-3) = (4y+1)(2x-3)$.

As $(4y+1)(2x-3) = 0$ must be true for all y, we must have $2x - 3 = 0$, hence $\boxed{x = \frac32 \text{ (D)}}$.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 12	Followed by Problem 14
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2002 AMC 10B Problems/Problem 14

Problem

The number $5^{64} \cdot 8^{25}$ is the square of a positive integer N. In decimal representation, the sum of the digits of N is

$\mathrm{(A) \ } 7 \qquad \mathrm{(B) \ } 14 \qquad \mathrm{(C) \ } 21 \qquad \mathrm{(D) \ } 28 \qquad \mathrm{(E) \ } 35$

Solution

Since, $N = 5^{64} \cdot 8^{25} = 5^{64} \cdot (2^3)^{25} = 5^{64} \cdot 2^{75}$.

Combing the 2's and 5's gives us, $(2 \cdot 5)^{64} \cdot 2^{(75-64)} = (2 \cdot 5)^{64} \cdot 2^{11} = 10^{64} \cdot 2^{11}$.

This is 2048 with sixty-four, 0's on the end. So, the sum of the digits of N is $2+4+8 = 14 \Longrightarrow \boxed{\mathrm{(B) \ } 14}$

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Number Theory Problems

2002 AMC 12B Problems/Problem 11

The following problem is from both the 2002 AMC 12B #11 and 2002 AMC 10B #15, so both problems redirect to this page.

Problem

The positive integers $A, B, A-B,$ and $A+B$ are all prime numbers. The sum of these four primes is

$\mathrm{(A)}\ \mathrm{even} \qquad\mathrm{(B)}\ \mathrm{divisible\ by\ }3 \qquad\mathrm{(C)}\ \mathrm{divisible\ by\ }5 \qquad\mathrm{(D)}\ \mathrm{divisible\ by\ }7 \qquad\mathrm{(E)}\ \mathrm{prime}$

Solution

Since $A-B$ and $A+B$ must have the same parity, and since there is only one even prime number, it follows that $A-B$ and $A+B$ are both odd. Thus one of A, B is odd and the other even. Since $A+B > A > A-B > 2$, it follows that A (as a prime greater than 2) is odd. Thus $B=2$, and $A-2, A, A+2$ are consecutive odd primes. At least one of $A-2, A, A+2$ is divisible by 3, from which it follows that $A-2=3$ and $A=5$. The sum of these numbers is thus 17, a prime, so the answer is $\boxed{\mathrm{(E)}\ \mathrm{prime}}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12B Problems/Problem 12

The following problem is from both the 2002 AMC 12B #12 and 2002 AMC 10B #16, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See also

Problem

For how many integers n is $\frac{n}{20-n}$ the square of an integer?

$\mathrm{(A)}\ 1 \qquad\mathrm{(B)}\ 2 \qquad\mathrm{(C)}\ 3 \qquad\mathrm{(D)}\ 4 \qquad\mathrm{(E)}\ 10$

Solution

Solution 1

Let $x^2 = \frac{n}{20-n}$, with $x \geq 0$ (note that the solutions $x < 0$ do not give any additional solutions for n). Then rewriting, $n = \frac{20x^2}{x^2 + 1}$. Since $\text{gcd}(x^2, x^2+1) = 1$, it follows that $x^2 + 1$ divides 20. Listing the factors of 20, we find that $x = 0, 1, 2, 3$ are the only $\boxed{\mathrm{(D)}\ 4}$ solutions (respectively yielding $n = 0, 10, 16, 18$).

Solution 2

For $n<0$ and $n>20$ the fraction is negative, for $n=20$ it is not defined, and for $n\in\{1,\dots,9\}$ it is between 0 and 1.

Thus we only need to examine $n=0$ and $n\in\{10,\dots,19\}$.

For $n=0$ and $n=10$ we obviously get the squares 0 and 1 respectively.

For prime n the fraction will not be an integer, as the denominator will not contain the prime in the numerator.

This leaves $n\in\{12,14,15,16,18\}$, and a quick substitution shows that out of these only $n=16$ and $n=18$ yield a square. Therefore, there are only $\boxed{\mathrm{(D)}\ 4}$ solutions (respectively yielding $n=0,10,16,18$).

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
Preceded by Problem 15	Followed by Problem 17
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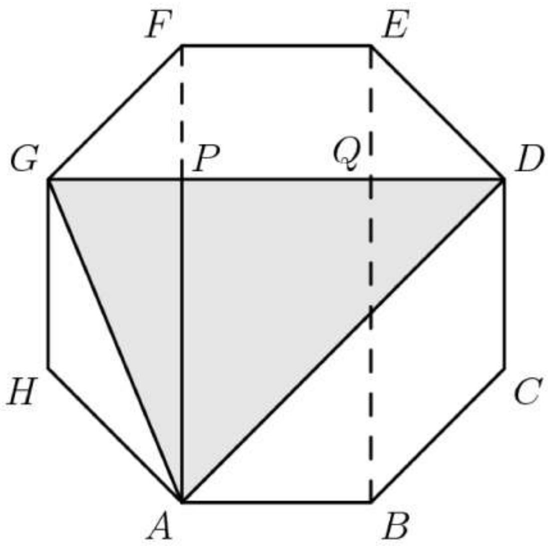
2002 AMC 10B Problems/Problem 17

Problem

A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

$\textbf{(A)}\ 4 + 2\sqrt2 \qquad \textbf{(B)}\ 6 + \sqrt2 \qquad \textbf{(C)}\ 4 + 3\sqrt2 \qquad \textbf{(D)}\ 3 + 4\sqrt2 \qquad \textbf{(E)}\ 8 + \sqrt2$

Solution



The area of the triangle ADG can be computed as $\frac{DG \cdot AP}{2}$. We will now find DG and AP.

Clearly, PFG is a right isosceles triangle with hypotenuse of length 2, hence $PG = \sqrt{2}$. The same holds for triangle QED and its leg QD. The length of PQ is equal to $FE = 2$. Hence $GD = 2 + 2\sqrt{2}$, and $AP = PD = 2 + \sqrt{2}$.

Then the area of ADG equals $\frac{DG \cdot AP}{2} = \frac{(2+2\sqrt{2})(2+\sqrt{2})}{2} = \frac{8+6\sqrt{2}}{2} = \boxed{\textbf{(C)} 4+3\sqrt{2}}$.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Categories: Introductory Geometry Problems | Area Problems

2002 AMC 12B Problems/Problem 14

The following problem is from both the 2002 AMC 12B #14 and 2002 AMC 10B #18, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 See also

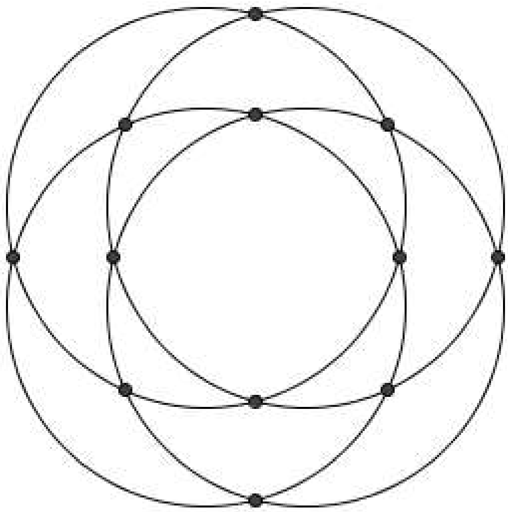
Problem

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

$\mathrm{(A)}\ 8\qquad\mathrm{(B)}\ 9\qquad\mathrm{(C)}\ 10\qquad\mathrm{(D)}\ 12\qquad\mathrm{(E)}\ 16$

Solution 1

For any given pair of circles, they can intersect at most 2 times. Since there are $\binom{4}{2} = 6$ pairs of circles, the maximum number of possible intersections is $6 \cdot 2 = 12$. We can construct such a situation as below, so the answer is $\boxed{\mathrm{(D)}\ 12}$.



Solution 2

Because a pair or circles can intersect at most 2 times, the first circle can intersect the second at 2 points, the third can intersect the first two at 4 points, and the fourth can intersect the first three at 6 points. This means that our answer is $2+4+6 = \boxed{\mathrm{(D)}\ 12}$.

See also

2002 AMC 10B Problems/Problem 19

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 Solution 3
- 5 See Also

Problem

Suppose that $\{a_n\}$ is an arithmetic sequence with

$$a_1 + a_2 + \cdots + a_{100} = 100 \text{ and } a_{101} + a_{102} + \cdots + a_{200} = 200.$$

What is the value of $a_2 - a_1$?

$\text{(A) } 0.0001 \qquad \text{(B) } 0.001 \qquad \text{(C) } 0.01 \qquad \text{(D) } 0.1 \qquad \text{(E) } 1$

Solution 1

We should realize that the two equations are 100 terms apart, so by subtracting the two equations in a form like...

$$(a_{101} - a_1) + (a_{102} - a_2) + \ldots + (a_{200} - a_{100}) = 200 - 100 = 100$$

We can find the value of the common difference every hundred terms. But we forgot that it happens hundred times. So we have to divide the answer by hundred...

$$\frac{100}{100} = 1$$

The answer yields us the common difference of every hundred terms. So you have to simply divide the answer by hundred again to find the common difference between one term, therefore...

$$\frac{1}{100} = \boxed{(\text{C}) .01}$$

Solution 2

Adding the two given equations together gives

$a_1 + a_2 + \ldots + a_{200} = 300.$

Now, let the common difference be d. Notice that $a_2 - a_1 = d$, so we merely need to find d to get the answer. The formula for an arithmetic sum is

$\frac{n}{2}(2a_1 + d(n-1)),$

where a_1 is the first term, n is the number of terms, and d is the common difference. Now we use this formula to find a closed form for the first given equation and the sum of the given equations. For the first equation, we have $n = 100$. Therefore, we have

$50(2a_1 + 99d) = 100,$

or

$2a_1 + 99d = 2. \ *(1)$

For the sum of the equations (shown at the beginning of the solution) we have $n = 200$, so

$100(2a_1 + 199d) = 300$

or

$2a_1 + 199d = 3 \ *(2)$

Now we have a system of equations in terms of a_1 and d. Subtracting (1) from (2) eliminates a_1, yielding $100d = 1$, and $d = a_2 - a_1 = \frac{1}{100} = \boxed{(\text{C}) .01}$.

Solution 3

Subtracting the 2 given equations yields

$$(a_{101} - a_1) + (a_{102} - a_2) + (a_{103} - a_3) + \ldots + (a_{200} - a_{100}) = 100$$

Now express each a_n in terms of first term a_1 and common difference x between consecutive terms

$$((a_1+100x)-(a_1))+((a_1+101x)-(a_1+x))+((a_1+102x)-(a_1+2x))+...+((a_1+199x)-(a_1+99x))=100$$

Simplifying and canceling a_1 and x terms gives

$$100x+100x+100x+...+100x=100$$

$$100x\times 100=100$$

$$100x=1$$

$$x=0.01=\boxed{(\mathrm{C})0.01}$$

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Algebra Problems

2002 AMC 10B Problems/Problem 20

Problem

Let a, b, and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

$\text{(A)}\ 0 \qquad \text{(B)}\ 1 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 8$

Solution

Rearranging, we get $a + 8c = 7b + 4$ and $8a - c = 7 - 4b$

Squaring both, $a^2 + 16ac + 64c^2 = 49b^2 + 56b + 16$ and $64a^2 - 16ac + c^2 = 16b^2 - 56b + 49$ are obtained.

Adding the two equations and dividing by 65 gives $a^2 + c^2 = b^2 + 1$, so $a^2 - b^2 + c^2 = \boxed{\text{(B)} 1}$.

See Also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 12B Problems/Problem 17

The following problem is from both the 2002 AMC 12B #17 and 2002 AMC 10B #21, so both problems redirect to this page.

Problem

Andy’ s lawn has twice as much area as Beth’ s lawn and three times as much area as Carlos’ lawn. Carlos’ lawn mower cuts half as fast as Beth’ s mower and one third as fast as Andy’ s mower. If they all start to mow their lawns at the same time, who will finish first?

$\mathrm{(A)}\ \text{Andy} \qquad\mathrm{(B)}\ \text{Beth} \qquad\mathrm{(C)}\ \text{Carlos} \qquad\mathrm{(D)}\ \text{Andy and Carlos tie for first.} \qquad\mathrm{(E)}\ \text{All three tie.}$

Solution

We say Andy's lawn has an area of x. Beth's lawn thus has an area of $\frac{x}{2}$, and Carlos's lawn has an area of $\frac{x}{3}$.

We say Andy's lawn mower cuts at a speed of y. Carlos's cuts at a speed of $\frac{y}{3}$, and Beth's cuts at a speed $\frac{2y}{3}$.

Each person's lawn is cut at a speed of $\frac{\text{area}}{\text{rate}}$, so Andy's is cut in $\frac{x}{y}$ time, as is Carlos's. Beth's is cut in $\frac{3}{4}\times\frac{x}{y}$, so the first one to finish is $\boxed{\mathrm{(B)}\ \text{Beth}}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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Category: Introductory Algebra Problems

2002 AMC 12B Problems/Problem 20

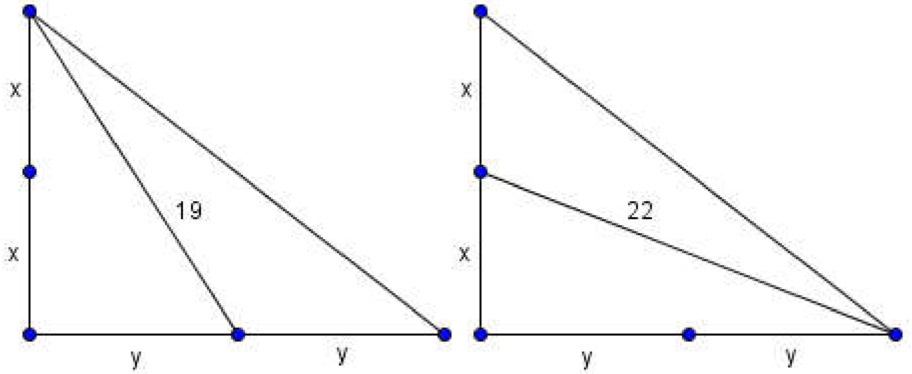
The following problem is from both the 2002 AMC 12B #20 and 2002 AMC 10B #22, so both problems redirect to this page.

Problem

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

$\mathrm{(A) \ } 24 \qquad \mathrm{(B) \ } 26 \qquad \mathrm{(C) \ } 28 \qquad \mathrm{(D) \ } 30 \qquad \mathrm{(E) \ } 32$

Solution



Let $OM = x$, $ON = y$. By the Pythagorean Theorem on $\triangle XON, MOY$ respectively,

$$(2x)^2 + y^2 = 19^2$$
$$x^2 + (2y)^2 = 22^2$$

Summing these gives $5x^2 + 5y^2 = 845 \Longrightarrow x^2 + y^2 = 169$.

By the Pythagorean Theorem again, we have

$$(2x)^2 + (2y)^2 = XY^2 \Longrightarrow XY = \sqrt{4(x^2+y^2)} = \sqrt{4(169)} = \sqrt{676} = \boxed{\mathrm{(B)}\ 26}$$

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 10B Problems/Problem 23

Contents

- 1 Problem 23
- 2 Solution 1
- 3 Solution 2
- 4 Additional Comment
- 5 See also

Problem 23

Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n. Then a_{12} is

$\mathrm{(A) \ } 45 \qquad \mathrm{(B) \ } 56 \qquad \mathrm{(C) \ } 67 \qquad \mathrm{(D) \ } 78 \qquad \mathrm{(E) \ } 89$

Solution 1

First of all, write a_3 and a_4 in terms of a_2.

$$a_3 = a_{1+2} = 1 + a_2 + 2 = a_2 + 3$$
$$a_4 = a_{2+2} = a_2 + a_2 + 4 = 2a_2 + 4$$

a_6 can be represented by a_2 in 2 different ways.

$$a_{2+4} = a_2 + a_4 + 8 = a_2 + 2a_2 + 4 + 8 = 3a_2 + 12$$
$$a_{3+3} = 2a_3 + 9 = 6 + 2a_2 + 9 = 2a_2 + 15$$

Since both are equal to a_6, you can set them equal to each other.

$$3a_2 + 12 = 2a_2 + 15$$
$$a_2 = 3$$

Substitute the value of a_2 back into a_6, and substitute that into a_{12}.

$$a_6 = 2a_2 + 15 = 6 + 15 = 21$$

$$a_{12} = a_{6+6} = a_6 + a_6 + 36 = 21 + 21 + 36 = \boxed{\mathrm{(D) \ } 78}$$

Solution 2

Substituting $n = 1$ into $a_{m+n} = a_m + a_n + mn$: $a_{m+1} = a_m + a_1 + m$. Since $a_1 = 1$, $a_{m+1} = a_m + m + 1$. Therefore, $a_m = a_{m-1} + m, a_{m-1} = a_{m-2} + (m-1), a_{m-2} = a_{m-3} + (m-2)$, and so on until $a_2 = a_1 + 2$. Adding the Left Hand Sides of all of these equations gives $a_m + a_{m-1} + a_{m-2} + a_{m-3} + \cdots + a_2$; adding the Right Hand Sides of these equations gives $(a_{m-1} + a_{m-2} + a_{m-3} + \cdots + a_1) + (m + (m-1) + (m-2) + \cdots + 2)$. These two expressions must be equal; hence $a_m + a_{m-1} + a_{m-2} + a_{m-3} + \cdots + a_2 = (a_{m-1} + a_{m-2} + a_{m-3} + \cdots + a_1) + (m + (m-1) + (m-2) + \cdots + 2$ and $a_m = a_1 + (m + (m-1) + (m-2) + \cdots + 2)$. Substituting $a_1 = 1$: $a_m = 1 + (m + (m-1) + (m-2) + \cdots + 2) = 1 + 2 + 3 + 4 + \cdots + m = \frac{(m+1)(m)}{2}$. Thus we have a general formula for a_m and substituting $m = 12$: $a_{12} = \frac{(13)(12)}{2} = (13)(6) = \boxed{\mathrm{(D) \ } 78}$.

Additional Comment

This is also the formula for the triangular numbers $T_n = 1 + 2 + 3 + ... + n$, as seen in Solution 2

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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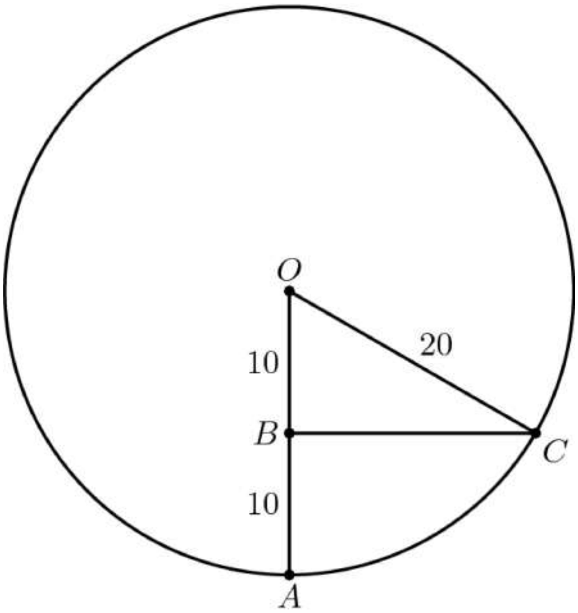
2002 AMC 10B Problems/Problem 24

Problem 24

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

$\mathrm{(A) \ } 5\qquad \mathrm{(B) \ } 6\qquad \mathrm{(C) \ } 7.5\qquad \mathrm{(D) \ } 10\qquad \mathrm{(E) \ } 15$

Solution



We can let this circle represent the ferris wheel with center O, and C represent the desired point 10 feet above the bottom. Draw a diagram like the one above. We find out $\triangle OBC$ is a $30-60-90$ triangle. That means $\angle BOC=60^{\circ}$ and the ferris wheel has made $\frac{60}{360}=\frac{1}{6}$ of a revolution. Therefore, the time it takes to travel that much of a distance is $\frac{1}{6}\text{th}$ of a minute, or 10 seconds. The answer is $\boxed{\mathrm{(D) \ } 10}$. Alternatively, we could also say that $\triangle ABC$ is congruent to $\triangle OBC$ by SAS, so AC is 20, and $\triangle AOC$ is equilateral, and $\angle BOC=60^{\circ}$

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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2002 AMC 10B Problems/Problem 25

Problem

When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

$\mathrm{(A) \ } 4\qquad \mathrm{(B) \ } 5\qquad \mathrm{(C) \ } 6\qquad \mathrm{(D) \ } 7\qquad \mathrm{(E) \ } 8$

Solution

Let x be the sum of the integers and y be the number of elements in the list. Then we get the equations $\frac{x+15}{y+1}=\frac{x}{y}+2$ and $\frac{x+15+1}{y+1+1}=\frac{x+16}{y+2}=\frac{x}{y}+2-1=\frac{x}{y}+1$. With a lot of algebra, the solution is found to be $y=\boxed{\textbf{(A)}\ 4}$.

See also

2002 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2002))	
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