2003 AMC 12A Problems/Problem 1

The following problem is from both the 2003 AMC 12A #1 and 2003 AMC 10A #1, so both problems redirect to this page.

Contents

Problem

What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

$\mathrm{(A) \ } 0\qquad \mathrm{(B) \ } 1\qquad \mathrm{(C) \ } 2\qquad \mathrm{(D) \ } 2003\qquad \mathrm{(E) \ } 4006$

Solution

Solution 1

The first 2003 even counting numbers are $2,4,6,...,4006$.

The first 2003 odd counting numbers are $1,3,5,...,4005$.

Thus, the problem is asking for the value of $(2+4+6+...+4006)-(1+3+5+...+4005)$.

$(2+4+6+...+4006)-(1+3+5+...+4005) = (2-1)+(4-3)+(6-5)+...+(4006-4005)$

$= 1+1+1+...+1 = \boxed{\mathrm{(D)}\ 2003}$

Solution 2

Using the sum of an arithmetic progression formula, we can write this as
$\frac{2003}{2}(2+4006) - \frac{2003}{2}(1+4005) = \frac{2003}{2} \cdot 2 = \boxed{\mathrm{(D)}\ 2003}$.

Solution 3

The formula for the sum of the first n even numbers, is $S_E = n^2 + n$, (E standing for even).

Sum of first n odd numbers, is $S_O = n^2$, (O standing for odd).

Knowing this, plug 2003 for n,

$S_E - S_O = (2003^2 + 2003) - (2003^2) = 2003 \Rightarrow \boxed{\mathrm{(D)}\ 2003}$.

See also

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2003 AMC 12A Problems/Problem 2

The following problem is from both the 2003 AMC 12A #2 and 2003 AMC 10A #2, so both problems redirect to this page.

Problem

Members of the Rockham Soccer League buy socks and T-shirts. Socks cost $4 per pair and each T-shirt costs $5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is $2366, how many members are in the League?

$\mathrm{(A) \ } 77\qquad \mathrm{(B) \ } 91\qquad \mathrm{(C) \ } 143\qquad \mathrm{(D) \ } 182\qquad \mathrm{(E) \ } 286$

Solution

Since T-shirts cost 5 dollars more than a pair of socks, T-shirts cost $5+4=9$ dollars.

Since each member needs 2 pairs of socks and 2 T-shirts, the total cost for 1 member is $2(4+9)=26$ dollars.

Since 2366 dollars was the cost for the club, and 26 was the cost per member, the number of members in the League is $2366\div 26=\boxed{\mathrm{(B)}\ 91}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 12A Problems/Problem 3

The following problem is from both the 2003 AMC 12A #3 and 2003 AMC 10A #3, so both problems redirect to this page.

Problem

A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

$\mathrm{(A)}\ 4.5\% \qquad\mathrm{(B)}\ 9\% \qquad\mathrm{(C)}\ 12\% \qquad\mathrm{(D)}\ 18\% \qquad\mathrm{(E)}\ 24\%$

Solution

The volume of the original box is $15\cdot10\cdot8=1200$

The volume of each cube that is removed is $3\cdot3\cdot3=27$

Since there are 8 corners on the box, 8 cubes are removed.

So the total volume removed is $8\cdot27=216$.

Therefore, the desired percentage is $\frac{216}{1200}\cdot100 = \boxed{\mathrm{(D)}\ 18\%}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Geometry Problems

2003 AMC 12A Problems/Problem 4

The following problem is from both the 2003 AMC 12A #4 and 2003 AMC 10A #4, so both problems redirect to this page.

Contents

Problem

It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to her home along the same route. What is her average speed, in km/hr, for the round trip?

$\mathrm{(A) \ } 3\qquad \mathrm{(B) \ } 3.125\qquad \mathrm{(C) \ } 3.5\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 4.5$

Solution

Solution 1

Since she walked 1 km to school and 1 km back home, her total distance is $1+1=2$ km.

Since she spent 30 minutes walking to school and 10 minutes walking back home, her total time is $30+10=40$ minutes = $\frac{40}{60}=\frac{2}{3}$ hours.

Therefore her average speed in km/hr is $\frac{2}{\frac{2}{3}}=\boxed{\mathrm{(A)}\ 3}$.

Solution 2

The average speed of two speeds that travel the same distance is the harmonic mean of the speeds, or $\frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2xy}{x+y}$ (for speeds x and y). Mary's speed going to school is $2\,\mathrm{km/hr}$, and her speed coming back is $6\,\mathrm{km/hr}$. Plugging the numbers in, we get that the average speed is $\frac{2\times6\times2}{6+2} = \frac{24}{8} = \boxed{\mathrm{(A)}\ 3}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 5

Contents

Problem

Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of $(d-1)(e-1)$?

$\mathrm{(A)}\ -\frac{5}{2} \qquad \mathrm{(B)}\ 0 \qquad \mathrm{(C)}\ 3 \qquad \mathrm{(D)}\ 5 \qquad \mathrm{(E)}\ 6$

Solution

Solution 1

Using factoring:

$2x^2 + 3x - 5 = 0$

$(2x+5)(x-1) = 0$

$x = -\frac{5}{2}$ or $x = 1$

So d and e are $-\frac{5}{2}$ and 1.

Therefore the answer is $\left(-\frac{5}{2} - 1\right)(1-1) = \left(-\frac{7}{2}\right)(0) = \boxed{\mathrm{(B)}\ 0}$

Solution 2

We can use the sum and product of a quadratic:

$$(d-1)(e-1) = de - (d+e) + 1 \implies \text{product} - \text{sum} + 1 \implies \frac{c}{a} - \left(-\frac{b}{a}\right) + 1 \implies \frac{b+c}{a} + 1 = \frac{5}{-5} + 1 = \boxed{\mathrm{(B)}\ 0}$$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Algebra Problems

2003 AMC 12A Problems/Problem 6

The following problem is from both the 2003 AMC 12A #6 and 2003 AMC 10A #6, so both problems redirect to this page.

Problem

Define $x\heartsuit y$ to be $|x-y|$ for all real numbers x and y. Which of the following statements is not true?

$\mathrm{(A)}\ x\heartsuit y=y\heartsuit x$ for all x and y

$\mathrm{(B)}\ 2(x\heartsuit y)=(2x)\heartsuit(2y)$ for all x and y

$\mathrm{(C)}\ x\heartsuit 0=x$ for all x

$\mathrm{(D)}\ x\heartsuit x=0$ for all x

$\mathrm{(E)}\ x\heartsuit y>0$ if $x\neq y$

Solution

Examining statement C:

$x\heartsuit 0=|x-0|=|x|$

$|x|\neq x$ when $x<0$, but statement C says that it does for all x.

Therefore the statement that is not true is $\boxed{\mathrm{(C)}\ x\heartsuit 0=x\ \text{for all}\ x}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 12A Problems/Problem 7

The following problem is from both the 2003 AMC 12A #7 and 2003 AMC 10A #7, so both problems redirect to this page.

Problem

How many non-congruent triangles with perimeter 7 have integer side lengths?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

By the triangle inequality, no side may have a length greater than the semiperimeter, which is $\frac{1}{2}\cdot7=3.5$.

Since all sides must be integers, the largest possible length of a side is 3. Therefore, all such triangles must have all sides of length 1, 2, or 3. Since $2+2+2=6<7$, at least one side must have a length of 3. Thus, the remaining two sides have a combined length of $7-3=4$. So, the remaining sides must be either 3 and 1 or 2 and 2. Therefore, the number of triangles is $\boxed{\textbf{(B)}\ 2}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Geometry Problems

2003 AMC 12A Problems/Problem 8

The following problem is from both the 2003 AMC 12A #8 and 2003 AMC 10A #8, so both problems redirect to this page.

Contents

Problem

What is the probability that a randomly drawn positive factor of 60 is less than 7?

$\mathrm{(A)}\ \frac{1}{10}\qquad\mathrm{(B)}\ \frac{1}{6}\qquad\mathrm{(C)}\ \frac{1}{4}\qquad\mathrm{(D)}\ \frac{1}{3}\qquad\mathrm{(E)}\ \frac{1}{2}$

Solution

Solution 1

For a positive number n which is not a perfect square, exactly half of the positive factors will be less than $\sqrt{n}$.

Since 60 is not a perfect square, half of the positive factors of 60 will be less than $\sqrt{60}\approx 7.746$.

Clearly, there are no positive factors of 60 between 7 and $\sqrt{60}$.

Therefore half of the positive factors will be less than 7.

So the answer is $\boxed{\mathrm{(E)}\ \frac{1}{2}}$.

Solution 2

Testing all numbers less than 7, numbers $1, 2, 3, 4, 5$, and 6 divide 60. The prime factorization of 60 is $2^2\cdot 3\cdot 5$. Using the formula for the number of divisors, the total number of divisors of 60 is $(3)(2)(2)=12$. Therefore, our desired probability is $\frac{6}{12}=\boxed{\mathrm{(E)}\ \frac{1}{2}}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 9

Contents

Problem

Simplify $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}$.

$\mathrm{(A)}\ \sqrt{x}\qquad\mathrm{(B)}\ \sqrt[3]{x^2}\qquad\mathrm{(C)}\ \sqrt[27]{x^2}\qquad\mathrm{(D)}\ \sqrt[54]{x}\qquad\mathrm{(E)}\ \sqrt[81]{x^{80}}$

Solution 1

$$\sqrt[3]{x\sqrt{x}} = \sqrt[3]{(\sqrt{x})^2\cdot\sqrt{x}} = \sqrt[3]{(\sqrt{x})^3} = \sqrt{x}.$$

Therefore:

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}} = \sqrt[3]{x\sqrt[3]{x\sqrt{x}}} = \sqrt[3]{x\sqrt{x}} = \sqrt{x} \Rightarrow \boxed{\mathrm{(A)}\ \sqrt{x}}$$

Solution 2

We know that $\sqrt[3]{x\sqrt{x}} = \sqrt[3]{x\cdot(x^{\frac{1}{2}})} = \sqrt[3]{x^{\frac{3}{2}}} = x^{\frac{1}{2}}$ We plug this into $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}$ and get $\sqrt[3]{x\sqrt[3]{x\cdot(x^{\frac{1}{2}})}}$. Again, we substitute and get $\sqrt[3]{x\cdot(x^{\frac{1}{2}})}$. We substitute one more time and get $x^{\frac{1}{2}} = \boxed{(A)\sqrt{x}}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Algebra Problems

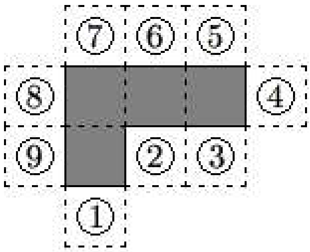
2003 AMC 12A Problems/Problem 13

The following problem is from both the 2003 AMC 12A #13 and 2003 AMC 10A #10, so both problems redirect to this page.

Contents

Problem

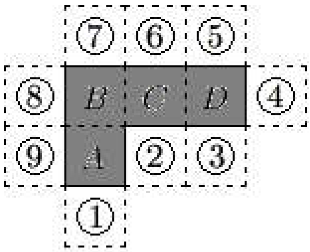
The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



$\mathrm{(A)}\ 2\qquad\mathrm{(B)}\ 3\qquad\mathrm{(C)}\ 4\qquad\mathrm{(D)}\ 5\qquad\mathrm{(E)}\ 6$

Solution

Solution 1



Let the squares be labeled A, B, C, and D.

When the polygon is folded, the "right" edge of square A becomes adjacent to the "bottom edge" of square C, and the "bottom" edge of square A becomes adjacent to the "bottom" edge of square D.

So, any "new" square that is attatched to those edges will prevent the polygon from becoming a cube with one face missing.

Therefore, squares 1, 2, and 3 will prevent the polygon from becoming a cube with one face missing.

Squares 4, 5, 6, 7, 8, and 9 will allow the polygon to become a cube with one face missing when folded.

Thus the answer is $\boxed{\mathrm{(E)}\ 6}$.

Solution 2

Another way to think of it is that a cube missing one face has 5 of its 6 faces. Since the shape has 4 faces already, we need another face. The only way to add another face is if the added square does not overlap any of the others. $1, 2,$ and 3 overlap, while squares 4 to 9 do not. The answer is $\boxed{\mathrm{(E)}\ 6}$

See Also

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2003 AMC 12A Problems/Problem 5

The following problem is from both the 2003 AMC 12A #5 and 2003 AMC 10A #11, so both problems redirect to this page.

Problem

The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A+M+C$?

$\mathrm{(A) \ } 10\qquad \mathrm{(B) \ } 11\qquad \mathrm{(C) \ } 12\qquad \mathrm{(D) \ } 13\qquad \mathrm{(E) \ } 14$

Solution

$AMC10+AMC12=123422$

$AMC00+AMC00=123400$

$AMC+AMC=1234$

$2\cdot AMC=1234$

$AMC=\frac{1234}{2}=617$

Since A, M, and C are digits, $A=6$, $M=1$, $C=7$.

Therefore, $A+M+C = 6+1+7 = \boxed{\mathrm{(E)}\ 14}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 12

Problem

A point (x, y) is randomly picked from inside the rectangle with vertices $(0,0)$, $(4,0)$, $(4,1)$, and $(0,1)$. What is the probability that $x < y$?

$\mathrm{(A)}\ \frac{1}{8} \qquad \mathrm{(B)}\ \frac{1}{4} \qquad \mathrm{(C)}\ \frac{3}{8} \qquad \mathrm{(D)}\ \frac{1}{2} \qquad \mathrm{(E)}\ \frac{3}{4}$

Solution

The rectangle has a width of 4 and a height of 1.

The area of this rectangle is $4 \cdot 1 = 4$.

The line $x = y$ intersects the rectangle at $(0,0)$ and $(1,1)$.

The area which $x < y$ is the right isosceles triangle with side length 1 that has vertices at $(0,0)$, $(1,1)$, and $(0,1)$.

The area of this triangle is $\frac{1}{2} \cdot 1^2 = \frac{1}{2}$

Therefore, the probability that $x < y$ is $\frac{\frac{1}{2}}{4} = \frac{1}{8} \Rightarrow \boxed{\mathrm{(A)}\ \frac{1}{8}}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 13

Problem

The sum of three numbers is 20. The first is four times the sum of the other two. The second is seven times the third. What is the product of all three?

$\mathrm{(A) \ } 28\qquad \mathrm{(B) \ } 40\qquad \mathrm{(C) \ } 100\qquad \mathrm{(D) \ } 400\qquad \mathrm{(E) \ } 800$

Contents

Solution

Solution 1

Let the numbers be x, y, and z in that order. The given tells us that

$$\begin{aligned} y &= 7z \\ x &= 4(y+z)=4(7z+z)=4(8z)=32z \\ x+y+z &= 32z+7z+z=40z=20 \\ z &= \frac{20}{40}=\frac{1}{2} \\ y &= 7z=7\cdot\frac{1}{2}=\frac{7}{2} \\ x &= 32z=32\cdot\frac{1}{2}=16 \end{aligned}$$

Therefore, the product of all three numbers is $xyz=16\cdot\frac{7}{2}\cdot\frac{1}{2}=28 \Rightarrow \boxed{\mathrm{(A)}\ 28}$.

Solution 2

Alternatively, we can set up the system in matrix form:

$$\begin{aligned} 1x+1y+1z &= 20 \\ 1x-4y-4z &= 0 \\ 0x+1y-7z &= 0 \end{aligned}$$

Or, in matrix form $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & -4 \\ 0 & 1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$

To solve this matrix equation, we can rearrange it thus:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & -4 \\ 0 & 1 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

Solving this matrix equation by using inverse matrices and matrix multiplication yields

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{7}{2} \\ 16 \end{bmatrix}$$

Which means that $x = \frac{1}{2}$, $y = \frac{7}{2}$, and $z = 16$. Therefore, $xyz = \frac{1}{2} \cdot \frac{7}{2} \cdot 16 = 28 \Rightarrow \boxed{\text{(A) } 28}$

See also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 14

Problem

Let n be the largest integer that is the product of exactly 3 distinct prime numbers d, e, and $10d+e$, where d and e are single digits. What is the sum of the digits of n?

$\mathrm{(A) \ } 12\qquad \mathrm{(B) \ } 15\qquad \mathrm{(C) \ } 18\qquad \mathrm{(D) \ } 21\qquad \mathrm{(E) \ } 24$

Solution

Since d is a single digit prime number, the set of possible values of d is $\{2,3,5,7\}$.

Since e is a single digit prime number and is the units digit of the prime number $10d+e$, the set of possible values of e is $\{3,7\}$.

Using these values for d and e, the set of possible values of $10d+e$ is $\{23,27,33,37,53,57,73,77\}$

Out of this set, the prime values are $\{23,37,53,73\}$

Therefore the possible values of n are:

$2\cdot3\cdot23=138$

$3\cdot7\cdot37=777$

$5\cdot3\cdot53=795$

$7\cdot3\cdot73=1533$

The largest possible value of n is 1533.

So, the sum of the digits of n is $1+5+3+3=12 \Rightarrow \boxed{\mathrm{(A)}\ 12}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Number Theory Problems

2003 AMC 10A Problems/Problem 15

Contents

Problem

What is the probability that an integer in the set $\{1, 2, 3, ..., 100\}$ is divisible by 2 and not divisible by 3?

$\mathrm{(A)}\ \frac{1}{6} \qquad \mathrm{(B)}\ \frac{33}{100} \qquad \mathrm{(C)}\ \frac{17}{50} \qquad \mathrm{(D)}\ \frac{1}{2} \qquad \mathrm{(E)}\ \frac{18}{25}$

Solution

There are 100 integers in the set.

Since every 2^{nd} integer is divisible by 2, there are $\lfloor \frac{100}{2} \rfloor = 50$ integers divisible by 2 in the set.

To be divisible by both 2 and 3, a number must be divisible by $\mathrm{lcm}(2,3) = 6$.

Since every 6^{th} integer is divisible by 6, there are $\lfloor \frac{100}{6} \rfloor = 16$ integers divisible by both 2 and 3 in the set.

So there are $50 - 16 = 34$ integers in this set that are divisible by 2 and not divisible by 3.

Therefore, the desired probability is $\frac{34}{100} = \frac{17}{50} \Rightarrow \boxed{\mathrm{(C)}\ \frac{17}{50}}$

Controversy

Due to the wording of the question, it may be taken as "Find the probability that an integer in said set is divisible by 2 and not 3 EXISTS". One example would be 2, which is not a multiple of 3, thus the probability is 1.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 16

Problem

What is the units digit of 13^{2003}?

$\mathrm{(A)}\ 1\qquad \mathrm{(B)}\ 3\qquad \mathrm{(C)}\ 7\qquad \mathrm{(D)}\ 8\qquad \mathrm{(E)}\ 9$

Solution

$13^{2003} \equiv 3^{2003} \pmod{10}$

Since $3^4 = 81 \equiv 1 \pmod{10}$:

$3^{2003} = (3^4)^{500} \cdot 3^3 \equiv 1^{500} \cdot 27 \equiv 7 \pmod{10}$

Therefore, the units digit is $7 \Rightarrow \boxed{\mathrm{(C)}\ 7}$

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Number Theory Problems

2003 AMC 10A Problems/Problem 17

Problem

The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

$\mathrm{(A) \ } \frac{3\sqrt{2}}{\pi} \qquad \mathrm{(B) \ } \frac{3\sqrt{3}}{\pi} \qquad \mathrm{(C) \ } \sqrt{3} \qquad \mathrm{(D) \ } \frac{6}{\pi} \qquad \mathrm{(E) \ } \sqrt{3}\pi$

Solution

Let s be the length of a side of the equilateral triangle and let r be the radius of the circle.

In a circle with a radius r the side of an inscribed equilateral triangle is $r\sqrt{3}$.

So $s=r\sqrt{3}$.

The perimeter of the triangle is $3s=3r\sqrt{3}$

The area of the circle is πr^{2}

So: $\pi r^{2} = 3r\sqrt{3}$

$\pi r = 3\sqrt{3}$

$r = \frac{3\sqrt{3}}{\pi} \Rightarrow \boxed{\mathrm{(B)}\ \frac{3\sqrt{3}}{\pi}}$

See Also

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Category: Introductory Geometry Problems

2003 AMC 10A Problems/Problem 18

Contents

Problem

What is the sum of the reciprocals of the roots of the equation $\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$?

$\text{(A)}\ -\frac{2004}{2003} \qquad \text{(B)}\ -1 \qquad \text{(C)}\ \frac{2003}{2004} \qquad \text{(D)}\ 1 \qquad \text{(E)}\ \frac{2004}{2003}$

Solution 1

Multiplying both sides by x:

$$\frac{2003}{2004}x^2 + 1x + 1 = 0$$

Let the roots be a and b.

The problem is asking for $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

By Vieta's formulas:

$$a+b = (-1)^1 \frac{1}{\frac{2003}{2004}} = -\frac{2004}{2003}$$

$$ab = (-1)^2 \frac{1}{\frac{2003}{2004}} = \frac{2004}{2003}$$

So the answer is $\frac{a+b}{ab} = \frac{-\frac{2004}{2003}}{\frac{2004}{2003}} = -1 \Rightarrow \boxed{\text{(B)}\ -1}$.

Solution 2

Dividing both sides by x,

$$\frac{2003}{2004} + \frac{1}{x} + \frac{1}{x^2} = 0,$$

we see by Vieta's formulas that the sum of the roots is $-1 \Rightarrow \boxed{\text{(B)}\ -1}$.

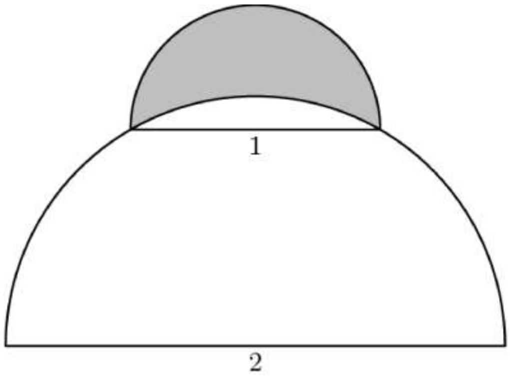
See Also

2003 AMC 12A Problems/Problem 15

The following problem is from both the 2003 AMC 12A #15 and 2003 AMC 10A #19, so both problems redirect to this page.

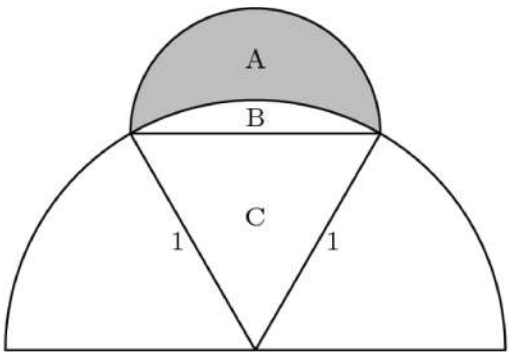
Problem

A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. Determine the area of this lune.



$\mathrm{(A)}\ \frac{1}{6}\pi-\frac{\sqrt{3}}{4} \qquad \mathrm{(B)}\ \frac{\sqrt{3}}{4}-\frac{1}{12}\pi \qquad \mathrm{(C)}\ \frac{\sqrt{3}}{4}-\frac{1}{24}\pi \qquad \mathrm{(D)}\ \frac{\sqrt{3}}{4}+\frac{1}{24}\pi \qquad \mathrm{(E)}\ \frac{\sqrt{3}}{4}+\frac{1}{12}\pi$

Solution



Let $[X]$ denote the area of region X in the figure above.

The shaded area $[A]$ is equal to the area of the smaller semicircle $[A+B]$ minus the area of a sector of the larger circle $[B+C]$ plus the area of a triangle formed by two radii of the larger semicircle and the diameter of the smaller semicircle $[C]$.

The area of the smaller semicircle is $[A+B]=\frac{1}{2}\pi\cdot(\frac{1}{2})^2=\frac{1}{8}\pi.$

Since the radius of the larger semicircle is equal to the diameter of the smaller semicircle, the triangle is an equilateral triangle and the sector measures 60°.

The area of the 60° sector of the larger semicircle is $[B+C]=\frac{60}{360}\pi\cdot(\frac{2}{2})^2=\frac{1}{6}\pi.$

The area of the triangle is $[C]=\frac{1^2\sqrt{3}}{4}=\frac{\sqrt{3}}{4}.$

So the shaded area is

$$[A]=[A+B]-[B+C]+[C]=\left(\frac{1}{8}\pi\right)-\left(\frac{1}{6}\pi\right)+\left(\frac{\sqrt{3}}{4}\right)=\boxed{\text{(C) }\frac{\sqrt{3}}{4}-\frac{1}{24}\pi}.$$

See Also

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Categories: Introductory Geometry Problems | Area Problems

2003 AMC 10A Problems/Problem 20

Problem 20

A base-10 three digit number n is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of n are both three-digit numerals?

$\mathrm{(A) \ } 0.3\qquad \mathrm{(B) \ } 0.4\qquad \mathrm{(C) \ } 0.5\qquad \mathrm{(D) \ } 0.6\qquad \mathrm{(E) \ } 0.7$

Solution

To be a three digit number in base-10:

$$10^{2} \leq n \leq 10^{3}-1$$

$$100 \leq n \leq 999$$

Thus there are 900 three-digit numbers in base-10

To be a three-digit number in base-9:

$$9^{2} \leq n \leq 9^{3}-1$$

$$81 \leq n \leq 728$$

To be a three-digit number in base-11:

$$11^{2} \leq n \leq 11^{3}-1$$

$$121 \leq n \leq 1330$$

So, $121 \leq n \leq 728$

Thus, there are 608 base-10 three-digit numbers that are three digit numbers in base-9 and base-11.

Therefore the desired probability is $\frac{608}{900} \approx 0.7 \Rightarrow \boxed{\mathrm{(E)}\ 0.7}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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Category: Introductory Number Theory Problems

2003 AMC 10A Problems/Problem 21

Contents

Problem

Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

$\mathrm{(A)}\ 22\qquad \mathrm{(B)}\ 25\qquad \mathrm{(C)}\ 27\qquad \mathrm{(D)}\ 28\qquad \mathrm{(E)}\ 729$

Solution 1

Let the ordered triplet (x,y,z) represent the assortment of x chocolate chip cookies, y oatmeal cookies, and z peanut butter cookies.

Using casework:

Pat selects 0 chocolate chip cookies:

Pat needs to select $6-0=6$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(0,6,0);(0,5,1);(0,4,2);(0,3,3);(0,2,4);(0,1,5);(0,0,6)\}\rightarrow 7$ assortments.

Pat selects 1 chocolate chip cookie:

Pat needs to select $6-1=5$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(1,5,0);(1,4,1);(1,3,2);(1,2,3);(1,1,4);(1,0,5)\}\rightarrow 6$ assortments.

Pat selects 2 chocolate chip cookies:

Pat needs to select $6-2=4$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(2,4,0);(2,3,1);(2,2,2);(2,1,3);(2,0,4)\}\rightarrow 5$ assortments.

Pat selects 3 chocolate chip cookies:

Pat needs to select $6-3=3$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(3,3,0);(3,2,1);(3,1,2);(3,0,3)\}\rightarrow 4$ assortments.

Pat selects 4 chocolate chip cookies:

Pat needs to select $6-4=2$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(4,2,0);(4,1,1);(4,0,2)\}\rightarrow 3$ assortments.

Pat selects 5 chocolate chip cookies:

Pat needs to select $6-5=1$ more cookies that are either oatmeal or peanut butter.

The assortments are: $\{(5,1,0);(5,0,1)\}\rightarrow 2$ assortments.

Pat selects 6 chocolate chip cookies:

Pat needs to select $6-6=0$ more cookies that are either oatmeal or peanut butter.

The only assortment is: $\{(6,0,0)\}\rightarrow 1$ assortment.

The total number of assortments of cookies that can be collected is
$7+6+5+4+3+2+1=28 \Rightarrow \boxed{\text{(D) } 28}$

Solution 2

It is given that it is possible to select at least 6 of each. Therefore, we can make a bijection to the number of ways to divide the six choices into three categories, since it is assumed that their order is unimportant. Using the ball and urns/sticks and stones/stars and bars formula, the number of ways to do this is
$\binom{8}{2} = 28 \Rightarrow \boxed{\text{(D) } 28}$

Solution 3

Suppose the six cookies to be chosen are the stars, as we attempt to implement stars and bars. We take two dividers, and place them between the cookies, such that the six cookies are split into 3 groups, where the groups are the number of chocolate chip, oatmeal, and peanut butter cookies, and each group can have 0. First, assume that the two dividers cannot go in between the same two cookies. By stars and bars, there are $\binom{7}{2}$ ways to make the groups.

Finally, since the two dividers can be together, we must add those cases where the two dividers are in the same space between cookies. There are 7 spaces, and hence 7 cases.

Our final answer is $21+7=\boxed{\text{(D) } 28}$

See also

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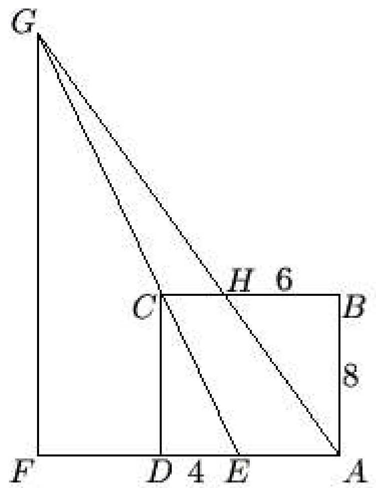
Category: Introductory Combinatorics Problems

2003 AMC 10A Problems/Problem 22

Contents

Problem

In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $GF \perp AF$. Find the length of GF.



$\text{(A) } 16 \qquad \text{(B) } 20 \qquad \text{(C) } 24 \qquad \text{(D) } 28 \qquad \text{(E) } 30$

Solution

Solution 1

$\angle GHC = \angle AHB$ (Opposite angles are equal).

$\angle F = \angle B$ (Both are 90 degrees).

$\angle BHA = \angle HAD$ (Alt. Interior Angles are congruent).

Therefore $\triangle GFA$ and $\triangle ABH$ are similar. $\triangle GCH$ and $\triangle GEA$ are also similar.

DA is 9, therefore EA must equal 5. Similarly, CH must equal 3.

Because GCH and GEA are similar, the ratio of $CH\ =\ 3$ and $EA\ =\ 5$, must also hold true for GH and HA. $\frac{GH}{GA} = \frac{3}{5}$, so HA is $\frac{2}{5}$ of GA. By Pythagorean theorem,
$(HA)^2\ =\ (HB)^2\ +\ (BA)^2 \ldots HA = 10.$

$$HA\ =\ 10 = \frac{2}{5} * (GA).$$

$$GA\ =\ 25.$$

So $\frac{GA}{HA} = \frac{GF}{BA}$.

$\frac{25}{10} = \frac{GF}{8}$.

Therefore $GF = \boxed{\text{(B) } 20}$.

Solution 2

Since $ABCD$ is a rectangle, $CD = AB = 8$.

Since $ABCD$ is a rectangle and $GF \perp AF$, $\angle GFE = \angle CDE = \angle ABC = 90^\circ$.

Since $ABCD$ is a rectangle, $AD || BC$.

So, AH is a transversal, and $\angle GAF = \angle AHB$.

This is sufficient to prove that $GFE \approx CDE$ and $GFA \approx ABH$.

Using ratios:

$$\frac{GF}{FE} = \frac{CD}{DE}$$

$$\frac{GF}{FD+4} = \frac{8}{4} = 2$$

$$GF = 2 \cdot (FD+4) = 2 \cdot FD + 8$$

$$\frac{GF}{FA} = \frac{AB}{BH}$$

$$\frac{GF}{FD+9} = \frac{8}{6} = \frac{4}{3}$$

$$GF = \frac{4}{3} \cdot (FD+9) = \frac{4}{3} \cdot FD + 12$$

Since GF can't have 2 different lengths, both expressions for GF must be equal.

$$2 \cdot FD + 8 = \frac{4}{3} \cdot FD + 12$$

$$\frac{2}{3} \cdot FD = 4$$

$$FD = 6$$

$$GF = 2 \cdot FD + 8 = 2 \cdot 6 + 8 = \boxed{\text{(B) } 20}$$

Solution 3

Since $ABCD$ is a rectangle, $CH = 3$, $EA = 5$, and $CD = 8$. From the Pythagorean Theorem, $CE^2 = CD^2 + DE^2 = 80 \Rightarrow CE = 4\sqrt{5}$.

Lemma

Statement: $GCH \approx GEA$

Proof: $\angle CGH = \angle EGA$, obviously.

$$\begin{aligned}\angle HCE &= 180^\circ - \angle CHG \\ \angle DCE &= \angle CHG - 90^\circ \\ \angle CEED &= 180 - \angle CHG \\ \angle GEA &= \angle GCH\end{aligned}$$

Since two angles of the triangles are equal, the third angles must equal each other. Therefore, the triangles are similar.

Let $GC = x$.

$$\begin{aligned}\frac{x}{3} &= \frac{x+4\sqrt{5}}{5} \\ 5x &= 3x + 12\sqrt{5} \\ 2x &= 12\sqrt{5} \\ x &= 6\sqrt{5}\end{aligned}$$

Also, $\triangle GFE \approx \triangle CDE$, therefore

$$\frac{8}{4\sqrt{5}} = \frac{GF}{10\sqrt{5}}$$

We can multiply both sides by $\sqrt{5}$ to get that GF is twice of 10, or $\boxed{\text{(B) } 20}$

Solution 4

We extend BC such that it intersects GF at X. Since $ABCD$ is a rectangle, it follows that $CD = 8$, therefore, $XF = 8$. Let $GX = y$. From the similarity of triangles GCH and GEA, we have the ratio $3:5$ (as $CH = 9 - 6 = 3$, and $EA = 9 - 4 = 5$). GX and GF are the altitudes of GCH and GEA, respectively. Thus, $y : y + 8 = 3 : 5$, from which we have $y = 12$, thus $GF = y + 8 = 12 + 8 = \boxed{\text{(B) } 20}$

Solution 5

Since $GF \perp AF$ and $AF \perp CD$, we have $GF \parallel CD \parallel AB$. Thus, $\triangle CDE \sim GFE$. Suppose $GF = x$ and $FD = y$. Thus, we have $\frac{x}{8} = \frac{y+4}{4}$. Additionally, now note that $\triangle GAF \sim AHB$, which is pretty obvious from insight, but can be proven by AA with extending BH to meet GF. From this new pair of similar triangles, we have $\frac{x}{8} = \frac{y+9}{6}$. Therefore, we have by combining those two equations,

$$\frac{y+9}{6} = \frac{y+4}{4}.$$

Solving, we have $y = 6$, and therefore $x = \boxed{\text{(B) } 20}$

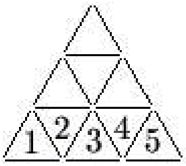
See Also

2003 AMC 10A Problems/Problem 23

Contents

Problem

A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles?



$\mathrm{(A)}\ 1,004,004 \qquad \mathrm{(B)}\ 1,005,006 \qquad \mathrm{(C)}\ 1,507,509 \qquad \mathrm{(D)}\ 3,015,018 \qquad \mathrm{(E)}\ 6,021,018$

Solution

Solution 1

There are $1+3+5+...+2003=1002^{2}=1004004$ small equilateral triangles.

Each small equilateral triangle needs 3 toothpicks to make it.

But, each toothpick that isn't one of the $1002\cdot3=3006$ toothpicks on the outside of the large equilateral triangle is a side for 2 small equilateral triangles.

So, the number of toothpicks on the inside of the large equilateral triangle is
$$\frac{10040004\cdot3-3006}{2}=1504503$$

Therefore the total number of toothpicks is $1504503+3006=\boxed{\mathrm{(C)}\ 1,507,509}$

Solution 2

We see that the bottom row of 2003 small triangles is formed from 1002 upward-facing triangles and 1001 downward-facing triangles. Since each upward-facing triangle uses three distinct toothpicks, and since the total number of upward-facing triangles is $1002+1001+...+1=\frac{1003\cdot1002}{2}=502503$, we have that the total number of toothpicks is $3\cdot502503=\boxed{\mathrm{(C)}\ 1,507,509}$

Solution 3

Experimenting a bit we find that the number of toothpicks needs for a triangle with 1, 2 and 3 rows is $1\cdot3$, $3\cdot3$ and $6\cdot3$ respectively. Since 1, 3 and 6 are triangular numbers we know that depending on how many rows there are in the triangle, the number we multiply by 3 to find total no.toothpicks is the corresponding triangular number. Since the triangle in question has $2n-1=2003 \implies n=1002$ rows, we can use $\frac{n(n+1)}{2}$ to find the triangular number for that row and multiply by 3, hence finding the total no. toothpicks; this is just $\frac{3 \cdot 1002 \cdot 1003}{2} = 3 \cdot 501 \cdot 1003 = \boxed{\mathrm{(C)}\ 1,507,509}$.

See Also

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Category: Introductory Combinatorics Problems

2003 AMC 12A Problems/Problem 12

The following problem is from both the 2003 AMC 12A #12 and 2003 AMC 10A #24, so both problems redirect to this page.

Problem

Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

$\mathrm{(A) \ } 8\qquad \mathrm{(B) \ } 9\qquad \mathrm{(C) \ } 10\qquad \mathrm{(D) \ } 11\qquad \mathrm{(E) \ } 12$

Solution

Let and B_j designate the red card numbered i and the blue card numbered j, respectively.

B_5 is the only blue card that R_5 evenly divides, so R_5 must be at one end of the stack and B_5 must be the card next to it.

R_1 is the only other red card that evenly divides B_5, so R_1 must be the other card next to B_5.

B_4 is the only blue card that R_4 evenly divides, so R_4 must be at one end of the stack and B_4 must be the card next to it.

R_2 is the only other red card that evenly divides B_4, so R_2 must be the other card next to B_4.

R_2 doesn't evenly divide B_3, so B_3 must be next to R_1, B_6 must be next to R_2, and R_3 must be in the middle.

This yields the following arrangement from top to bottom: $\{R_5,B_5,R_1,B_3,R_3,B_6,R_2,B_4,R_4\}$

Therefore, the sum of the numbers on the middle three cards is $3+3+6=\boxed{\mathrm{(E)}\ 12}$.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10A Problems/Problem 25

Contents

Problem

Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q+r$ divisible by 11?

$\mathrm{(A) \ } 8180\qquad \mathrm{(B) \ } 8181\qquad \mathrm{(C) \ } 8182\qquad \mathrm{(D) \ } 9000\qquad \mathrm{(E) \ } 9090$

Solution

Solution 1

When a 5-digit number is divided by 100, the first 3 digits become the quotient, q, and the last 2 digits become the remainder, r.

Therefore, q can be any integer from 100 to 999 inclusive, and r can be any integer from 0 to 99 inclusive.

For each of the $9\cdot10\cdot10=900$ possible values of q, there are at least $\left\lfloor\frac{100}{11}\right\rfloor=9$ possible values of r such that $q+r\equiv 0\pmod{11}$.

Since there is 1 "extra" possible value of r that is congruent to $0\pmod{11}$, each of the $\left\lfloor\frac{900}{11}\right\rfloor=81$ values of q that are congruent to $0\pmod{11}$ have 1 more possible value of r such that $q+r\equiv 0\pmod{11}$.

Therefore, the number of possible values of n such that $q+r\equiv 0\pmod{11}$ is $900\cdot9+81\cdot1=8181\Rightarrow B$.

Solution 2

Let n equal $\overline{abcde}$, where a through e are digits. Therefore,

$$q=\overline{abc}=100a+10b+c$$

$$r=\overline{de}=10d+e$$

We now take $q+r \bmod 11$:

$$q+r=100a+10b+c+10d+e\equiv a-b+c-d+e\equiv 0 \bmod 11$$

The divisor trick for 11 is as follows:

"Let $n=\overline{a_1a_2a_3\cdots a_x}$ be an x digit integer. If $a_1-a_2+a_3-\cdots+(-1)^{x-1}a_x$ is divisible by 11, then n is also divisible by 11."

Therefore, the five digit number n is divisible by 11. The 5-digit multiples of 11 range from $910\cdot 11$ to $9090\cdot 11$. There are $8181\Rightarrow \mathrm{(B)}$ divisors of 11 between those inclusive.

Solution 3

Since q is a quotient and r is a remainder when n is divided by 100. So we have $n = 100q + r$. Since we are counting choices where $q + r$ is divisible by 11, we have $n = 99q + q + r = 99q + 11k$ for some k. This means that n is the sum of two multiples of 11 and would thus itself be a multiple of 11. Then we can count all the four digit multiples of 11 as in Solution 2. (This solution is essentially the same as Solution 2, but it does not necessarily involve mods and so could potentially be faster.)

Notes

The part labeled "divisor trick" actually follows from the same observation we made in the previous step: $10 \equiv \underline{(-1)} \pmod{11}$, therefore $10^{2k} \equiv 1$ and $10^{2k+1} \equiv (-1)$ for all k. For a $5-$digit number $\overline{abcde}$ we get $\overline{abcde} \equiv a \cdot 1 + b \cdot (-1) + c \cdot 1 + d \cdot (-1) + e \cdot 1 = a - b + c - d + e$, as claimed.

Also note that in the "divisor trick" we actually want to assign the signs backwards - if we make sure that the last sign is a $+$, the result will have the same remainder modulo 11 as the original number.

See Also

2003 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
Preceded by Problem 24	Followed by Last Question
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