2003 AMC 12B Problems/Problem 1

The following problem is from both the 2003 AMC 12B #1 and 2003 AMC 10B #1, so both problems redirect to this page.

Problem

Which of the following is the same as

$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}?$$

$(\mathrm{A})\ -1 \qquad (\mathrm{B})\ -\frac{2}{3} \qquad (\mathrm{C})\ \frac{2}{3} \qquad (\mathrm{D})\ 1 \qquad (\mathrm{E})\ \frac{14}{3}$

Solution

The numbers in the numerator and denominator can be grouped like this:

$$\begin{aligned} 2+(-4+6)+(-8+10)+(-12+14) &= 2*4 \\ 3+(-6+9)+(-12+15)+(-18+21) &= 3*4 \\ \frac{2*4}{3*4} &= \frac{2}{3} \Rightarrow (\mathrm{C}) \end{aligned}$$

Alternatively, notice that each term in the numerator is $\frac{2}{3}$ of a term in the denominator, so the quotient has to be $\frac{2}{3}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 2

The following problem is from both the 2003 AMC 12B #2 and 2003 AMC 10B #2, so both problems redirect to this page.

Problem

Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs $\$1$ more than a pink pill, and Al's pills cost a total of $\$546$ for the two weeks. How much does one green pill cost?

$\textbf{(A)}\ \$7 \qquad\textbf{(B)}\ \$14 \qquad\textbf{(C)}\ \$19 \qquad\textbf{(D)}\ \$20 \qquad\textbf{(E)}\ \$39$

Solution

Because there are 14 days in two weeks, Al spends $546/14 = 39$ dollars per day for the cost of a green pill and a pink pill. If the green pill costs x dollars and the pink pill $x-1$ dollars, the sum of the two costs $2x-1$ should equal 39 dollars. Then the cost of the green pill x is $\boxed{\textbf{(D)}\ \$20}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 3

Problem

The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd counting numbers. What is the smallest of the even integers?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 14$

Solution

It is a well-known fact that the sum of the first n odd numbers is n^2. This makes the sum of the first 8 odd numbers equal to 64.

Let n be equal to the smallest of the 5 even integers. Then $(n+2)$ is the next highest, $(n+4)$ even higher, and so on.

This sets up the equation $n+(n+2)+(n+4)+(n+6)+(n+8)+4=64$

Now we solve:

$$5n+24=645n=40n=8$$

Thus, the smallest integer is $\boxed{\textbf{(B)}\ 8}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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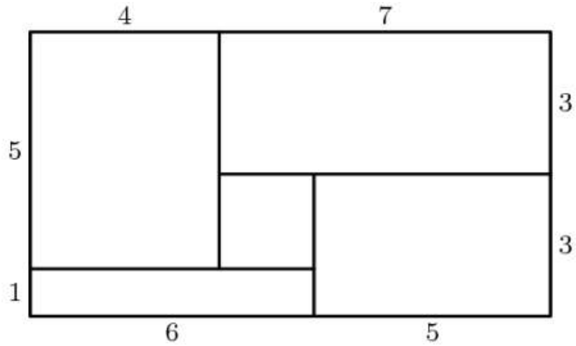
2003 AMC 10B Problems/Problem 4

The following problem is from both the 2003 AMC 12B #3 and 2003 AMC 10B #4, so both problems redirect to this page.

Problem

Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region. Asters cost $\$1$ each, begonias $\$1.50$ each, cannas $\$2$ each, dahlias $\$2.50$ each, and Easter lilies $\$3$ each. What is the least possible cost, in dollars, for her garden?



$\mathrm{(A) \ } 108\qquad \mathrm{(B) \ } 115\qquad \mathrm{(C) \ } 132\qquad \mathrm{(D) \ } 144\qquad \mathrm{(E) \ } 156$

Solution

The areas of the five regions from greatest to least are $21, 20, 15, 6$ and 4.

If we want to minimize the cost, we want to maximize the area of the cheapest flower and minimize the area of the most expensive flower. Doing this, the cost is $1\cdot21 + 1.50\cdot20 + 2\cdot15 + 2.50\cdot6 + 3\cdot4$, which simplifies to $\$108$. Therefore the answer is $\boxed{\mathrm{(A)}\ 108}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 5

The following problem is from both the 2003 AMC 12B #4 and 2003 AMC 10B #5, so both problems redirect to this page.

Problem

Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. He walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow the lawn.

$\textbf{(A)}\ 0.75 \qquad \textbf{(B)}\ 0.8 \qquad \textbf{(C)}\ 1.35 \qquad \textbf{(D)}\ 1.5 \qquad \textbf{(E)}\ 3$

Solution

Since the swath Moe actually mows is 24 inches, or 2 feet wide, he mows 10000 square feet in one hour. His lawn has an area of 13500, so it will take Moe 1.35 hours to finish mowing the lawn. Thus the answer is $\boxed{\textbf{(C)}\ 1.35}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 6

The following problem is from both the 2003 AMC 12B #5 and 2003 AMC 10B #6, so both problems redirect to this page.

Problem

Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is $4:3$. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 20.5 \qquad \textbf{(C)}\ 21 \qquad \textbf{(D)}\ 21.5 \qquad \textbf{(E)}\ 22$

Solution

If you divide the television screen into two right triangles, the legs are in the ratio of $4:3$, and we can let one leg be $4x$ and the other be $3x$. Then we can use the Pythagorean Theorem.

$$\begin{aligned}(4x)^2+(3x)^2&=27^2\\16x^2+9x^2&=729\\25x^2&=729\\x^2&=\frac{729}{25}\\x&=\frac{27}{5}\\x&=5.4\end{aligned}$$

The horizontal length is $5.4\times 4=21.6$, which is closest to $\boxed{\textbf{(D)}\ \ 21.5}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 7

Problem

The symbolism $\lfloor x \rfloor$ denotes the largest integer not exceeding x. For example, $\lfloor 3 \rfloor = 3,$ and $\lfloor 9/2 \rfloor = 4.$ Compute

$$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor.$$

$\textbf{(A)}\ 35 \qquad \textbf{(B)}\ 38 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 136$

Solution

The first three values in the sum are equal to $1,$ the next five equal to $2,$ the next seven equal to $3,$ and the last one equal to $4.$ For example, since $2^2 = 4$ any square root of a number less than 4 must be less than $2.$ Sum them all together to get

$$3 \cdot 1 + 5 \cdot 2 + 7 \cdot 3 + 1 \cdot 4 = 3 + 10 + 21 + 4 = \boxed{\textbf{(B)}\ \ 38}$$

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 8

The following problem is from both the 2003 AMC 12B #6 and 2003 AMC 10B #8, so both problems redirect to this page.

Problem

The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

$$\textbf{(A)}\ -\sqrt{3} \qquad \textbf{(B)}\ -\frac{2\sqrt{3}}{3} \qquad \textbf{(C)}\ -\frac{\sqrt{3}}{3} \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 3$$

Solution

Let the first term be a and the common difference be r. Therefore,

$$ar = 2 \;\; (1) \qquad \text{and} \qquad ar^3 = 6 \;\; (2)$$

Dividing (2) by (1) eliminates the a, yielding $r^2 = 3$, so $r = \pm\sqrt{3}$.

Now, since $ar = 2$, $a = \frac{2}{r}$, so $a = \frac{2}{\pm\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$.

We therefore see that $\boxed{\textbf{(B)}\ -\frac{2\sqrt{3}}{3}}$ is a possible first term.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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Category: Introductory Algebra Problems

2003 AMC 10B Problems/Problem 9

Problem

Find the value of x that satisfies the equation $25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$.

$\textbf{(A) } 2 \qquad \textbf{(B) } 3 \qquad \textbf{(C) } 5 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 9$

Solution

Manipulate the powers of 5 in order to get a clean expression.

$$\frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} \cdot 25^{\frac{17}{x}}} = \frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} \cdot 5^{\frac{34}{x}}} = 5^{\frac{48}{x} - (\frac{26}{x} + \frac{34}{x})} = 5^{-\frac{12}{x}}$$

$$25^{-2} = (5^2)^{-2} = 5^{-4}$$

$$5^{-4} = 5^{-\frac{12}{x}}$$

If two numbers are equal, and their bases are equal, then their exponents are equal as well. Set the two exponents equal to each other.

$$\begin{aligned} -4 &= \frac{-12}{x} \\ -4x &= -12 \\ x &= \boxed{\textbf{(B) } 3} \end{aligned}$$

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 10

Problem

Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of the three letters followed by three digits. By how many times is the number of possible license plates increased?

$\textbf{(A)}\ \frac{26}{10} \qquad \textbf{(B)}\ \frac{26^2}{10^2} \qquad \textbf{(C)}\ \frac{26^2}{10} \qquad \textbf{(D)}\ \frac{26^3}{10^3} \qquad \textbf{(E)}\ \frac{26^3}{10^2}$

Solution

There are 26 letters and 10 digits. There were $26 \cdot 10^4$ old license plates. There are $26^3 \cdot 10^3$ new license plates. The number of license plates increased by

$$\frac{26^3 \cdot 10^3}{26 \cdot 10^4} = \boxed{\textbf{(C)}\ \frac{26^2}{10}}$$

See Also

2003 AMC 10B Problems/Problem 11

Problem

A line with slope 3 intersects a line with slope 5 at point $(10,15)$. What is the distance between the x-intercepts of these two lines?

$\textbf{(A) } 2 \qquad \textbf{(B) } 5 \qquad \textbf{(C) } 7 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 20$

Solution

Using the point-slope formula, the equation of each line is

$$y-15=3(x-10) \longrightarrow y=3x-15$$

$$y-15=5(x-10) \longrightarrow y=5x-35$$

Substitute in $y=0$ to find the x-intercepts.

$$0=3x-15 \longrightarrow x=5$$

$$0=5x-35 \longrightarrow x=7$$

The difference between them is $7-5=\boxed{\textbf{(A)}\ 2}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 12

Problem

Al, Betty, and Clare split $\$1000$ among them to be invested in different ways. Each begins with a different amount. At the end of one year, they have a total of $\$1500$ dollars. Betty and Clare have both doubled their money, whereas Al has managed to lose $\$100$ dollars. What was Al's original portion?

$\textbf{(A)}\ \$250 \qquad \textbf{(B)}\ \$350 \qquad \textbf{(C)}\ \$400 \qquad \textbf{(D)}\ \$450 \qquad \textbf{(E)}\ \$500$

Solution

For this problem, we will have to write a three-variable equation, but not necessarily solve it. Let $a, b,$ and c represent the original portions of Al, Betty, and Clare, respectively. At the end of one year, they each have $a-100, 2b,$ and $2c$. From this, we can write two equations.

$$a+b+c = 10002a+2b+2c = 2000a-100+2b+2c = 1500a+2b+2c = 1600$$

Since all we need to find is $a,$ subtract the second equation from the first equation to get $a = 400.$

Al's original portion was $\boxed{\textbf{(C)}\ \$400}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 12B Problems/Problem 8

The following problem is from both the 2003 AMC 12B #8 and 2003 AMC 10B #13, so both problems redirect to this page.

Problem

Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

$\textbf{(A) } 3 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 9 \qquad \textbf{(E) } 10$

Solution

Let a and b be the digits of x,

$$\clubsuit(\clubsuit(x)) = a + b = 3$$

Clearly $\clubsuit(x)$ can only be $3, 12, 21,$ or 30 and only 3 and 12 are possible to have two digits sum to.

If $\clubsuit(x)$ sums to 3, there are 3 different solutions : $12, 21, \text{or } 30$

If $\clubsuit(x)$ sums to 12, there are 7 different solutions: $39, 48, 57, 66, 75, 84, \text{or } 93$

The total number of solutions is $3 + 7 = 10 \Rightarrow (E)$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 14

Problem

Given that $3^8 \cdot 5^2 = a^b$, where both a and b are positive integers, find the smallest possible value for $a + b$.

$\textbf{(A)}\ 25 \qquad\textbf{(B)}\ 34 \qquad\textbf{(C)}\ 351 \qquad\textbf{(D)}\ 407 \qquad\textbf{(E)}\ 900$

Solution

$$3^8 \cdot 5^2 = (3^4)^2 \cdot 5^2 = (3^4 \cdot 5)^2 = 405^2$$

405 is not a perfect power, so the smallest possible value of $a + b$ is $405 + 2 = \boxed{\textbf{(D)}\ 407}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 15

Contents

Problem

There are 100 players in a single tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The match continues until only one player remains unbeaten. The total number of matches played is

$\textbf{(A)}$ a prime number

$\textbf{(B)}$ divisible by 2

$\textbf{(C)}$ divisible by 5

$\textbf{(D)}$ divisible by 7

$\textbf{(E)}$ divisible by 11

Solution

Solution 1

Notice that 99 players need to be eliminated for there to be declared a winner. Notice also that every match eliminates exactly one person. Therefore, 99 matches are needed to eliminate 99 people and therefore declare a winner. The rest of the information is irrelevant. Therefore, the total number of matches is $\text{divisible by } 11, \boxed{\text{E}}$.

Solution 2

28 people receive byes, so in the first round there are 36 matches played. In the second round there are $28+36=64$ people, so there are 32 matches. In the subsequent rounds, there are $16, 8, 4, 2, 1$ matches played, for a total of $36+32+16+8+4+2+1=99$ matches. Divisible by 11 $\Rightarrow \boxed{(E)}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 16

Problem

A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that a restaurant should offer so that a customer could have a different dinner each night in the year 2003?

$\textbf{(A)}\ 4 \qquad\textbf{(B)}\ 5 \qquad\textbf{(C)}\ 6 \qquad\textbf{(D)}\ 7 \qquad\textbf{(E)}\ 8$

Solution

Let m be the number of main courses the restaurant serves, so $2m$ is the number of appetizers. Then the number of dinner combinations is $2m\times m\times 3=6m^2$. Since the customer wants to eat a different dinner in all 365 days of 2003, we must have

$$\begin{aligned} 6m^2 &\geq 365 \\ m^2 &\geq 60.83\ldots. \end{aligned}$$

The smallest integer value that satisfies this is $\boxed{\textbf{(E)}\ 8}$.

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 10B Problems/Problem 17

Problem

An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius? (Note: a cone with radius r and height h has volume $\pi r^2 h/3$ and a sphere with radius r has volume $4\pi r^3/3$.)

$\mathrm{(A) \ } 2:1 \qquad \mathrm{(B) \ } 3:1 \qquad \mathrm{(C) \ } 4:1 \qquad \mathrm{(D) \ } 16:3 \qquad \mathrm{(E) \ } 6:1$

Solution

Let r be the radius of both the cone and the sphere of ice cream. The cone can hold a maximum of $\pi r^2 h/3$ melted ice cream. The volume of the frozen ice cream is $4\pi r^3/3.$ The volume of the melted ice cream is 75% of that, which is $\pi r^3.$ Since the melted ice cream fills the cone exactly,

$$\begin{aligned}\pi r^3 &= \pi r^2 h/3 \\ r &= h/3 \\ 3 &= h/r\end{aligned}$$

Then the ratio of the cone's height to its radius is $h:r = \boxed{\mathrm{(B)}\ 3:1}$

See Also

2003 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2003))	
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2003 AMC 12B Problems/Problem 12

The following problem is from both the 2003 AMC 12B #12 and 2003 AMC 10B #18, so both problems redirect to this page.

Problem

What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

for all positive even integers n?

$\mathrm{(A) \ } 3 \qquad \mathrm{(B) \ } 5 \qquad \mathrm{(C) \ } 11 \qquad \mathrm{(D) \ } 15 \qquad \mathrm{(E) \ } 165$

Solution

Since for all consecutive odd integers, one of every five is a multiple of 5 and one of every three is a multiple of 3, the answer is $3*5=15$, so $\boxed{\mathrm{D}}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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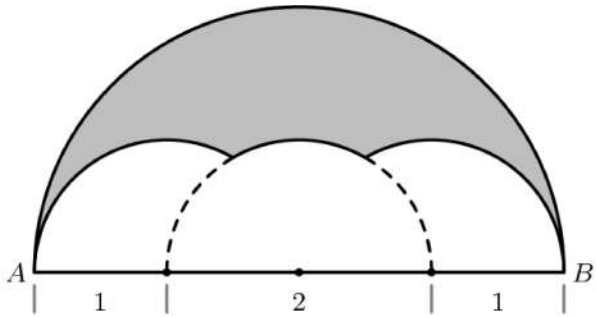
2003 AMC 10B Problems/Problem 19

The following problem is from both the 2003 AMC 12B #16 and 2003 AMC 10B #19, so both problems redirect to this page.

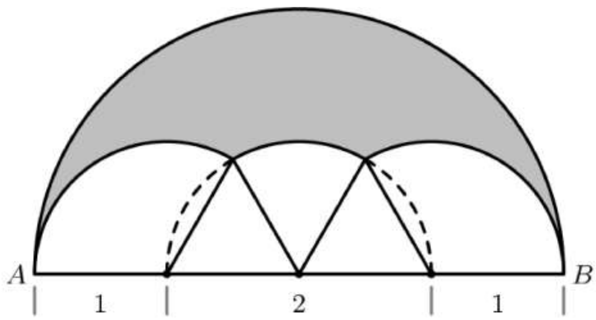
Problem

Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



$\textbf{(A) } \pi-\sqrt{3} \qquad \textbf{(B) } \pi-\sqrt{2} \qquad \textbf{(C) } \frac{\pi+\sqrt{2}}{2} \qquad \textbf{(D) } \frac{\pi+\sqrt{3}}{2} \qquad \textbf{(E) } \frac{7}{6}\pi-\frac{\sqrt{3}}{2}$

Solution



By drawing four lines from the intersect of the semicircles to their centers, we have split the white region into $\frac{5}{6}$ of a circle with radius 1 and two equilateral triangles with side length 1. This gives the area of the white region as $\frac{5}{6}\pi+\frac{2\cdot\sqrt{3}}{4}=\frac{5}{6}\pi+\frac{\sqrt{3}}{2}$. The area of the shaded region is the area of the white region subtracted from the area of the large semicircle. This is equivalent to $2\pi-(\frac{5}{6}\pi+\frac{\sqrt{3}}{2})=\frac{7}{6}\pi-\frac{\sqrt{3}}{2}$.

Thus the answer is $\boxed{\textbf{(E) } \frac{7}{6}\pi-\frac{\sqrt{3}}{2}}$.

See Also

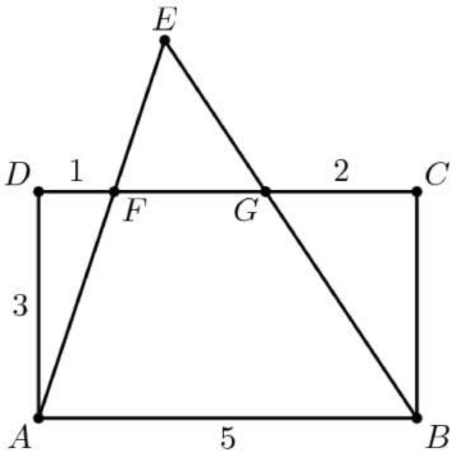
2003 AMC 10B Problems/Problem 20

The following problem is from both the 2003 AMC 12B #14 and 2003 AMC 10B #20, so both problems redirect to this page.

Contents

Problem

In rectangle $ABCD, AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ \frac{21}{2} \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ \frac{25}{2} \qquad \textbf{(E)}\ 15$

Solution 1

$\triangle EFG \sim \triangle EAB$ because $FG \parallel AB.$ The ratio of $\triangle EFG$ to $\triangle EAB$ is $2:5$ since $AB=5$ and $FG=2$ from subtraction. If we let h be the height of $\triangle EAB,$

$$\frac{2}{5} = \frac{h-3}{h}$$

$$2h = 5h-15$$

$$3h = 15$$

$$h = 5$$

The height is 5 so the area of $\triangle EAB$ is $\frac{1}{2}(5)(5) = \boxed{\textbf{(D)}\ \frac{25}{2}}$.

Solution 2

We can look at this diagram as if it were a coordinate plane with point A being $(0,0)$. This means that the equation of the line AE is $y = 3x$ and the equation of the line EB is $y = \frac{-3}{2}x + \frac{15}{2}$. From this we can set of the follow equation to find the x coordinate of point E:

$$3x = \frac{-3}{2}x + \frac{15}{2}$$

$$6x = -3x + 15$$

$$9x = 15$$

$$x = \frac{5}{3}$$

We can plug this into one of our original equations to find that the y coordinate is 5, meaning the area of $\triangle EAB$ is $\frac{1}{2}(5)(5) = \boxed{\textbf{(D)}\ \frac{25}{2}}$

See Also

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2003 AMC 10B Problems/Problem 21

Problem

A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after three such replacements?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{5}{32} \qquad \textbf{(C)}\ \frac{9}{32} \qquad \textbf{(D)}\ \frac{3}{8} \qquad \textbf{(E)}\ \frac{7}{16}$

Solution

We can divide the case of all beads in the bag being red after three replacements into three cases.

The first case is in which the two green beads are the first two beads to be chosen. The probability for this is

$$\frac{2}{4} \times \frac{1}{4} \times \frac{4}{4} = \frac{1}{8}$$

The second case is in which the green beads are chosen first and third. The probability for this is

$$\frac{2}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$$

The third case is in which the green beads are chosen second and third. The probability for this is

$$\frac{2}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{1}{16}$$

Add all these cases together

$$\frac{1}{8} + \frac{3}{32} + \frac{1}{16} = \frac{4}{32} + \frac{3}{32} + \frac{2}{32} = \boxed{\textbf{(C)}\ \frac{9}{32}}$$

See Also

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2003 AMC 10B Problems/Problem 22

Problem

A clock chimes once at 30 minutes past each hour and chimes on the hour according to the hour. For example, at 1PM there is one chime and at noon and midnight there are twelve chimes. Starting at $11:15\text{AM}$ on $\text{February } 26, 2003,$ on what date will the 2003^{rd} chime occur?

$\textbf{(A) } \text{March 8} \qquad\textbf{(B) } \text{March 9} \qquad\textbf{(C) } \text{March 10} \qquad\textbf{(D) } \text{March 20} \qquad\textbf{(E) } \text{March 21}$

Solution

First, find how many chimes will have already happened before midnight (the beginning of the day) of $\text{February 27, 2003}.$ 13 half-hours have passed, and the number of chimes according to the hour is $1+2+3+\cdots+12.$ The total number of chimes is $13+78=91$

Every day, there will be 24 half-hours and $2(1+2+3+\cdots+12)$ chimes according to the arrow, resulting in $24+156=180$ total chimes.

On $\text{February 27},$ the number of chimes that still need to occur is $2003-91=1912.$ $1912 \div 180 = 10R112.$ Round up, and it is 11 days past $\text{February 27},$ which is $\boxed{\textbf{(B) } \text{March 9}}$

See Also

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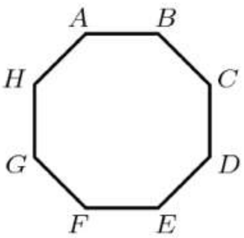
2003 AMC 10B Problems/Problem 23

The following problem is from both the 2003 AMC 12B #15 and 2003 AMC 10B #23, so both problems redirect to this page.

Contents

Problem

A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?



$\mathrm{(A)}\ 1-\frac{\sqrt{2}}{2}\qquad\mathrm{(B)}\ \frac{\sqrt{2}}{4}\qquad\mathrm{(C)}\ \sqrt{2}-1\qquad\mathrm{(D)}\ \frac{1}{2}\qquad\mathrm{(E)}\ \frac{1+\sqrt{2}}{4}$

Solution

Solution 1

Here is an easy way to look at this, where p is the perimeter, and a is the apothem:

Area of Octagon: $\frac{ap}{2}=1.$

Area of Rectangle: $\frac{p}{8}\times 2a=\frac{ap}{4}.$

You can see from this that the octagon's area is twice as large as the rectangle's area is $\boxed{\mathrm{(D)}\ \frac{1}{2}}$.

Solution 2

Here is a less complicated way than that of the user above. If you draw a line segment from each vertex to the center of the octagon and draw the rectangle ABEF, you can see that two of the triangles share the same base and height with half the rectangle. Therefore, the rectangle's area is the same as 4 of the 8 triangles, and is $\boxed{\mathrm{(D)}\ \frac{1}{2}}$ the area of the octagon.

Solution 3

Drawing lines AD, BG, CF, and EH, we can see that the octagon is comprised of 1 square, 4 rectangles, and 4 triangles. The triangles each are 45-45-90 triangles, and since their diagonal is x, each of their sides is x*sqrt(2)/2. The area of the entire figure is, likewise, x^2 (the square) + 4*x*x*sqrt(2)/2 (the 4 rectangles) + 2* (x*sqrt(2)/2)^2 (the triangles), which simplifies to 2x^2 + 2*sqrt(2)x^2. The area of ABEF is just x*(x+(2*x*sqrt(2)/2)), = x^2 + sqrt(2)x^2, which we can see is the area of ABCDEFGH/2 = $\boxed{\textbf{(D)}\ \frac{1}{2}}$ the area of the octagon.

See Also

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2003 AMC 10B Problems/Problem 24

Problem

The first four terms in an arithmetic sequence are $x+y, x-y$, xy , and $\frac{x}{y}$, in that order. What is the fifth term?

$\textbf{(A)}\ -\frac{15}{8}\qquad\textbf{(B)}\ -\frac{6}{5}\qquad\textbf{(C)}\ 0\qquad\textbf{(D)}\ \frac{27}{20}\qquad\textbf{(E)}\ \frac{123}{40}$

Solution

The difference between consecutive terms is $(x-y)-(x+y)=-2y.$ Therefore we can also express the third and fourth terms as $x-3y$ and $x-5y.$ Then we can set them equal to xy and $\frac{x}{y}$ because they are the same thing.

$$\begin{aligned} xy &= x-3y \\ xy-x &= -3y \\ x(y-1) &= -3y \\ x &= \frac{-3y}{y-1} \end{aligned}$$

Substitute into our other equation.

$$\frac{x}{y}=x-5y$$

$$\frac{-3}{y-1}=\frac{-3y}{y-1}-5y$$

$$-3=-3y-5y(y-1)$$

$$0=5y^2-2y-3$$

$$0=(5y+3)(y-1)$$

$$y=-\frac{3}{5},1$$

But y cannot be 1 because then every term would be equal to $x.$ Therefore $y=-\frac{3}{5}.$ Substituting the value for y into any of the equations, we get $x=-\frac{9}{8}.$ Finally,

$$\frac{x}{y}-2y=\frac{9\cdot5}{8\cdot3}+\frac{6}{5}=\boxed{\textbf{(E)}\ \frac{123}{40}}$$

See Also

2003 AMC 10B Problems/Problem 25

Contents

Problem

How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

$\mathrm{(A) \ } 27 \qquad \mathrm{(B) \ } 30 \qquad \mathrm{(C) \ } 33 \qquad \mathrm{(D) \ } 81 \qquad \mathrm{(E) \ } 90$

Solution

Solution 1

To test if a number is divisible by three, you add up the digits of the number. If the sum is divisible by three, then the original number is a multiple of three. If the sum is too large, you can repeat the process until you can tell whether it is a multiple of three. This is the basis for the solution. Since the last two digits are 23, the sum of the digits is $2+3=5$ (therefore it is not divisible by three). However certain numbers can be added to make the sum of the digits a multiple of three.

$5+1=6$,

$5+4=9$, and so on.

However since the largest four-digit number ending with 23 is 9923, the maximum sum is

$5+18=23$.

Using that process we can fairly quickly compile a list of the sum of the first two digits of the number.

$$\{1,4,7,10,13,16\}$$

Now we find all the two-digit numbers that have any of the sums shown above. We can do this by listing all the two digit numbers xy in separate cases.

$$I. x+y=1, \{10\}=1$$

$$II. x+y=4, \{13,22,31,40\}=4$$

$$III. x+y=7, \{16,25,34,43,52,61,70\}=7$$

$$IV. x+y=10, \{19,28,37,46,55,64,73,82,91\}=9$$

$$V. x+y=13, \{49,58,67,76,85,94\}=6$$

$$VI. x+y=16, \{79,88,97\}=3$$

And finally, we add the number of elements in each set.

$$1+4+7+9+6+3=\boxed{\textbf{(B)}\ 30}$$

Solution 2

A number divisible by 3 has all its digits add to a multiple of $3.$ The last two digits are 2 and 3 and add up to $5\equiv 2\pmod 3).$ Therefore the first two digits must add up to $1 \pmod 3).$ 4 digits (including 0) are $0 \pmod 3),$ 3 are $1 \pmod 3),$ and 3 are $2 \pmod 3).$ The following combinations are equivalent to $1 \pmod 3)$:

$$0 \pmod 3)+1 \pmod 3)\equiv 1 \pmod 3)+0 \pmod 3)\equiv 2 \pmod 3)+2 \pmod 3)$$

Let the first term in each combination represent the thousands digit and the second term represent the hundreds digit. We can use this to find the total amount of four-digit numbers.

$$3\cdot 3+3\cdot 4+3\cdot 3=9+12+9=\boxed{\textbf{(B)}\ 30}$$

Solution 3

We have the following: $n\equiv 0\pmod{30}$ and $n\equiv 32\pmod{100}$. Then $n=3a=100b+23$ for some integer a and b. Taking mod 3 gives: $0\equiv b+2\pmod 3 \implies b\equiv 1\pmod 3$ so $b=3c+1$ for some integer c. But $N=100b+23=100(3c+1)+23$ so $n=300c+123$. Bounding this gives us: $999<300c+123<10000$ so $876<300c<9877$. Dividing by 300 gives $2.92<c<32.9222$ so $3\le c\le 32$. This gives $32-3+1=\boxed{\textbf{(B)}\ 30}$

See also

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