

2004 AMC 10A Problems/Problem 1

Problem

You and five friends need to raise **1500** dollars in donations for a charity, dividing the fundraising equally. How many dollars will each of you need to raise?

- (A) 250 (B) 300 (C) 1500 (D) 7500 (E) 9000

Solution

There are **6** people to split the **1500** dollars among, so each person must raise $\frac{1500}{6} = 250$ dollars.

⇒ (A) 250

See also

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2004 AMC 10A Problems/Problem 2

Problem

For any three real numbers a , b , and c , with $b \neq c$, the operation $\otimes$ is defined by:

$$\otimes(a,b,c) = \frac{a}{b-c}$$

What is $\otimes(\otimes(1,2,3), \otimes(2,3,1), \otimes(3,1,2))$?

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) 0 (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

Solution

$$\otimes\left(\frac{1}{2-3}, \frac{2}{3-1}, \frac{3}{1-2}\right) = \otimes(-1, 1, -3) = \frac{-1}{1+3} = -\frac{1}{4} \Rightarrow \boxed{\text{(B)} \quad -\frac{1}{4}}$$

See also

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2004 AMC 12A Problems/Problem 1

The following problem is from both the 2004 AMC 12A #1 and 2004 AMC 10A #3, so both problems redirect to this page.

Problem

Alicia earns 20 dollars per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia’s wages are used to pay local taxes?

(A) 0.0029 (B) 0.029 (C) 0.29 (D) 2.9 (E) 29

Solution

20 dollars is the same as 2000 cents, and 1.45% of 2000 is $0.0145 \times 2000 = 29$ cents. $\Rightarrow$ (E) 29.

See also

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2004 AMC 10A Problems/Problem 4

Problem

What is the value of x if $|x - 1| = |x - 2|$?

- (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2

Solution

$|x - 1|$ is the distance between x and 1; $|x - 2|$ is the distance between x and 2.

Therefore, the given equation says x is equidistant from 1 and 2, so $x = \frac{1+2}{2} = \frac{3}{2} \Rightarrow \boxed{\text{(D)} \frac{3}{2}}$.

Alternatively, we can solve by casework (a method which should work for any similar problem involving absolute values of real numbers). If $x \leq 1$, then $|x - 1| = 1 - x$ and $|x - 2| = 2 - x$, so we must solve $1 - x = 2 - x$, which has no solutions. Similarly, if $x \geq 2$, then $|x - 1| = x - 1$ and $|x - 2| = x - 2$, so we must solve $x - 1 = x - 2$, which also has no solutions. Finally, if $1 \leq x \leq 2$, then $|x - 1| = x - 1$ and $|x - 2| = 2 - x$, so we must solve $x - 1 = 2 - x$, which has the unique solution $x = \frac{3}{2}$.

See also

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2004 AMC 10A Problems/Problem 5

Problem

A set of three points is randomly chosen from the grid shown. Each three point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?



- (A) $\frac{1}{21}$ (B) $\frac{1}{14}$ (C) $\frac{2}{21}$ (D) $\frac{1}{7}$ (E) $\frac{2}{7}$

Solution

There are $\binom{9}{3}$ ways to choose three points out of the 9 there. There are 8 combinations of dots such that they lie in a straight line: three vertical, three horizontal, and the diagonals.

$$\frac{8}{\binom{9}{3}} = \frac{8}{84} = \frac{2}{21} \Rightarrow \boxed{\text{(C) } \frac{2}{21}}$$

See also

- AoPS topic (<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131315>)

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2004 AMC 12A Problems/Problem 4

The following problem is from both the 2004 AMC 12A #4 and 2004 AMC 10A #6, so both problems redirect to this page.

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Problem

Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

(A) 22 (B) 23 (C) 24 (D) 25 (E) 26

Solution

Since Bertha has 6 daughters, she has $30 - 6 = 24$ granddaughters, of which none have daughters. Of Bertha's daughters, $\frac{24}{6} = 4$ have daughters, so $6 - 4 = 2$ do not have daughters. Therefore, of Bertha's daughters and granddaughters, $24 + 2 = 26$ do not have daughters. (E) 26

Solution 2

Draw a tree diagram and see that the answer can be found in the sum of $6 + 6$ granddaughters, $5 + 5$ daughters, and 4 more daughters. Adding them together gives the answer of (E) 26.

See also

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2004 AMC 10A Problems/Problem 7

Problem

A grocer stacks oranges in a pyramid-like stack whose rectangular base is **5** oranges by **8** oranges. Each orange above the first level rests in a pocket formed by four oranges below. The stack is completed by a single row of oranges. How many oranges are in the stack?

(A) 96 (B) 98 (C) 100 (D) 101 (E) 134

Solution

There are $5 \times 8 = 40$ oranges on the **1st** layer of the stack. The **2nd** layer that is added on top of the first will be a layer of $4 \times 7 = 28$ oranges. When the third layer is added on top of the **2nd**, it will be a layer of $3 \times 6 = 18$ oranges, etc.

Therefore, there are $5 \times 8 + 4 \times 7 + 3 \times 6 + 2 \times 5 + 1 \times 4 = 40 + 28 + 18 + 10 + 4 = 100$ oranges in the stack. **(C) 100**

See also

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2004 AMC 12A Problems/Problem 7

The following problem is from both the 2004 AMC 12A #7 and 2004 AMC 10A #8, so both problems redirect to this page.

Problem

A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players *A*, *B*, and *C* start with **15**, **14**, and **13** tokens, respectively. How many rounds will there be in the game?

- (A) 36 (B) 37 (C) 38 (D) 39 (E) 40

Solution

Look at a set of **3** rounds, where the players have $x + 1$, x , and $x - 1$ tokens. Each of the players will gain two tokens from the others and give away **3** tokens, so overall, each player will lose **1** token. Therefore, after **12** sets of **3** rounds, or **36** rounds, the players will have **3**, **2**, and **1** tokens, respectively. After **1** more round, player *A* will give away his last **3** tokens and the game will end. (B) 37

See also

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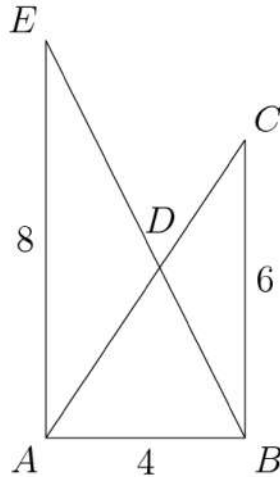
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2004 AMC 12A Problems/Problem 8

The following problem is from both the 2004 AMC 12A #8 and 2004 AMC 10A #9, so both problems redirect to this page.

Problem

In the overlapping triangles $\triangle ABC$ and $\triangle ABE$ sharing common side AB , $\angle EAB$ and $\angle ABC$ are right angles, $AB = 4$, $BC = 6$, $AE = 8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D . What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?



- (A) 2 (B) 4 (C) 5 (D) 8 (E) 9

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Solution 1

Since $AE \perp AB$ and $BC \perp AB$, $AE \parallel BC$. By alternate interior angles and $AA \sim$, we find that $\triangle ADE \sim \triangle CDB$, with side length ratio $\frac{4}{3}$. Their heights also have the same ratio, and since the two heights add up to 4, we have that $h_{ADE} = 4 \cdot \frac{4}{7} = \frac{16}{7}$ and $h_{CDB} = 3 \cdot \frac{4}{7} = \frac{12}{7}$. Subtracting the areas, $\frac{1}{2} \cdot 8 \cdot \frac{16}{7} - \frac{1}{2} \cdot 6 \cdot \frac{12}{7} = 4 \Rightarrow \boxed{(B) 4}$.

Solution 2

Let $[X]$ represent the area of figure X . Note that $[\triangle BEA] = [\triangle ABD] + [\triangle ADE]$ and $[\triangle BCA] = [\triangle ABD] + [\triangle BDC]$.

$$[\triangle ADE] - [\triangle BDC] = [\triangle BEA] - [\triangle BCA] = \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 6 \times 4 = 16 - 12 = 4 \Rightarrow \boxed{(B) 4}$$

See also

- AoPS topic (<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131320>)

2004 AMC 10A Problems/Problem 10

Problem

Coin **A** is flipped three times and coin **B** is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

- (A) $\frac{29}{128}$ (B) $\frac{23}{128}$ (C) $\frac{1}{4}$ (D) $\frac{35}{128}$ (E) $\frac{1}{2}$

Solution

There are **4** ways that the same number of heads will be obtained; **0**, **1**, **2**, or **3** heads.

The probability of both getting **0** heads is $\left(\frac{1}{2}\right)^3 \binom{3}{0} \left(\frac{1}{2}\right)^4 \binom{4}{0} = \frac{1}{128}$

The probability of both getting **1** head is $\left(\frac{1}{2}\right)^3 \binom{3}{1} \left(\frac{1}{2}\right)^4 \binom{4}{1} = \frac{12}{128}$

The probability of both getting **2** heads is $\left(\frac{1}{2}\right)^3 \binom{3}{2} \left(\frac{1}{2}\right)^4 \binom{4}{2} = \frac{18}{128}$

The probability of both getting **3** heads is $\left(\frac{1}{2}\right)^3 \binom{3}{3} \left(\frac{1}{2}\right)^4 \binom{4}{3} = \frac{4}{128}$

Therefore, the probability of flipping the same number of heads is: $\frac{1 + 12 + 18 + 4}{128} = \frac{35}{128} \Rightarrow \boxed{\text{(D)} \frac{35}{128}}$

See also

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2004 AMC 12A Problems/Problem 9

The following problem is from both the 2004 AMC 12A #9 and 2004 AMC 10A #11, so both problems redirect to this page.

Problem

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by **25%** without altering the volume, by what percent must the height be decreased?

(A) 10 (B) 25 (C) 36 (D) 50 (E) 60

Solution

When the diameter is increased by **25%**, it is increased by $\frac{5}{4}$, so the area of the base is increased by $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$.

To keep the volume the same, the height must be $\frac{1}{\frac{25}{16}} = \frac{16}{25}$ of the original height, which is a **36%** reduction.

(C) 36

See also

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2004 AMC 10A Problems/Problem 12

Problem

Henry’s Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

- (A) 24 (B) 256 (C) 768 (D) 40,320 (E) 120,960

Solution

For each condiment, a customer may either order it or not. There are 8 condiments. Therefore, there are $2^8 = 256$ ways to order the condiments. There are also 3 choices for the meat, making a total of $256 \times 3 = 768$ possible hamburgers. (C) 768

See also

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2004 AMC 10A Problems/Problem 13

Problem

At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

- (A) 8
- (B) 12
- (C) 16
- (D) 18
- (E) 24

Solution

If each man danced with **3** women, then there will be a total of $3 \times 12 = 36$ pairs of men and women. However, each woman only danced with **2** men, so there must have been $\frac{36}{2} \Rightarrow \boxed{\text{(D) } 18}$ women.

See also

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2004 AMC 12A Problems/Problem 11

The following problem is from both the 2004 AMC 12A #11 and 2004 AMC 10A #14, so both problems redirect to this page.

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Problem

The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is **20** cents. If she had one more quarter, the average value would be **21** cents. How many dimes does she have in her purse?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Solution 1

Let the total value (in cents) of the coins Paula has originally be x , and the number of coins she has be n . Then $\frac{x}{n} = 20 \implies x = 20n$ and $\frac{x + 25}{n + 1} = 21$. Substituting yields $20n + 25 = 21(n + 1)$, implying $n = 4$, $x = 80$. It is easy to see that Paula has **3** quarters and **1** nickel, so she has (A) 0 dimes.

Solution 2

If the new coin was worth **20** cents, adding it would not change the mean. The additional **5** cents raise the mean by **1**, thus the new number of coins must be **5**. Therefore there were **4** coins worth a total of $4 \times 20 = 80$ cents. As in the previous solution, we conclude that the only way to get **80** cents using **4** coins is $25 + 25 + 25 + 5$.

(A) 0

See also

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2004 AMC 10A Problems/Problem 15

Problem

Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $\frac{x+y}{x}$?

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Solution

Rewrite $\frac{(x+y)}{x}$ as $\frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$.

We also know that $\frac{y}{x} < 0$ because x and y are of opposite sign.

Therefore, $1 + \frac{y}{x}$ is maximized when $\left|\frac{y}{x}\right|$ is minimized, which occurs when $|x|$ is the largest and $|y|$ is the smallest.

This occurs at $(-4, 2)$, so $\frac{x+y}{x} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \boxed{\text{(D)} \frac{1}{2}}$.

See also

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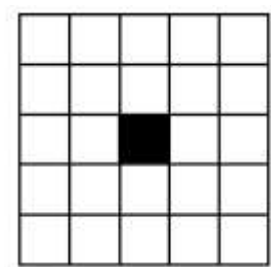
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Problem

The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5 . How many of these squares contain the black center square?



- (A) 12
- (B) 15
- (C) 17
- (D) 19
- (E) 20

Solution 1

There are:

- 1 of the 1×1 squares containing the black square,
- 4 of the 2×2 squares containing the black square,
- 9 of the 3×3 squares containing the black square,
- 4 of the 4×4 squares containing the black square,
- 1 of the 5×5 squares containing the black square.

Thus, the answer is $1 + 4 + 9 + 4 + 1 = 19 \Rightarrow$ **(D) 19**.

Solution 2

We use complementary counting. There are only 2×2 and 1×1 squares that do not contain the black square. Counting, there are **12** 2×2 , and $25 - 1 = 24$ 1×1 squares that do not contain the black square. That gives **12 + 24 = 36** squares that don't contain it. There are a total of **25 + 16 + 9 + 4 + 1 = 55** squares possible, therefore there are **55 - 36 = 19** squares that contains the black square, which is **(D) 19**.

See also

2004 AMC 10A (Problems • Answer Key • Resources)	
(http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
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2004 AMC 12A Problems/Problem 15

The following problem is from both the 2004 AMC 12A #15 and 2004 AMC 10A #17, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 See also

Problem

Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

(A) 250 (B) 300 (C) 350 (D) 400 (E) 500

Solution 1

Call the length of the race track x . When they meet at the first meeting point, Brenda has run 100 meters, while Sally has run $\frac{x}{2} - 100$ meters. By the second meeting point, Sally has run 150 meters, while Brenda has run

$x - 150$ meters. Since they run at a constant speed, we can set up a proportion: $\frac{100}{x - 150} = \frac{\frac{x}{2} - 100}{150}$.

Cross-multiplying, we get that $x = 350 \Rightarrow \boxed{\text{(C) } 350}$.

Sidenote by carlos8:

Since they run at constant speeds, Brenda must've ran 200 meters to get to the second meeting point, therefore we can make an equation $200 = x - 150$, solving for x , gives us our answer 350.

Solution 2

The total distance the girls run between the start and the first meeting is one half of the track length. The total distance they run between the two meetings is the track length. As the girls run at constant speeds, the interval between the meetings is twice as long as the interval between the start and the first meeting. Thus between the meetings Brenda will run $2 \times 100 = 200$ meters. Therefore the length of the track is $150 + 200 = 350$

meters $\Rightarrow \boxed{\text{(C) } 350}$

See also

2004 AMC 12A (Problems • Answer Key • Resources)	
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2004 AMC 12A Problems/Problem 14

The following problem is from both the 2004 AMC 12A #14 and 2004 AMC 10A #18, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 See also

Problem

A sequence of three real numbers forms an arithmetic progression with a first term of **9**. If **2** is added to the second term and **20** is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term in the geometric progression?

(A) 1 (B) 4 (C) 36 (D) 49 (E) 81

Solution 1

Let d be the common difference. Then **9**, $9 + d + 2 = 11 + d$, $9 + 2d + 20 = 29 + 2d$ are the terms of the geometric progression. Since the middle term is the geometric mean of the other two terms, $(11 + d)^2 = 9(2d + 29) \implies d^2 + 4d - 140 = (d + 14)(d - 10) = 0$. The smallest possible value occurs when $d = -14$, and the third term is $2(-14) + 29 = 1 \Rightarrow \boxed{\text{(A) } 1}$.

Solution 2

Let d be the common difference and r be the common ratio. Then the arithmetic sequence is **9**, $9 + d$, and $9 + 2d$. The geometric sequence (when expressed in terms of d) has the terms **9**, $11 + d$, and $29 + 2d$. Thus, we get the following equations:

$$9r = 11 + d \Rightarrow d = 9r - 11$$

$$9r^2 = 29 + 2d$$

Plugging in the first equation into the second, our equation becomes

$$9r^2 = 29 + 18r - 22 \implies 9r^2 - 18r - 7 = 0.$$

By the quadratic formula, r can either be $-\frac{1}{3}$ or $\frac{7}{3}$. If r is $-\frac{1}{3}$, the third term (of the geometric sequence) would be **1**, and if r is $\frac{7}{3}$, the third term would be **49**.

Clearly the minimum possible value for the third term of the geometric sequence is $\boxed{\text{(A) } 1}$.

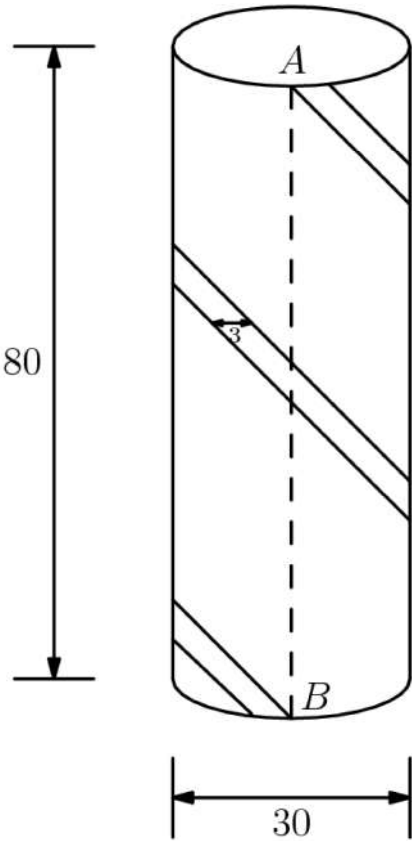
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2004 AMC 10A Problems/Problem 19

Problem

A white cylindrical silo has a diameter of 30 feet and a height of 80 feet. A red stripe with a horizontal width of 3 feet is painted on the silo, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?



- (A) 120 (B) 180 (C) 240 (D) 360 (E) 480

Solution

The cylinder can be “unwrapped” into a rectangle, and we see that the stripe is a parallelogram with base **3** and height **80**. $3 \times 80 = 240 \Rightarrow \boxed{\text{(C) } 240}$

See also

2004 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
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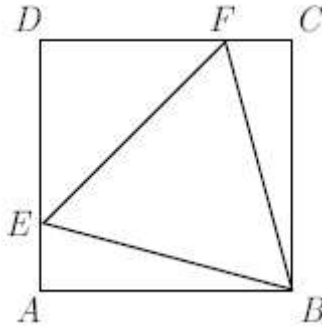
2004 AMC 10A Problems/Problem 20

Contents

- 1 Problem
- 2 Solution
- 3 Solution 2 (Non-trig)
- 4 See also

Problem

Points E and F are located on square $ABCD$ so that $\triangle BEF$ is equilateral. What is the ratio of the area of $\triangle DEF$ to that of $\triangle ABE$?



- (A) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2 (E) $1 + \sqrt{3}$

Solution

Since triangle BEF is equilateral, $EA = FC$, and EAB and FCB are SAS congruent. Thus, triangle DEF is an isosceles right triangle. So we let $DE = x$. Thus $EF = EB = FB = x\sqrt{2}$. If we go angle chasing, we find out that $\angle AEB = 75^\circ$, thus $\angle ABE = 15^\circ$. $\frac{AE}{EB} = \sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$. Thus $\frac{AE}{x\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$, or $AE = \frac{x(\sqrt{3} - 1)}{2}$. Thus $AB = \frac{x(\sqrt{3} + 1)}{2}$, and $[ABE] = \frac{x^2}{4}$, and $[DEF] = \frac{x^2}{2}$. Thus the ratio of the areas is (D) 2

Solution 2 (Non-trig)

Without loss of generality let the side length of $ABCD$ be 1. Let $DE = x$ and $AE = 1 - x$. Then triangles ABE and CBF are clearly congruent by HL, so $CF = AE$ and $DE = DF$. We find that $BE = EF = x\sqrt{2}$, and so, by the Pythagorean Theorem, we have $(1 - x)^2 + 1 = 2x^2$. This yields $x^2 + 2x = 2$, so $x^2 = 2 - 2x$. Thus, the desired ratio of areas is

$$\frac{\frac{x^2}{2}}{\frac{1-x}{2}} = \frac{x^2}{1-x} = 2.$$

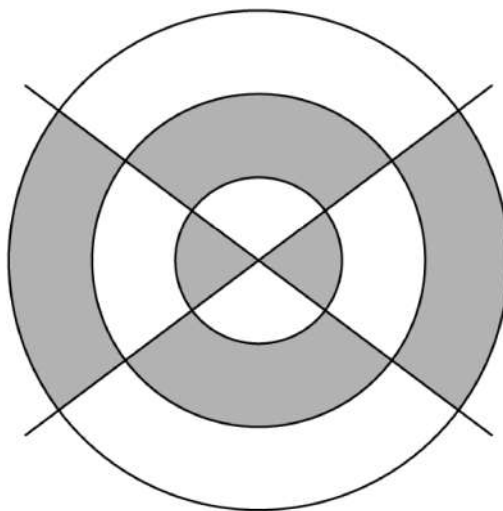
See also

- AoPS topic (<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131332>)

2004 AMC 10A Problems/Problem 21

Problem

Two distinct lines pass through the center of three concentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is $\frac{8}{13}$ of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines? (Note: π radians is 180 degrees.)



- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{7}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{4}$

Solution

Let the area of the shaded region be S , the area of the unshaded region be U , and the acute angle that is formed by the two lines be θ . We can set up two equations between S and U :

$$S + U = 9\pi$$

$$S = \frac{8}{13}U$$

$$\text{Thus } \frac{21}{13}U = 9\pi, \text{ and } U = \frac{39\pi}{7}, \text{ and thus } S = \frac{8}{13} \cdot \frac{39\pi}{7} = \frac{24\pi}{7}.$$

Now we can make a formula for the area of the shaded region in terms of θ :

$$\frac{2\theta}{2\pi} \cdot \pi + \frac{2(\pi - \theta)}{2\pi} \cdot (4\pi - \pi) + \frac{2\theta}{2\pi}(9\pi - 4\pi) = \theta + 3\pi - 3\theta + 5\theta = 3\theta + 3\pi = \frac{24\pi}{7}$$

$$\text{Thus } 3\theta = \frac{3\pi}{7} \Rightarrow \theta = \frac{\pi}{7} \Rightarrow \boxed{\text{(B)} \frac{\pi}{7}}$$

See also

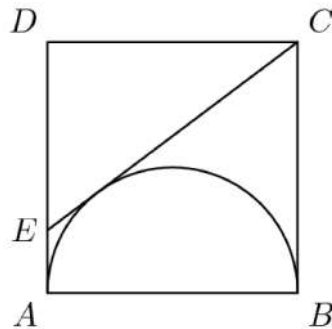
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2004 AMC 12A Problems/Problem 18

The following problem is from both the 2004 AMC 12A #18 and 2004 AMC 10A #22, so both problems redirect to this page.

Problem

Square $ABCD$ has side length 2 . A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E . What is the length of $\overline{CE}$?

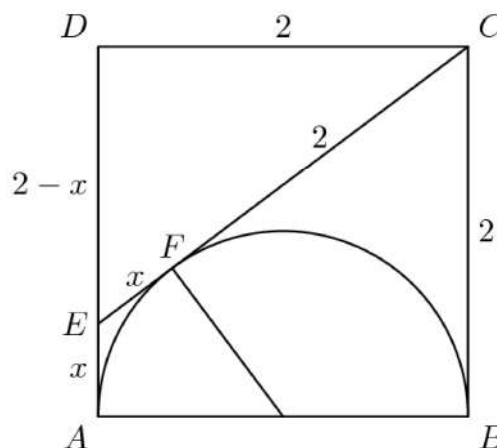


- (A) $\frac{2 + \sqrt{5}}{2}$ (B) $\sqrt{5}$ (C) $\sqrt{6}$ (D) $\frac{5}{2}$ (E) $5 - \sqrt{5}$

Contents

- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 Solution 3
- 5 Solution 4
- 6 See also

Solution 1



Let the point of tangency be F . By the Two Tangent Theorem $BC = FC = 2$ and $AE = EF = x$. Thus $DE = 2 - x$. The Pythagorean Theorem on $\triangle CDE$ yields

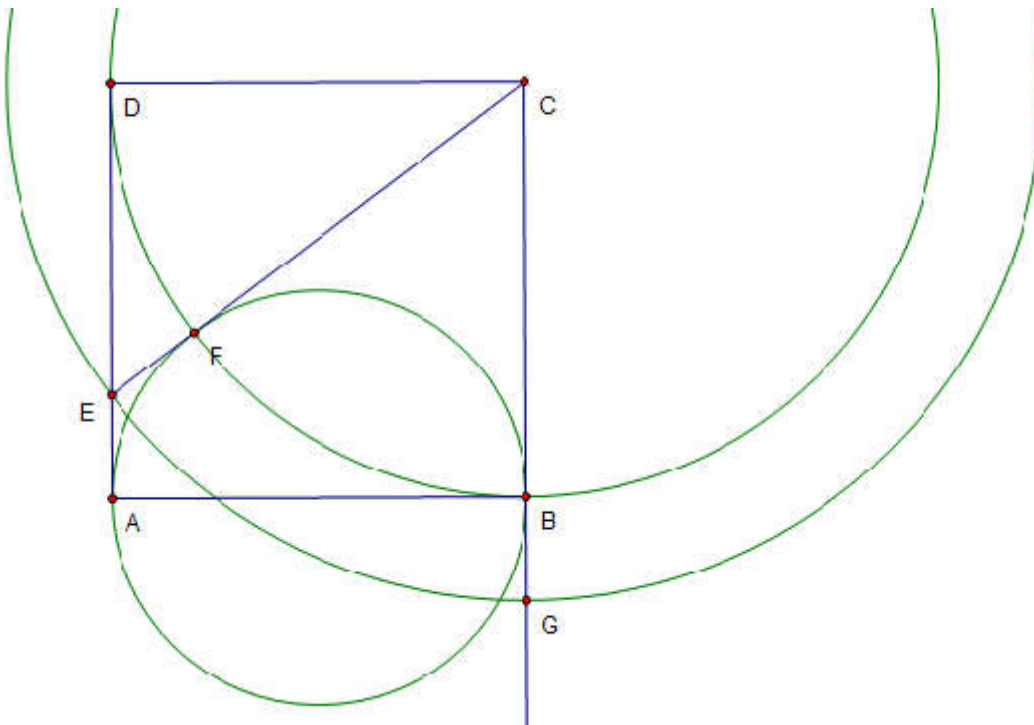
$$\begin{aligned}
 DE^2 + CD^2 &= CE^2 \\
 (2-x)^2 + 2^2 &= (2+x)^2 \\
 x^2 - 4x + 8 &= x^2 + 4x + 4 \\
 x &= \frac{1}{2}
 \end{aligned}$$

Hence $CE = FC + x = \frac{5}{2} \Rightarrow \boxed{\text{(D)} \frac{5}{2}}$.

Solution 2

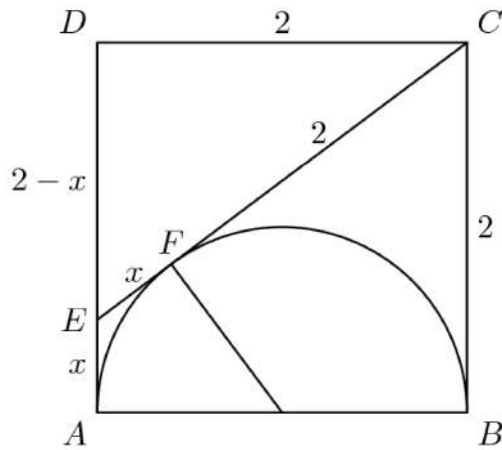
Call the point of tangency point F and the midpoint of AB as G . $CF = 2$ by the pythagorean theorem. Notice that $\angle EGF = \frac{180 - 2 \cdot \angle CGF}{2} = 90 - \angle CGF$. Thus, $EF = \cot \theta = \frac{1}{2}$. Adding, the answer is $\frac{5}{2}$.

Solution 3



Clearly, $EA = EF = BG$. Thus, the sides of right triangle CDE are in arithmetic progression. Thus it is similar to the triangle $3-4-5$ and since $DC = 2$, $CE = 5/2$.

Solution 4



Let us call the midpoint of side AB , point G . Since the semicircle has radius 1, we can do the Pythagorean theorem on sides GB, BC, GC . We get $GC = \sqrt{5}$. We then know that $CF = 2$ by Pythagorean theorem. Then by connecting EG , we get similar triangles EFG and GFC . Solving the ratios, we get $x = \frac{1}{2}$, so the answer is $\frac{5}{2} \Rightarrow \boxed{(D) \frac{5}{2}}$.

- Solution by [AOPS12142015](#)

See also

- AoPS topic (<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131334>)

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Category: Intermediate Geometry Problems

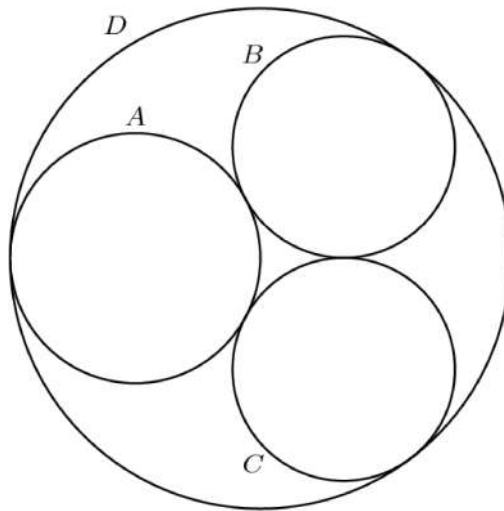
2004 AMC 10A Problems/Problem 23

Contents

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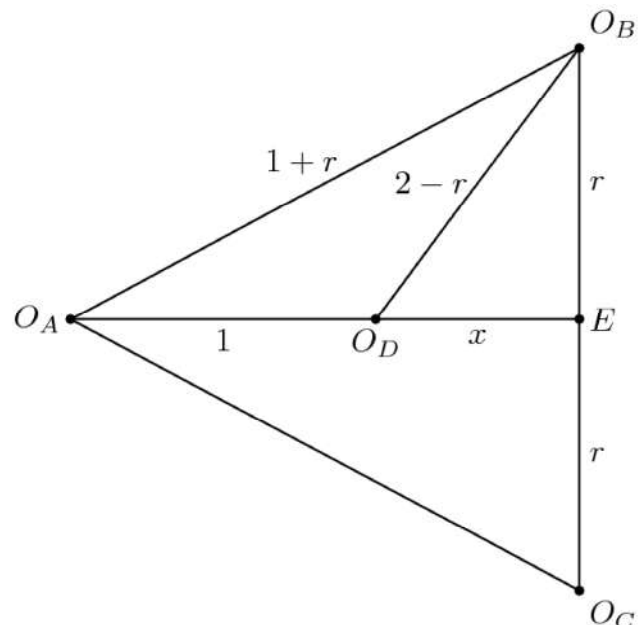
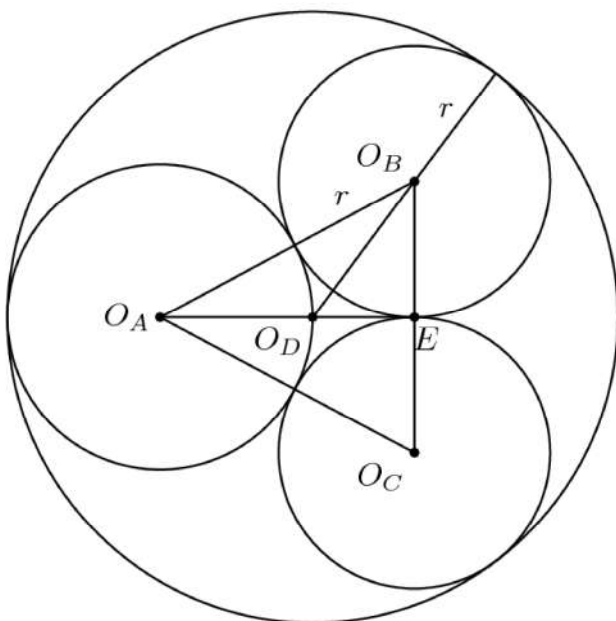
Problem

Circles A , B , and C are externally tangent to each other and internally tangent to circle D . Circles B and C are congruent. Circle A has radius 1 and passes through the center of D . What is the radius of circle B ?



- (A) $\frac{2}{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{7}{8}$ (D) $\frac{8}{9}$ (E) $\frac{1+\sqrt{3}}{3}$

Solution 1



Let O_i be the center of circle i for all $i \in \{A, B, C, D\}$ and let E be the tangent point of B, C . Since the radius of D is the diameter of A , the radius of D is 2 . Let the radius of B, C be r and let $O_D E = x$. If we connect O_A, O_B, O_C , we get an isosceles triangle with lengths $1 + r, 2r$. Then right triangle $O_D O_B O_E$ has legs r, x and hypotenuse $2 - r$. Solving for x , we get $x^2 = (2 - r)^2 - r^2 \implies x = \sqrt{4 - 4r}$.

Also, right triangle $O_A O_B O_E$ has legs $r, 1 + x$, and hypotenuse $1 + r$. Solving,

$$\begin{aligned} r^2 + (1 + \sqrt{4 - 4r})^2 &= (1 + r)^2 \\ 1 + 4 - 4r + 2\sqrt{4 - 4r} &= 2r + 1 \\ 1 - r &= \left(\frac{6r - 4}{4}\right)^2 \\ \frac{9}{4}r^2 - 2r &= 0 \\ r &= \frac{8}{9} \end{aligned}$$

So the answer is (D) $\frac{8}{9}$.

Solution 2

Using Descartes' Circle Formula, $\left(1 - \frac{1}{2} + \frac{1}{r} + \frac{1}{r}\right)^2 = 2\left(\frac{1}{4} + 1 + \frac{1}{r^2} + \frac{1}{r^2}\right)$. Solving this gives us linear equation with $r = \text{(D) } \frac{8}{9}$.

See also

- <url>viewtopic.php?t=131335 AoPS topic</url>

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Category: Introductory Geometry Problems

2004 AMC 12A Problems/Problem 17

The following problem is from both the 2004 AMC 12A #17 and 2004 AMC 10A #24, so both problems redirect to this page.

Problem

Let f be a function with the following properties:

- (i) $f(1) = 1$, and
- (ii) $f(2n) = n \times f(n)$, for any positive integer n .

What is the value of $f(2^{100})$?

- (A) 1 (B) 2^{99} (C) 2^{100} (D) 2^{4950} (E) 2^{9999}

Solution

$$\begin{aligned} f(2^{100}) &= f(2 \times 2^{99}) = 2^{99} \times f(2^{99}) = 2^{99} \cdot 2^{98} \times f(2^{98}) = \dots = 2^{99} 2^{98} \dots 2^1 \cdot 1 \cdot f(1) \\ &= 2^{99+98+\dots+2+1} = 2^{\frac{99(100)}{2}} = 2^{4950} \Rightarrow \text{(D)}. \end{aligned}$$

See also

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Category: Introductory Algebra Problems

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2004 AMC 12A Problems/Problem 22

The following problem is from both the 2004 AMC 12A #22 and 2004 AMC 10A #25, so both problems redirect to this page.

Problem

Three mutually tangent spheres of radius **1** rest on a horizontal plane. A sphere of radius **2** rests on them. What is the distance from the plane to the top of the larger sphere?

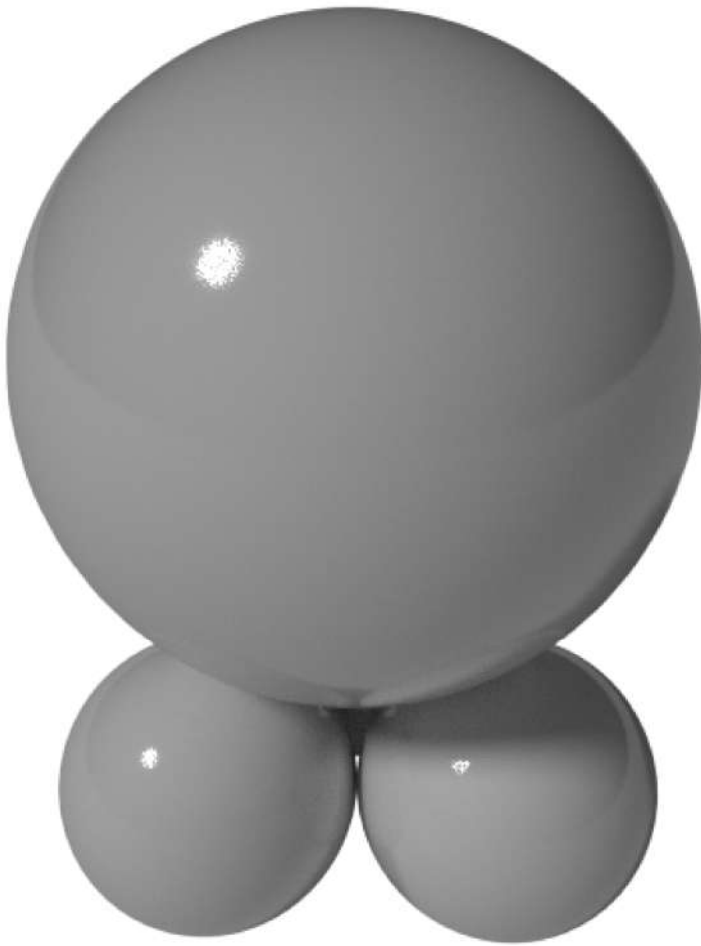
- (A) $3 + \frac{\sqrt{30}}{2}$ (B) $3 + \frac{\sqrt{69}}{3}$ (C) $3 + \frac{\sqrt{123}}{4}$ (D) $\frac{52}{9}$ (E) $3 + 2\sqrt{2}$

Solution

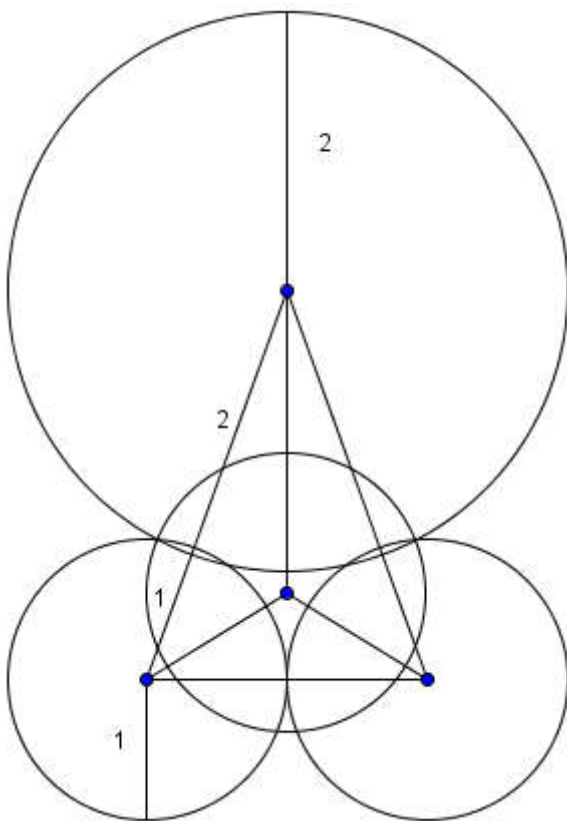
We draw the three spheres of radius **1**:



And then add the sphere of radius **2**:

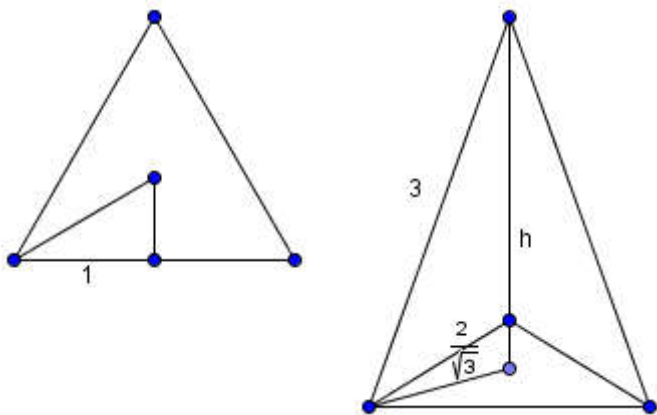


The height from the center of the bottom sphere to the plane is **1**, and from the center of the top sphere to the tip is **2**.



We now need the vertical height of the centers. If we connect the centers, we get a triangular pyramid with an equilateral triangle base. The distance from the vertex of the equilateral triangle to its centroid can be found by

$30 - 60 - 90\triangle_s$ to be $\frac{2}{\sqrt{3}}$



By the Pythagorean Theorem, we have $\left(\frac{2}{\sqrt{3}}\right)^2 + h^2 = 3^2 \implies h = \frac{\sqrt{69}}{3}$. Adding the heights up, we get

$\frac{\sqrt{69}}{3} + 1 + 2 \Rightarrow \boxed{\text{(B) } 3 + \frac{\sqrt{69}}{3}}$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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Categories: Introductory Geometry Problems | 3D Geometry Problems