2006 AMC 12A Problems/Problem 1

The following problem is from both the 2006 AMC 12A #1 and 2006 AMC 10A #1, so both problems redirect to this page.

Problem

Sandwiches at Joe's Fast Food cost $\$3$ each and sodas cost $\$2$ each. How many dollars will it cost to purchase 5 sandwiches and 8 sodas?

$\mathrm{(A) \ } 31\qquad \mathrm{(B) \ } 32\qquad \mathrm{(C) \ } 33\qquad \mathrm{(D) \ } 34\qquad \mathrm{(E) \ } 35$

Solution

The 5 sandwiches cost $5\cdot 3=15$ dollars. The 8 sodas cost $8\cdot 2=16$ dollars. In total, the purchase costs $15+16=31$ dollars. The answer is $\mathrm{(A)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 2

The following problem is from both the 2006 AMC 12A #2 and 2006 AMC 10A #2, so both problems redirect to this page.

Problem

Define $x \otimes y = x^3 - y$. What is $h \otimes (h \otimes h)$?

$\mathrm{(A)}\ -h\qquad\mathrm{(B)}\ 0\qquad\mathrm{(C)}\ h\qquad\mathrm{(D)}\ 2h\qquad\mathrm{(E)}\ h^3$

Solution

By the definition of $\otimes$, we have $h \otimes h = h^3 - h$. Then $h \otimes (h \otimes h) = h \otimes (h^3 - h) = h^3 - (h^3 - h) = h$. The answer is $\mathrm{(C)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 3

The following problem is from both the 2006 AMC 12A #3 and 2006 AMC 10A #3, so both problems redirect to this page.

Problem

The ratio of Mary's age to Alice's age is $3:5$. Alice is 30 years old. How old is Mary?

$\mathrm{(A)}\ 15\qquad \mathrm{(B)}\ 18\qquad \mathrm{(C)}\ 20\qquad \mathrm{(D)}\ 24\qquad \mathrm{(E)}\ 50$

Solution

Let m be Mary's age. Then $\frac{m}{30}=\frac{3}{5}$. Solving for m, we obtain $m=18$. The answer is $\mathrm{(B)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 4

The following problem is from both the 2006 AMC 12A #4 and 2008 AMC 10A #4, so both problems redirect to this page.

Problem

A digital watch displays hours and minutes with AM and PM. What is the largest possible sum of the digits in the display?

$\mathrm{(A)}\ 17\qquad\mathrm{(B)}\ 19\qquad\mathrm{(C)}\ 21\qquad\mathrm{(D)}\ 22\qquad\mathrm{(E)}\ 23$

Solution

From the greedy algorithm, we have 9 in the hours section and 59 in the minutes section.
$9+5+9=23 \Rightarrow \mathrm{(E)}$

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Number Theory Problems

2006 AMC 12A Problems/Problem 5

The following problem is from both the 2006 AMC 12A #5 and 2006 AMC 10A #5, so both problems redirect to this page.

Problem

Doug and Dave shared a pizza with 8 equally-sized slices. Doug wanted a plain pizza, but Dave wanted anchovies on half the pizza. The cost of a plain pizza was 8 dollars, and there was an additional cost of 2 dollars for putting anchovies on one half. Dave ate all the slices of anchovy pizza and one plain slice. Doug ate the remainder. Each paid for what he had eaten. How many more dollars did Dave pay than Doug?

$\mathrm{(A) \ } 1\qquad \mathrm{(B) \ } 2\qquad \mathrm{(C) \ } 3\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 5$

Solution

Dave and Doug paid $8+2=10$ dollars in total. Doug paid for three slices of plain pizza, which cost $\frac{3}{8}\cdot 8=3$. Dave paid $10-3=7$ dollars. Dave paid $7-3=4$ more dollars than Doug. The answer is $\mathrm{(D)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_12A_Problems/Problem_5&oldid=53344"

Category: Introductory Algebra Problems

2006 AMC 10A Problems/Problem 6

Problem

What non-zero real value for x satisfies $(7x)^{14} = (14x)^7$?

$\mathrm{(A)}\ \frac{1}{7} \qquad \mathrm{(B)}\ \frac{2}{7} \qquad \mathrm{(C)}\ 1 \qquad \mathrm{(D)}\ 7 \qquad \mathrm{(E)}\ 14$

Solution

Taking the seventh root of both sides, we get $(7x)^2 = 14x$.

Simplifying the LHS gives $49x^2 = 14x$, which then simplifies to $7x = 2$.

Thus, $x = \frac{2}{7}$, and the answer is $\mathrm{(B)}$.

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10A_Problems/Problem_6&oldid=54286"

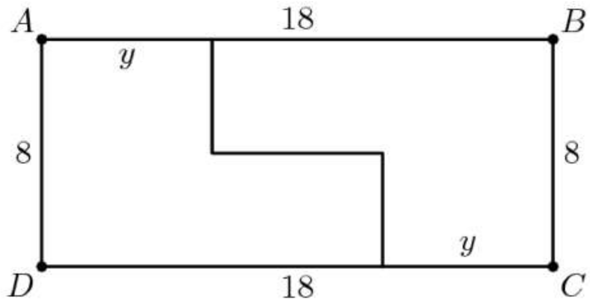
Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 6

The following problem is from both the 2006 AMC 12A #6 and 2006 AMC 10A #7, so both problems redirect to this page.

Problem

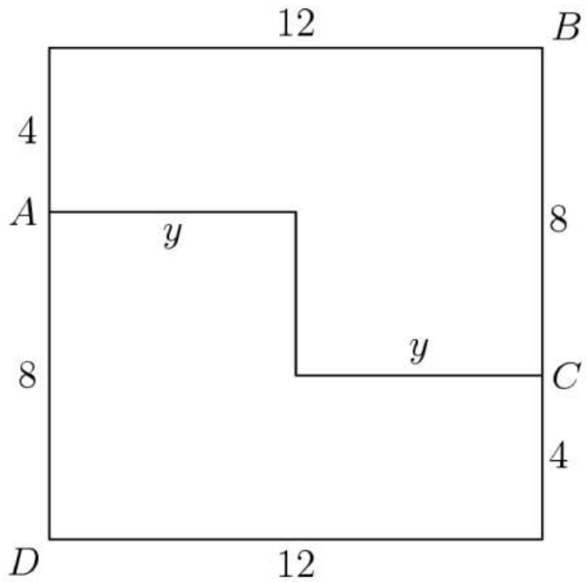
The 8×18 rectangle $ABCD$ is cut into two congruent hexagons, as shown, in such a way that the two hexagons can be repositioned without overlap to form a square. What is y?



$\mathrm{(A)}\ 6\qquad\mathrm{(B)}\ 7\qquad\mathrm{(C)}\ 8\qquad\mathrm{(D)}\ 9\qquad\mathrm{(E)}\ 10$

Solution

Since the two hexagons are going to be repositioned to form a square without overlap, the area will remain the same. The rectangle's area is $18 \cdot 8 = 144$. This means the square will have four sides of length 12. The only way to do this is shown below.



As you can see from the diagram, the line segment denoted as y is half the length of the side of the square, which leads to $y = \frac{12}{2} = 6 \Longrightarrow \mathrm{(A)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A Problems/Problem 8

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
  - 2.3 Solution 3
- 3 See also

Problem

A parabola with equation $y = x^2 + bx + c$ passes through the points $(2,3)$ and $(4,3)$. What is c?

$\mathrm{(A) \ } 2\qquad \mathrm{(B) \ } 5\qquad \mathrm{(C) \ } 7\qquad \mathrm{(D) \ } 10\qquad \mathrm{(E) \ } 11$

Solution

Solution 1

Substitute the points $(2,3)$ and $(4,3)$ into the given equation for (x,y).

Then we get a system of two equations:

$3 = 4 + 2b + c$

$3 = 16 + 4b + c$

Subtracting the first equation from the second we have:

$0 = 12 + 2b$

$b = -6$

Then using $b = -6$ in the first equation:

$0 = 1 + -12 + c$

$c = 11 \Longrightarrow \mathrm{(E)}$ is the answer.

Solution 2

Alternatively, notice that since the equation is that of a monic parabola, the vertex is likely $(3,2)$. Thus, the form of the equation of the parabola is $y - 2 = (x-3)^2$. Expanding this out, we find that $c = 11$.

Solution 3

The points given have the same y-value, so the vertex lies on the line $x = \frac{2+4}{2} = 3$.

The x-coordinate of the vertex is also equal to $\frac{-b}{2a}$, so set this equal to 3 and solve for b, given that $a = 1$:

$x = \frac{-b}{2a}$

$3 = \frac{-b}{2}$

$6 = -b$

$b = -6$

Now the equation is of the form $y = x^2 - 6x + c$. Now plug in the point $(2, 3)$ and solve for c:

$$y = x^2 - 6x + c$$

$$3 = 2^2 - 6(2) + c$$

$$3 = 4 - 12 + c$$

$$3 = -8 + c$$

$$\boxed{c = 11 (\text{E})}$$

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 8

The following problem is from both the 2006 AMC 12A #8 and 2008 AMC 10A #9, so both problems redirect to this page.

Problem

How many sets of two or more consecutive positive integers have a sum of 15?

$\mathrm{(A) \ } 1\qquad \mathrm{(B) \ } 2\qquad \mathrm{(C) \ } 3\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 5$

Solution

Notice that if the consecutive positive integers have a sum of 15, then their average (which could be a fraction) must be a divisor of 15. If the number of integers in the list is odd, then the average must be either 1, 3, or 5, and 1 is clearly not possible. The other two possibilities both work:

- $1 + 2 + 3 + 4 + 5 = 15$
- $4 + 5 + 6 = 15$

If the number of integers in the list is even, then the average will have a $\frac{1}{2}$. The only possibility is $\frac{15}{2}$, from which we get:

- $15 = 7 + 8$

Thus, the correct answer is 3, answer choice $\mathrm{(C)}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 12A Problems/Problem 10

The following problem is from both the 2006 AMC 12A #10 and 2006 AMC 10A #10, so both problems redirect to this page.

Problem

For how many real values of x is $\sqrt{120-\sqrt{x}}$ an integer?

$\mathrm{(A)}\ 3 \qquad \mathrm{(B)}\ 6 \qquad \mathrm{(C)}\ 9 \qquad \mathrm{(D)}\ 10 \qquad \mathrm{(E)}\ 11$

Solution

For $\sqrt{120-\sqrt{x}}$ to be an integer, $120-\sqrt{x}$ must be a perfect square.

Since $\sqrt{x}$ can't be negative, $120-\sqrt{x} \leq 120$.

The perfect squares that are less than or equal to 120 are $\{0,1,4,9,16,25,36,49,64,81,100\}$, so there are 11 values for $120-\sqrt{x}$.

Since every value of $120-\sqrt{x}$ gives one and only one possible value for x, the number of values of x is $11 \Rightarrow E$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10A Problems/Problem 11

Problem

Which of the following describes the graph of the equation $(x+y)^2 = x^2 + y^2$?

$\mathrm{(A)}\ \text{the empty set}\qquad\mathrm{(B)}\ \text{one point}\qquad\mathrm{(C)}\ \text{two lines}\qquad\mathrm{(D)}\ \text{a circle}\qquad\mathrm{(E)}\ \text{the entire plane}$

Solution

Expanding the left side, we have

$$x^2 + 2xy + y^2 = x^2 + y^2 \Longrightarrow 2xy = 0 \Longrightarrow xy = 0 \Longrightarrow x = 0 \text{ or } y = 0$$

Thus there are two lines described in this graph, the horizontal line $y = 0$ and the vertical line $x = 0$. Thus, our answer is $\mathrm{(C)}$.

See also

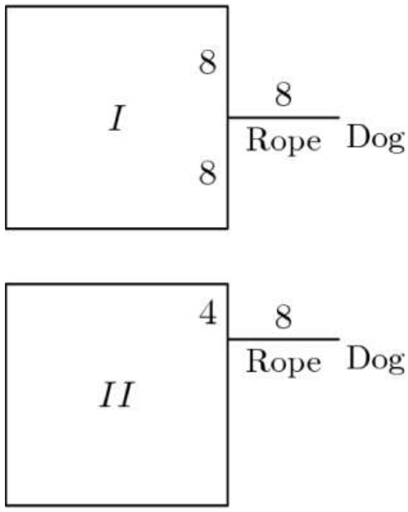
2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10A Problems/Problem 12

Problem

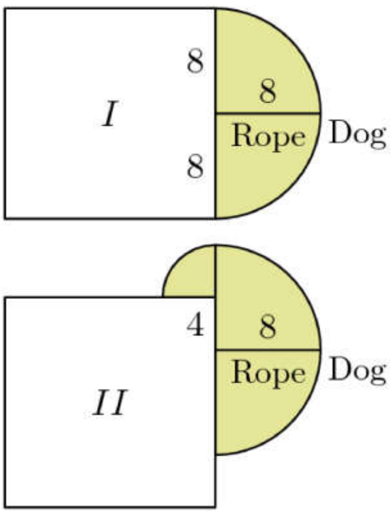
Rolly wishes to secure his dog with an 8-foot rope to a square shed that is 16 feet on each side. His preliminary drawings are shown.



Which of these arrangements give the dog the greater area to roam, and by how many square feet?

$\text{(A) } I,\ \textrm{ by } 8\pi\qquad \text{(B) } I,\ \textrm{ by } 6\pi\qquad \text{(C) } II,\ \textrm{ by } 4\pi\qquad \text{(D) } II,\ \textrm{ by } 8\pi\qquad \text{(E) } II,\ \textrm{ by } 10\pi$

Solution



Let us first examine the area of both possible arrangements. The rope outlines a circular boundary that the dog may dwell in. Arrangement I allows the dog $\frac{1}{2}\cdot(\pi\cdot 8^2)=32\pi$ square feet of area. Arrangement II allows 32π square feet plus a little more on the top part of the fence. So we already know that Arrangement II allows more freedom - only thing left is to find out how much. The extra area can be represented by a quarter of a circle with radius 4. So the extra area is $\frac{1}{4}\cdot(\pi\cdot 4^2)=4\pi$. Thus the answer is $\boxed{\text{(C)}}$.

See also

2006 AMC 10A Problems/Problem 13

Problem

A player pays $\$5$ to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number on the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning times the amount won is what the player should pay.)

$\mathrm{(A)}\ \$12 \qquad \mathrm{(B)}\ \$30 \qquad \mathrm{(C)}\ \$50 \qquad \mathrm{(D)}\ \$60 \qquad \mathrm{(E)}\ \$100$

Solution

The probability of rolling an even number on the first turn is $\frac{1}{2}$ and the probability of rolling the same number on the next turn is $\frac{1}{6}$The probability of winning is $\frac{1}{12}$. If the game is to be fair, the amount paid, 5 dollars, must be $\frac{1}{12}$ the amount of prize money, so the answer is D. 60

See also

<table>
<tr><td colspan="2">2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))</td></tr>
<tr><td>Preceded by
Problem 12</td><td>Followed by
Problem 14</td></tr>
<tr><td colspan="2">1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25</td></tr>
<tr><td colspan="2">All AMC 10 Problems and Solutions</td></tr>
</table>

Category: Introductory Combinatorics Problems

2006 AMC 12A Problems/Problem 12

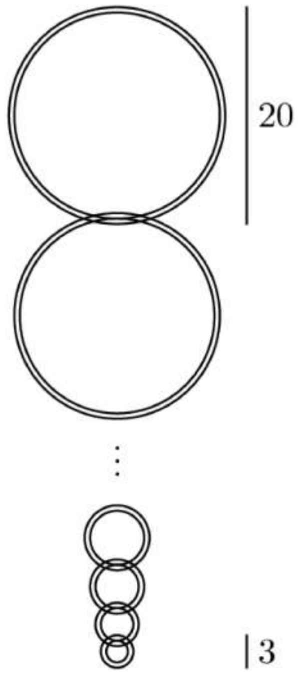
The following problem is from both the 2004 AMC 12A #12 and 2004 AMC 10A #14, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solutions
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See Also

Problem

A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the outer rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?



$\mathrm{(A)}\ 171\qquad\mathrm{(B)}\ 173\qquad\mathrm{(C)}\ 182\qquad\mathrm{(D)}\ 188\qquad\mathrm{(E)}\ 210$

Solutions

Solution 1

The inside diameters of the rings are the positive integers from 1 to 18. The total distance needed is the sum of these values plus 2 for the top of the first ring and the bottom of the last ring. Using the formula for the sum of an arithmetic series, the answer is $\frac{18\cdot 19}{2}+2=173\Rightarrow\mathrm{(B)}$.

Solution 2

Alternatively, the sum of the consecutive integers from 3 to 20 is $\frac{1}{2}(18)(3+20)=207$. However, the 17 intersections between the rings must be subtracted, and we also get $207-2(17)=173$.

See Also

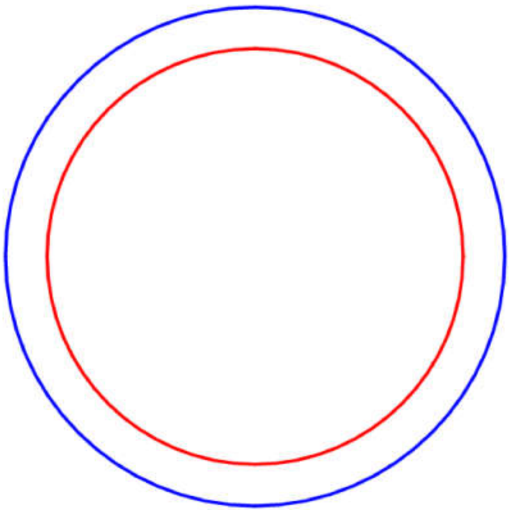
2006 AMC 10A Problems/Problem 15

Problem

Odell and Kershaw run for 30 minutes on a circular track. Odell runs clockwise at 250 m/min and uses the inner lane with a radius of 50 meters. Kershaw runs counterclockwise at 300 m/min and uses the outer lane with a radius of 60 meters, starting on the same radial line as Odell. How many times after the start do they pass each other?

$\mathrm{(A) \ } 29\qquad \mathrm{(B) \ } 42\qquad \mathrm{(C) \ } 45\qquad \mathrm{(D) \ } 47\qquad \mathrm{(E) \ } 50$

Solution



Since $d = rt$, we note that Odell runs one lap in $\frac{2 \cdot 50\pi}{250} = \frac{2\pi}{5}$ minutes, while Kershaw also runs one lap in $\frac{2 \cdot 60\pi}{300} = \frac{2\pi}{5}$ minutes. They take the same amount of time to run a lap, and since they are running in opposite directions they will meet exactly twice per lap (once at the starting point, the other at the half-way point). Thus, there are $\frac{30}{\frac{2\pi}{5}} \approx 23.8$ laps run by both, or $\lfloor 2 \cdot 23.8 \rfloor = 23 \cdot 2 + 1 = 47$ meeting points $\Longrightarrow \mathrm{(D)}$.

See Also

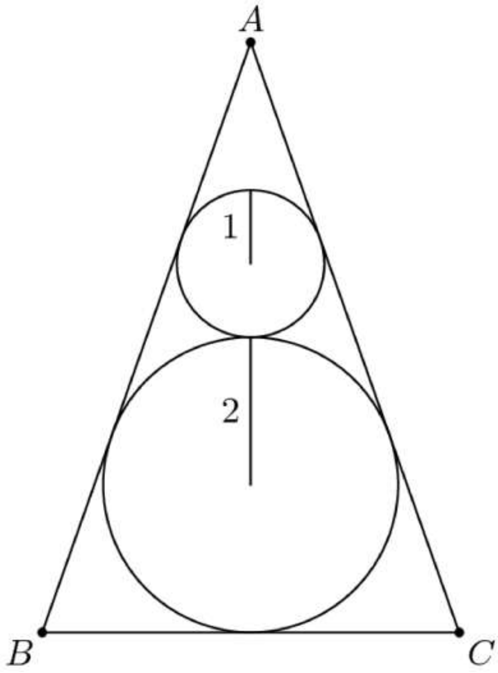
2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Algebra Problems | Introductory Geometry Problems

2006 AMC 10A Problems/Problem 16

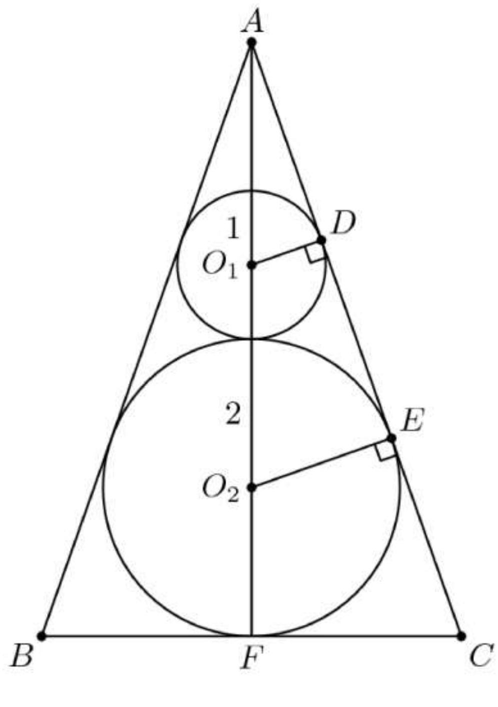
Problem

A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides $\overline{AB}$ and $\overline{AC}$ are congruent. What is the area of $\triangle ABC$?



$$\text{(A) } \frac{35}{2} \qquad \text{(B) } 15\sqrt{2} \qquad \text{(C) } \frac{64}{3} \qquad \text{(D) } 16\sqrt{2} \qquad \text{(E) } 24$$

Solution



Note that $\triangle ADO_1 \sim \triangle AEO_2 \sim \triangle AFC$. Using the first pair of similar triangles, we write the proportion:

$$\frac{AO_1}{AO_2} = \frac{DO_1}{EO_2} \implies \frac{AO_1}{AO_1 + 3} = \frac{1}{2} \implies AO_1 = 3$$

By the Pythagorean Theorem we have that $AD = \sqrt{3^2 - 1^2} = \sqrt{8}$.

Now using $\triangle ADO_1 \sim \triangle AFC$,

$$\frac{AD}{AF} = \frac{DO_1}{CF} \Longrightarrow \frac{2\sqrt{2}}{8} = \frac{1}{CF} \Longrightarrow CF = 2\sqrt{2}$$

The area of the triangle is
$\frac{1}{2} \cdot AF \cdot BC = \frac{1}{2} \cdot AF \cdot (2 \cdot CF) = AF \cdot CF = 8\left(2\sqrt{2}\right) = 16\sqrt{2}$ (D).

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

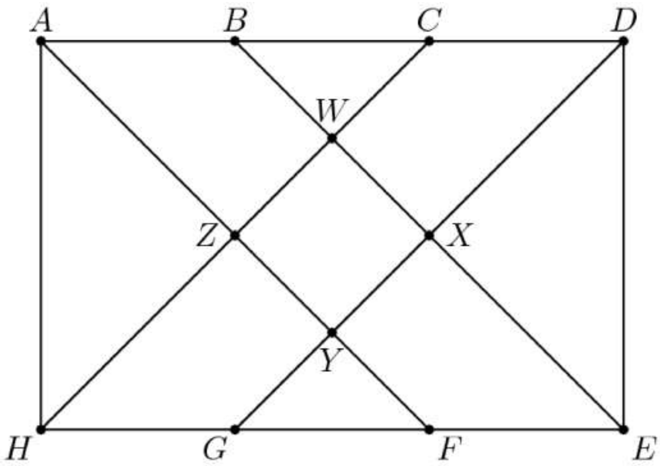
Categories: Introductory Geometry Problems | Circle Problems | Similar Triangles Problems

2006 AMC 10A Problems/Problem 17

Problem

In rectangle $ADEH$, points B and C trisect $\overline{AD}$, and points G and F trisect $\overline{HE}$. In addition, $AH = AC = 2$, and $AD = 3$. What is the area of quadrilateral $WXYZ$ shown in the figure?

$\text{(A)}\ \frac{1}{2} \qquad \text{(B)}\ \frac{\sqrt{2}}{2} \qquad \text{(C)}\ \frac{\sqrt{3}}{2} \qquad \text{(D)}\ \frac{2\sqrt{2}}{2} \qquad \text{(E)}\ \frac{2\sqrt{3}}{3}$



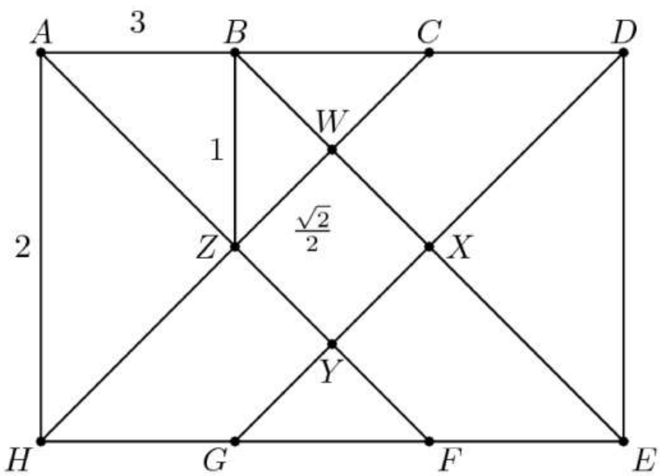
Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See Also

Solution

Solution 1

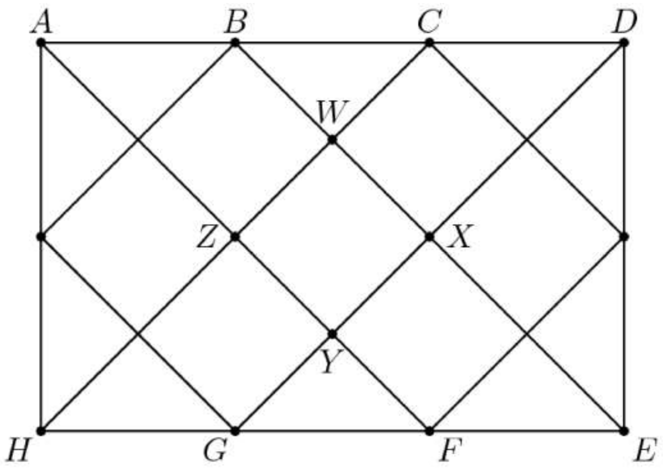
It is not difficult to see by symmetry that $WXYZ$ is a square.



Draw $\overline{BZ}$. Clearly $BZ = \frac{1}{2}AH = 1$. Then $\triangle BWZ$ is isosceles, and is a $45-45-90\triangle$. Hence $WZ = \frac{1}{\sqrt{2}}$, and $[WXYZ] = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$ $\mathrm{(A)}$.

There are many different similar ways to come to the same conclusion using different 45-45-90 triangles.

Solution 2



Draw the lines as shown above, and count the squares. There are 12, so we have $\frac{2 \cdot 3}{12} = \frac{1}{2}$.

See Also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Geometry Problems | Area Problems

2006 AMC 10A Problems/Problem 18

Problem

A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

$\mathrm{(A) \ } 10^4\times 26^2\qquad \mathrm{(B) \ } 10^3\times 26^3\qquad \mathrm{(C) \ } 5\times 10^4\times 26^2\qquad \mathrm{(D) \ } 10^2\times 26^4\qquad \mathrm{(E) \ } 5\times 10^3\times 26^3$

Solution

There are $10\cdot10\cdot10\cdot10=10^4$ ways to choose 4 digits.

There are $26\cdot26=26^2$ ways to choose the 2 letters.

For the letters to be next to each other, they can be the 1st and 2nd, 2nd and 3rd, 3rd and 4th, 4th and 5th, or the 5th and 6th characters. So, there are $6-1=5$ choices for the position of the letters.

Therefore, the number of distinct license plates is $5\times10^4\times26^2 \Longrightarrow \mathrm{C}.$

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Combinatorics Problems

2006 AMC 10A Problems/Problem 19

Problem

How many non-similar triangles have angles whose degree measures are distinct positive integers in arithmetic progression?

$\mathrm{(A) \ } 0\qquad \mathrm{(B) \ } 1\qquad \mathrm{(C) \ } 59\qquad \mathrm{(D) \ } 89\qquad \mathrm{(E) \ } 178$

Solution

The sum of the angles of a triangle is 180 degrees. For an arithmetic progression with an odd number of terms, the middle term is equal to the average of the sum of all of the terms, making it $\frac{180}{3}=60$ degrees. The minimum possible value for the smallest angle is 1 and the highest possible is 59 (since the numbers are distinct), so there are 59 possibilities $\Longrightarrow \mathrm{C}$.

See also

<table>
<tr><td colspan="2">2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))</td></tr>
<tr><td>Preceded by
Problem 18</td><td>Followed by
Problem 20</td></tr>
<tr><td colspan="2">1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25</td></tr>
<tr><td colspan="2">All AMC 10 Problems and Solutions</td></tr>
</table>

Category: Introductory Geometry Problems

2006 AMC 10A Problems/Problem 20

Problem

Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

$\textbf{(A) } \frac{1}{2}\qquad\textbf{(B) } \frac{3}{5}\qquad\textbf{(C) } \frac{2}{3}\qquad\textbf{(D) } \frac{4}{5}\qquad\textbf{(E) } 1$

Solution

For two numbers to have a difference that is a multiple of 5, the numbers must be congruent $\bmod 5$ (their remainders after division by 5 must be the same).

$0, 1, 2, 3, 4$ are the possible values of numbers in $\bmod 5$. Since there are only 5 possible values in $\bmod 5$ and we are picking 6 numbers, by the Pigeonhole Principle, two of the numbers must be congruent $\bmod 5$.

Therefore the probability that some pair of the 6 integers has a difference that is a multiple of 5 is $1 \Longrightarrow \mathrm{E}$.

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10A_Problems/Problem_20&oldid=54296"

Category: Introductory Number Theory Problems

2006 AMC 10A Problems/Problem 21

Problem

How many four-digit positive integers have at least one digit that is a 2 or a 3?

$\textbf{(A) } 2439 \qquad\textbf{(B) } 4096 \qquad\textbf{(C) } 4903 \qquad\textbf{(D) } 4904 \qquad\textbf{(E) } 5416$

Solution

Since we are asked for the number of positive 4-digit integers with at least 2 or 3 in it, we can find this by finding the total number of 4-digit integers and subtracting off those which do not have any 2s or 3s as digits.

The total number of 4-digit integers is $9 \cdot 10 \cdot 10 \cdot 10 = 9000$, since we have 10 choices for each digit except the first (which can't be 0).

Similarly, the total number of 4-digit integers without any 2 or 3 is $7 \cdot 8 \cdot 8 \cdot 8 = 3584$.

Therefore, the total number of positive 4-digit integers that have at least one 2 or 3 in their decimal representation is $9000 - 3584 = 5416 \Longrightarrow \textbf{(E)}$

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Combinatorics Problems

2006 AMC 12A Problems/Problem 14

The following problem is from both the 2006 AMC 12A #14 and 2006 AMC 10A #22, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solutions
  - 2.1 Solution 1
  - 2.2 Solution 2
  - 2.3 Solution 3
- 3 See also

Problem

Two farmers agree that pigs are worth 300 dollars and that goats are worth 210 dollars. When one farmer owes the other money, he pays the debt in pigs or goats, with "change" received in the form of goats or pigs as necessary. (For example, a 390 dollar debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?

$\mathrm{(A) \ } 5\qquad \mathrm{(B) \ } 10\qquad \mathrm{(C) \ } 30\qquad \mathrm{(D) \ } 90\qquad \mathrm{(E) \ } 210$

Solutions

Solution 1

The problem can be restated as an equation of the form $300p + 210g = x$, where p is the number of pigs, g is the number of goats, and x is the positive debt. The problem asks us to find the lowest x possible. p and g must be integers, which makes the equation a Diophantine equation. The Euclidean algorithm tells us that there are integer solutions to the Diophantine equation $am + bn = c$, where c is the greatest common divisor of a and b, and no solutions for any smaller c. Therefore, the answer is the greatest common divisor of 300 and 210, which is 30, $\mathrm{(C)}$

Solution 2

Alternatively, note that $300p + 210g = 30(10p + 7g)$ is divisible by 30 no matter what p and g are, so our answer must be divisible by 30. In addition, three goats minus two pigs give us $630 - 600 = 30$ exactly. Since our theoretical best can be achieved, it must really be the best, and the answer is $\mathrm{(C)}$. debt that can be resolved.

Solution 3

Let us simplify this problem. Dividing by 30, we get a pig to be: $\frac{300}{30} = 10$, and a goat to be $\frac{210}{30} = 7$. It becomes evident that if you exchange 5 pigs for 7 goats, we get the smallest positive difference - $5 \cdot 10 - 7 \cdot 7 = 50 - 49 = 1$. Since we originally divided by 30, we need to multiply again, thus getting the answer: $1 \cdot 30 = \mathrm{(C)} 30$

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

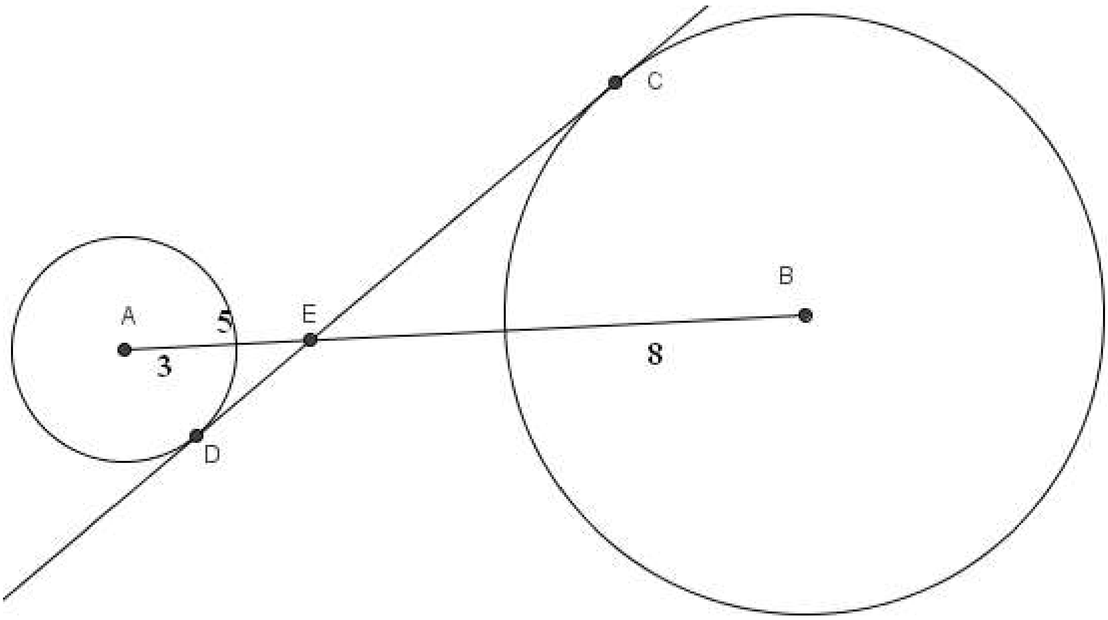
2006 AMC 12A Problems/Problem 16

The following problem is from both the 2006 AMC 12A #16 and 2006 AMC 10A #23, so both problems redirect to this page.

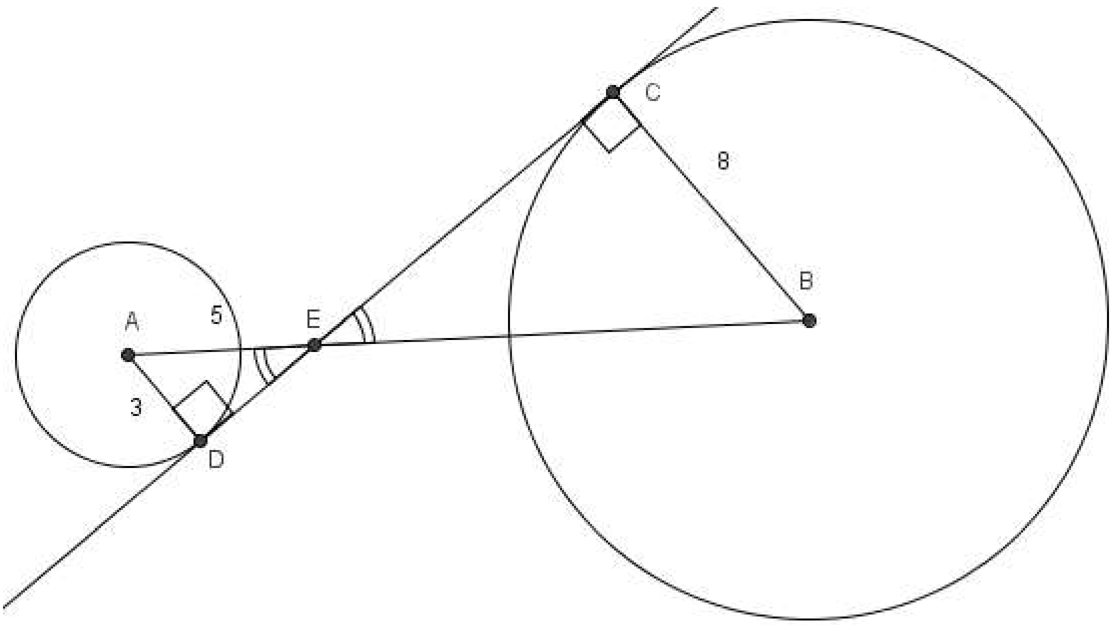
Problem

Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent intersects the circles at C and D, respectively. Lines AB and CD intersect at E, and $AE = 5$. What is CD?

$\mathrm{(A)}\ 13\qquad\mathrm{(B)}\ \frac{44}{3}\qquad\mathrm{(C)}\ \sqrt{221}\qquad\mathrm{(D)}\ \sqrt{255}\qquad\mathrm{(E)}\ \frac{55}{3}$



Solution



$\angle AED$ and $\angle BEC$ are vertical angles so they are congruent, as are angles $\angle ADE$ and $\angle BCE$ (both are right angles because the radius and tangent line at a point on a circle are always perpendicular). Thus, $\triangle ACE \sim \triangle BDE$.

By the Pythagorean Theorem, line segment $DE = 4$. The sides are proportional, so $\frac{DE}{AD} = \frac{CE}{BC} \Rightarrow \frac{4}{3} = \frac{CE}{8}$. This makes $CE = \frac{32}{3}$ and $CD = CE + DE = 4 + \frac{32}{3} = \frac{44}{3} \Longrightarrow \mathrm{B}$.

See also

2006 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2006))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2006 AMC 10A Problems/Problem 24

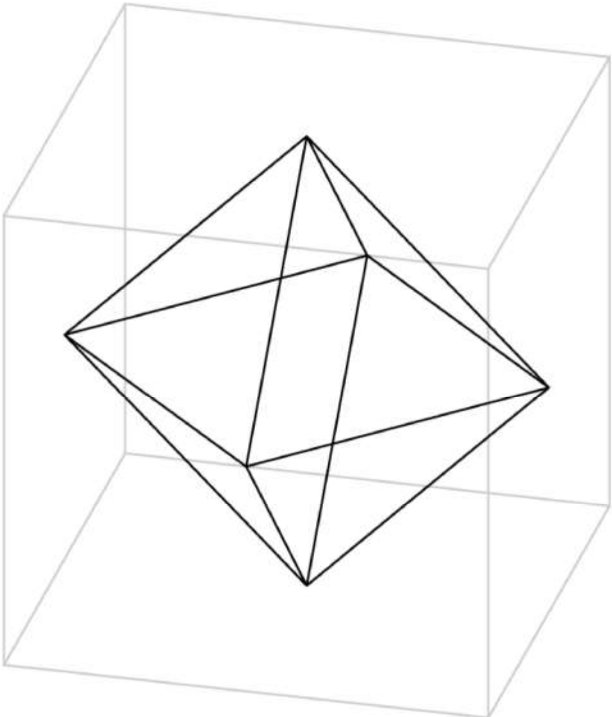
Problem

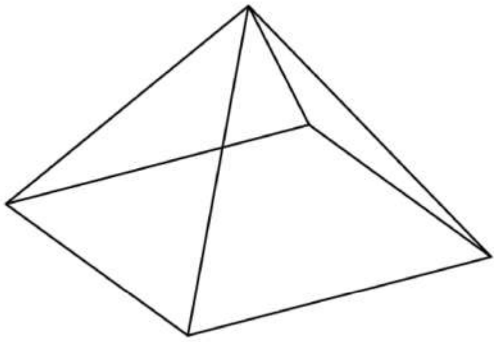
Centers of adjacent faces of a unit cube are joined to form a regular octahedron. What is the volume of this octahedron?

$\mathrm{(A)}\ \frac{1}{8}\qquad\mathrm{(B)}\ \frac{1}{6}\qquad\mathrm{(C)}\ \frac{1}{4}\qquad\mathrm{(D)}\ \frac{1}{3}\qquad\mathrm{(E)}\ \frac{1}{2}$

Solution

We can break the octahedron into two square pyramids by cutting it along a plane perpendicular to one of its internal diagonals.





The cube has edges of length 1 so all edges of the regular octahedron have length $\frac{\sqrt{2}}{2}$. Then the square base of the pyramid has area $\left(\frac{1}{2}\sqrt{2}\right)^2 = \frac{1}{2}$. We also know that the height of the pyramid is half the height of the cube, so it is $\frac{1}{2}$. The volume of a pyramid with base area B and height h is $A = \frac{1}{3}Bh$ so each of the pyramids has volume $\frac{1}{3}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$. The whole octahedron is twice this volume, so $\frac{1}{12}\cdot 2 = \frac{1}{6} \Longrightarrow \mathrm{(B)}$.

See also

2006 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2006 AMC 12A Problems/Problem 20

The following problem is from both the 2006 AMC 12A #20 and 2006 AMC 10A #25, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solutions
  - 2.1 Solution 1
  - 2.2 Solution 2 (using the answer choices)
- 3 See also

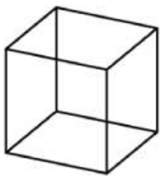
Problem

A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

$$\mathrm{(A)}\ \frac{1}{2187}\qquad\mathrm{(B)}\ \frac{1}{729}\qquad\mathrm{(C)}\ \frac{2}{243}\qquad\mathrm{(D)}\ \frac{1}{81}\qquad\mathrm{(E)}\ \frac{5}{243}$$

Solutions

Solution 1



Let us count the good paths. The bug starts at an arbitrary vertex, moves to a neighboring vertex (3 ways), and then to a new neighbor (2 more ways). So, without loss of generality, let the cube have vertices $ABCDEFGH$ such that $ABCD$ and $EFGH$ are two opposite faces with A above E and B above F. The bug starts at A and moves first to B, then to C.

From this point, there are two cases.

Case 1: the bug moves to D. From D, there is only one good move available, to H. From H, there are two ways to finish the trip, either by going $H \to G \to F \to E$ or $H \to E \to F \to G$. So there are 2 good paths in this case.

Case 2: the bug moves to G. Case 2a: the bug moves $G \to H$. In this case, there are 0 good paths because it will not be possible to visit both D and E without double-visiting some vertex. Case 2b: the bug moves $G \to F$. There is a unique good path in this case, $F \to E \to H \to D$.

Thus, all told we have 3 good paths after the first two moves, for a total of $3 \cdot 3 \cdot 2 = 18$ good paths. There were $3^7 = 2187$ possible paths the bug could have taken, so the probability a random path is good is the ratio of good paths to total paths, $\frac{18}{2187} = \frac{2}{243} \Rightarrow \boxed{(C)}$.

Solution 2 (using the answer choices)

As in Solution 1, the bug can move from its arbitrary starting vertex to a neighboring vertex in 3 ways. After this, the bug can move to a new neighbor in 2 ways (it cannot return to the first vertex). The total number of paths (as stated above) is 3^7 or 2187. Therefore, the probability of the bug following a good path is equal to $\frac{6x}{2187}$ for some positive integer x. The only answer choice which can be expressed in this form is $\frac{2}{243} \Rightarrow \boxed{(C)}$.