2006 AMC 10B Problems/Problem 1

Problem

What is $(-1)^1 + (-1)^2 + ... + (-1)^{2006}$?

$\textbf{(A)} \ -2006 \qquad \textbf{(B)} \ -1 \qquad \textbf{(C)} \ 0 \qquad \textbf{(D)} \ 1 \qquad \textbf{(E)} \ 2006$

Solution

Since -1 raised to an odd exponent is -1 and -1 raised to an even integer exponent is 1:

$$(-1)^1 + (-1)^2 + ... + (-1)^{2006} = (-1) + (1) + ... + (-1) + (1) = 0 \Longrightarrow \boxed{C}$$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by First Problem	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 2

Problem

For real numbers x and y, define $x\spadesuit y = (x+y)(x-y)$. What is $3\spadesuit(4\spadesuit 5)$?

$\mathrm{(A)}\ -72\qquad\mathrm{(B)}\ -27\qquad\mathrm{(C)}\ -24\qquad\mathrm{(D)}\ 24\qquad\mathrm{(E)}\ 72$

Solution

Since $x\spadesuit y = (x+y)(x-y)$:

$$3\spadesuit(4\spadesuit 5) = 3\spadesuit((4+5)(4-5)) = 3\spadesuit(-9) = (3+(-9))(3-(-9)) = -72 \Rightarrow A$$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 3

Problem

A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score?

$\mathrm{(A)}\ 10 \qquad \mathrm{(B)}\ 14 \qquad \mathrm{(C)}\ 17 \qquad \mathrm{(D)}\ 20 \qquad \mathrm{(E)}\ 24$

Solution

Let x be the number of points scored by the Cougars, and y be the number of points scored by the Panthers. The problem is asking for the value of y.

$x + y = 34$

$x - y = 14$

$2x = 48$

$x = 24$

$y = 10 \Rightarrow \boxed{A}$

See Also

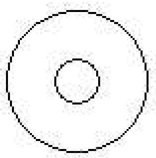
2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 4

Problem

Circles of diameter 1 inch and 3 inches have the same center. The smaller circle is painted red, and the portion outside the smaller circle and inside the larger circle is painted blue. What is the ratio of the blue-painted area to the red-painted area?



$\mathrm{(A) \ } 2\qquad \mathrm{(B) \ } 3\qquad \mathrm{(C) \ } 6\qquad \mathrm{(D) \ } 8\qquad \mathrm{(E) \ } 9$

Solution

The area painted red is equal to the area of the smaller circle and the area painted blue is equal to the area of the larger circle minus the area of the smaller circle.

So:

$$A_{red} = \pi(\frac{1}{2})^2 = \frac{\pi}{4}$$

$$A_{blue} = \pi(\frac{3}{2})^2 - \pi(\frac{1}{2})^2 = 2\pi$$

So the desired ratio is:

$$\frac{2\pi}{\frac{\pi}{4}} = 8 \Rightarrow D$$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2006 AMC 10B Problems/Problem 5

Problem

A 2×3 rectangle and a 3×4 rectangle are contained within a square without overlapping at any point, and the sides of the square are parallel to the sides of the two given rectangles. What is the smallest possible area of the square?

$\mathrm{(A)}\ 16 \qquad \mathrm{(B)}\ 25 \qquad \mathrm{(C)}\ 36 \qquad \mathrm{(D)}\ 49 \qquad \mathrm{(E)}\ 64$

Solution

By placing the 2×3 rectangle adjacent to the 3×4 rectangle with the 3 side of the 2×3 rectangle next to the 4 side of the 3×4 rectangle, we get a figure that can be completely enclosed in a square with a side length of 5. The area of this square is $5^2 = 25$.

Since the sum of the areas of the two rectangles is $2 \cdot 3 + 3 \cdot 4 = 18$, the area of a square cannot be less than 18. Therefore 16 is not possible.

So the answer is $25 \Rightarrow B$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

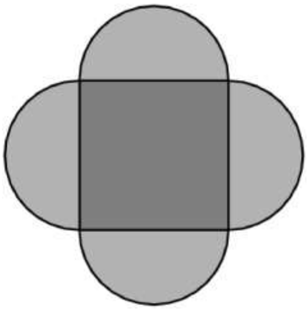
Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_5&oldid=54522"

Category: Introductory Geometry Problems

2006 AMC 10B Problems/Problem 6

Problem

A region is bounded by semicircular arcs constructed on the side of a square whose sides measure $\frac{2}{\pi}$, as shown. What is the perimeter of this region?



$\textbf{(A) }\frac{4}{\pi}\qquad\textbf{(B) }2\qquad\textbf{(C) }\frac{8}{\pi}\qquad\textbf{(D) }4\qquad\textbf{(E) }\frac{16}{\pi}$

Solution

Since the side of the square is the diameter of the semicircle, the radius of the semicircle is $\frac{1}{2}\cdot\frac{2}{\pi}=\frac{1}{\pi}$.

Since the length of one of the semicircular arcs is half the circumference of the corresponding circle, the length of one arc is $\frac{1}{2}\cdot 2\cdot\pi\cdot\frac{1}{\pi}=1$.

Since the desired perimeter is made up of four of these arcs, the perimeter is $4\cdot 1 = 4 \Longrightarrow \boxed{\textbf{(D)}}$.

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_6&oldid=54523"

Categories: Introductory Geometry Problems | Circle Problems

2006 AMC 10B Problems/Problem 7

Problem

Which of the following is equivalent to $\sqrt{\frac{x}{1-\frac{x-1}{x}}}$ when $x<0$?

$\mathrm{(A) \ } -x\qquad \mathrm{(B) \ } x\qquad \mathrm{(C) \ } 1\qquad \mathrm{(D) \ } \sqrt{\frac{x}{2}}\qquad \mathrm{(E) \ } x\sqrt{-1}$

Solution

$$\sqrt{\frac{x}{1-\frac{x-1}{x}}}=\sqrt{\frac{x}{\frac{x}{x}-\frac{x-1}{x}}}=\sqrt{\frac{x}{\frac{x-(x-1)}{x}}}=\sqrt{\frac{x}{\frac{1}{x}}}=\sqrt{x^2}=|x|$$

Since $x<0$

$|x|=-x\Rightarrow A$

See Also

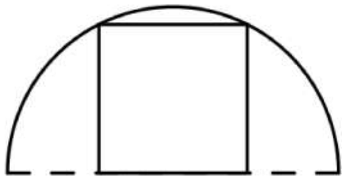
2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 8

Problem

A square of area 40 is inscribed in a semicircle as shown. What is the area of the semicircle?



$\text{(A) } 20\pi \qquad \text{(B) } 25\pi \qquad \text{(C) } 30\pi \qquad \text{(D) } 40\pi \qquad \text{(E) } 50\pi$

Solution

Since the area of the square is 40, the length of the side is $\sqrt{40} = 2\sqrt{10}$. The distance between the center of the semicircle and one of the bottom vertecies of the square is half the length of the side, which is $\sqrt{10}$.

Using the Pythagorean Theorem to find the square of radius, $r^2 = (2\sqrt{10})^2 + (\sqrt{10})^2 = 50$. So, the area of the semicircle is $\frac{1}{2} \cdot \pi \cdot 50 = 25\pi \Longrightarrow \boxed{\text{(B)}}$.

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Geometry Problems | Area Problems | Circle Problems

2006 AMC 10B Problems/Problem 9

Problem

Francesca uses 100 grams of lemon juce, 100 grams of sugar, and 400 grams of water to make lemonade. There are 25 calories in 100 grams of lemon juice and 386 calories in 100 grams of sugar. Water contains no calories. How many calories are in 200 grams of her lemonade?

$\mathrm{(A) \ } 129\qquad \mathrm{(B) \ } 137\qquad \mathrm{(C) \ } 174\qquad \mathrm{(D) \ } 233\qquad \mathrm{(E) \ } 411$

Solution

The calorie to gram ratio of Francesca's lemonade is $\frac{25+386+0}{100+100+400}=\frac{411\text{calories}}{600\text{grams}}=\frac{137\text{calories}}{200\text{grams}}$

So in 200grams of Francesca's lemonade there are $200\text{grams}\cdot\frac{137\text{calories}}{200\text{grams}}=137\text{calories}\Rightarrow B$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_9&oldid=54526"

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 10

Problem

In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

$\mathrm{(A)}\ 43 \qquad \mathrm{(B)}\ 44 \qquad \mathrm{(C)}\ 45 \qquad \mathrm{(D)}\ 46 \qquad \mathrm{(E)}\ 47$

Solution

Let x be the length of the first side.

The lengths of the sides are: x, $3x$, and 15.

By the Triangle Inequality,

$$3x < x + 15$$

$$2x < 15$$

$$x < \frac{15}{2}$$

The largest integer satisfing this inequality is 7.

So the largest perimeter is $7 + 3 \cdot 7 + 15 = 43 \Rightarrow A$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2006 AMC 10B Problems/Problem 11

Problem

What is the tens digit in the sum $7! + 8! + 9! + ... + 2006!$

$\mathrm{(A) \ } 1\qquad \mathrm{(B) \ } 3\qquad \mathrm{(C) \ } 4\qquad \mathrm{(D) \ } 6\qquad \mathrm{(E) \ } 9$

Solution

Since $10!$ is divisible by 100, any factorial greater than $10!$ is also divisible by 100. The last two digits of all factorials greater than $10!$ are 00, so the last two digits of $10! + 11! + ... + 2006!$ are 00. (*)

So all that is needed is the tens digit of the sum $7! + 8! + 9!$

$7! + 8! + 9! = 5040 + 40320 + 362880 = 408240$

So the tens digit is $4 \Rightarrow C$

(*) A slightly faster method would have to take the $\pmod{100}$ residue of $7!+8!+9!$. Since $7!=5040$, we can rewrite the sum as

$$5040+8\cdot 5040+72\cdot 5040\equiv 40+8\cdot 40+72\cdot 40=40+320+2880\equiv 40 \pmod{100}.$$

Since the last two digits of the sum is 40, the tens digit is 4.

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_11&oldid=72605"

Category: Introductory Number Theory Problems

2006 AMC 10B Problems/Problem 12

Problem

The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point $(1,2)$. What is $a+b$?

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ \frac{3}{4} \qquad (\mathrm{C})\ 1 \qquad (\mathrm{D})\ 2 \qquad (\mathrm{E})\ \frac{9}{4}$

Solution

Since $(1,2)$ is a solution to both equations, plugging in $x=1$ and $y=2$ will give the values of a and b.

$1 = \frac{1}{4} \cdot 2 + a$

$a = \frac{1}{2}$

$2 = \frac{1}{4} \cdot 1 + b$

$b = \frac{7}{4}$

So: $a + b = \frac{1}{2} + \frac{7}{4} = \frac{9}{4} \Rightarrow E$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

- Lines

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 13

Problem

Joe and JoAnn each bought 12 ounces of coffee in a 16 ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

$$\text{(A) } \frac{6}{7} \qquad \text{(B) } \frac{13}{14} \qquad \text{(C) } 1 \qquad \text{(D) } \frac{14}{13} \qquad \text{(E) } \frac{7}{6}$$

Solution

After drinking and adding cream, Joe's cup has 2 ounces of cream.

After adding cream to her cup, JoAnn's cup had 14 ounces of liquid. By drinking 2 ounces out of the 14 ounces of liquid, she drank $\frac{2}{14} = \frac{1}{7}$th of the cream. So there is $2 \cdot \frac{6}{7} = \frac{12}{7}$ ounces of cream left.

So the desired ratio is: $\frac{2}{\frac{12}{7}} = \frac{7}{6} \Rightarrow E$

This is problem 6.2.5 in the Art of Problem Solving, Introduction to Algebra

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_13&oldid=58431"

Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 14

Problem

Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 - px + q = 0$. What is q?

$\text{(A)}\ \frac{5}{2} \qquad \text{(B)}\ \frac{7}{2} \qquad \text{(C)}\ 4 \qquad \text{(D)}\ \frac{9}{2} \qquad \text{(E)}\ 8$

Solution

In a quadratic equation in the form $x^2 + bx + c = 0$, the product of the roots is c(Vieta's Formulas).

Using this property, we have that $ab = 2$ and

$$q = (a + \frac{1}{b}) \cdot (b + \frac{1}{a}) = \frac{ab+1}{b} \cdot \frac{ab+1}{a} = \frac{(ab+1)^2}{ab} = \frac{(2+1)^2}{2} = \frac{9}{2} \Rightarrow D$$

- Notice the fact that we never actually found the roots.

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

- Vieta's formulas

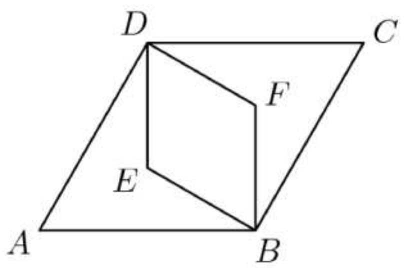
Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 15

Contents

Problem

Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24 and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$?



$\mathrm{(A) \ } 6 \qquad \mathrm{(B) \ } 4\sqrt{3} \qquad \mathrm{(C) \ } 8 \qquad \mathrm{(D) \ } 9 \qquad \mathrm{(E) \ } 6\sqrt{3}$

Solution

Using properties of a rhombus, $\angle DAB = \angle DCB = 60^\circ$ and $\angle ADC = \angle ABC = 120^\circ$. It is easy to see that rhombus $ABCD$ is made up of equilateral triangles DAB and DCB. Let the lengths of the sides of rhombus $ABCD$ be s.

The longer diagonal of rhombus $BFDE$ is BD. Since BD is a side of an equilateral triangle with a side length of s, $BD = s$. The longer diagonal of rhombus $ABCD$ is AC. Since AC is twice the length of an altitude of of an equilateral triangle with a side length of s, $AC = 2 \cdot \frac{s\sqrt{3}}{2} = s\sqrt{3}$

The ratio of the longer diagonal of rhombus $BFDE$ to rhombus $ABCD$ is $\frac{s}{s\sqrt{3}} = \frac{\sqrt{3}}{3}$. Therefore, the ratio of the area of rhombus $BFDE$ to rhombus $ABCD$ is $\left(\frac{\sqrt{3}}{3} \right)^2 = \frac{1}{3}$

Let x be the area of rhombus $BFDE$. Then $\frac{x}{24} = \frac{1}{3}$, so $x = 8 \Longrightarrow \boxed{\mathrm{(C)}}$.

Solution 2

Triangle DAB is equilateral so triangles DEA, AEB, BED, BFD, BFC and CFD are all congruent with angles 30°, 30° and 120° from which it follows that rhombus $BFDE$ has one third the area of rhombus $ABCD$ i.e. $8 \Longrightarrow \boxed{\mathrm{(C)}}$.

See Also

2006 AMC 10B Problems/Problem 16

Problem

Leap Day, February 29, 2004, occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020, occur?

$\mathrm{(A) \ } \text{Tuesday} \qquad \mathrm{(B) \ } \text{Wednesday} \qquad \mathrm{(C) \ } \text{Thursday} \qquad \mathrm{(D) \ } \text{Friday} \qquad \mathrm{(E) \ } \text{Saturday}$

Solution

There are 365 days in a year, plus 1 extra day if there is a Leap Day, which occurs on years that are multiples of 4 (with a few exceptions that don't affect this problem).

Therefore, the number of days between Leap Day 2004 and Leap Day 2020 is:

$$16 \cdot 365 + 4 \cdot 1 = 5844$$

Since the days of the week repeat every 7 days and $5844 \equiv -1 \bmod 7$, the day of the week Leap Day 2020 occurs is the day of the week the day before Leap Day 2004 occurs, which is $\text{Saturday} \Rightarrow E$.

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Number Theory Problems

2006 AMC 10B Problems/Problem 17

Problem

Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

$$\mathrm{(A)}\ \frac{1}{10}\qquad\mathrm{(B)}\ \frac{1}{6}\qquad\mathrm{(C)}\ \frac{1}{5}\qquad\mathrm{(D)}\ \frac{1}{3}\qquad\mathrm{(E)}\ \frac{1}{2}$$

Solution

Since there is the same amounts of all the balls in Alice's bag and Bob's bag, and there is an equal chance of each ball being selected, the color of the ball that Alice puts in Bob's bag doesn't matter. Without loss of generality, let the ball Alice puts in Bob's bag be red.

For both bags to have the same contents, Bob must select one of the 2 red balls out of the 6 balls in his bag.

So the desired probability is $\frac{2}{6}=\frac{1}{3}\Rightarrow D$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Combinatorics Problems

2006 AMC 10B Problems/Problem 18

Problem

Let $a_1, a_2, \dots$ be a sequence for which $a_1 = 2$, $a_2 = 3$, and $a_n = \frac{a_{n-1}}{a_{n-2}}$ for each positive integer $n \ge 3$. What is a_{2006}?

$\mathrm{(A)}\ \frac{1}{2} \qquad \mathrm{(B)}\ \frac{2}{3} \qquad \mathrm{(C)}\ \frac{3}{2} \qquad \mathrm{(D)}\ 2 \qquad \mathrm{(E)}\ 3$

Solution

Looking at the first few terms of the sequence:

$$a_1 = 2, a_2 = 3, a_3 = \frac{3}{2}, a_4 = \frac{1}{2}, a_5 = \frac{1}{3}, a_6 = \frac{2}{3}, a_7 = 2, a_8 = 3, \dots.$$

Clearly, the sequence repeats every 6 terms.

Since $2006 \equiv 2 \bmod 6$,

$a_{2006} = a_2 = 3 \Rightarrow E$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2006_AMC_10B_Problems/Problem_18&oldid=63290"

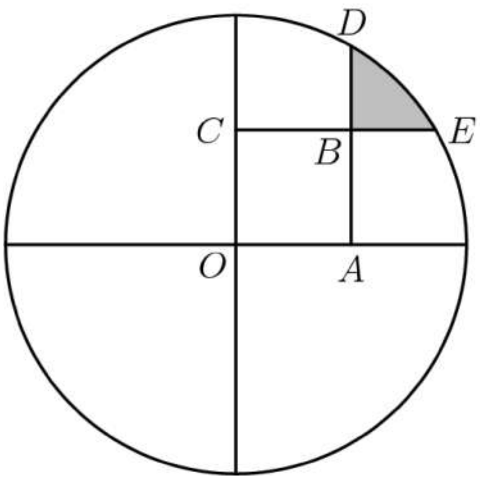
Category: Introductory Algebra Problems

2006 AMC 10B Problems/Problem 19

Contents

Problem

A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides AB and CB are extended past B to meet the circle at D and E, respectively. What is the area of the shaded region in the figure, which is bounded by BD, BE, and the minor arc connecting D and E?



$\mathrm{(A)}\ \frac{\pi}{3}+1-\sqrt{3}\qquad\mathrm{(B)}\ \frac{\pi}{2}(2-\sqrt{3})\qquad\mathrm{(C)}\ \pi(2-\sqrt{3})\qquad\mathrm{(D)}\ \frac{\pi}{6}+\frac{\sqrt{3}+1}{2}\qquad\mathrm{(E)}\ \frac{\pi}{3}-1+\sqrt{3}$

Solution 1

The shaded area is equivalent to the area of sector DOE, minus the area of triangle DOE plus the area of triangle DBE.

Using the Pythagorean Theorem, $(DA)^2=(CE)^2=2^2-1^2=3$ so $DA=CE=\sqrt{3}$.

Clearly, DOA and EOC are $30-60-90$ triangles with $\angle EOC=\angle DOA=60^\circ$. Since $OABC$ is a square, $\angle COA=90^\circ$.

$\angle DOE$ can be found by doing some subtraction of angles.

$\angle COA-\angle DOA=\angle EOA$

$90^\circ-60^\circ=\angle EOA=30^\circ$

$\angle DOA-\angle EOA=\angle DOE$

$60^\circ-30^\circ=\angle DOE=30^\circ$

So, the area of sector DOE is $\frac{30}{360}\cdot\pi\cdot2^2=\frac{\pi}{3}$.

The area of triangle DOE is $\frac{1}{2}\cdot2\cdot2\cdot\sin30^\circ=1$.

Since $AB = CB = 1$, $DB = ED = (\sqrt{3}-1)$. So, the area of triangle DBE is $\frac{1}{2}\cdot(\sqrt{3}-1)^2 = 2-\sqrt{3}$. Therefore, the shaded area is $(\frac{\pi}{3}) - (1) + (2-\sqrt{3}) = \frac{\pi}{3} + 1 - \sqrt{3} \Longrightarrow \boxed{(\text{A})}$

Solution 2

From the pythagorean theorem, we can see that DA is $\sqrt{3}$. Then, $DB = DA - BA = \sqrt{3} - 1$. The area of the shaded element is the area of sector DOE minus the areas of triangle DBO and triange EBO combined. Because $[DBO] = [EBO] = \frac{\sqrt{3}-1}{2}$ (Using the Base Altitude formula, where DB and BE are the bases and OA and CO are the altitudes, respectively), we have the area of sector DBE to be $\frac{\pi}{3} + 1 - \sqrt{3} \Longrightarrow \boxed{A}$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Geometry Problems | Area Problems | Circle Problems

2006 AMC 10B Problems/Problem 20

Contents

Problem

In rectangle $ABCD$, we have $A=(6,-22)$, $B=(2006,178)$, $D=(8,y)$, for some integer y. What is the area of rectangle $ABCD$?

$\mathrm{(A) \ } 4000\qquad \mathrm{(B) \ } 4040\qquad \mathrm{(C) \ } 4400\qquad \mathrm{(D) \ } 40,000\qquad \mathrm{(E) \ } 40,400$

Solution

Solution 1

Let the slope of AB be m_1 and the slope of AD be m_2.

$$m_1 = \frac{178-(-22)}{2006-6} = \frac{1}{10}$$

$$m_2 = \frac{y-(-22)}{8-6} = \frac{y+22}{2}$$

Since AB and AD form a right angle:

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = -10$$

$$\frac{y+22}{2} = -10$$

$$y = -42$$

Using the distance formula:

$$AB = \sqrt{(2006-6)^2 + (178-(-22))^2} = \sqrt{(2000)^2 + (200)^2} = 200\sqrt{101}$$

$$AD = \sqrt{(8-6)^2 + (-42-(-22))^2} = \sqrt{(2)^2 + (-20)^2} = 2\sqrt{101}$$

Therefore the area of rectangle $ABCD$ is $200\sqrt{101}\cdot2\sqrt{101} = 40,400 \Rightarrow E$

Solution 2

This solution is the same as Solution 1 up to the point where we find that $y=-42$.

We build right triangles so we can use the Pythagorean Theorem. The triangle with hypotenuse AB has legs 200 and 2000, while the triangle with hypotenuse AD has legs 2 and 20. Aha! The two triangles are similar, with one triangle having side lengths 100 times the other!

Let $AD=x$. Then from our reasoning above, we have $AB=100x$. Finally, the area of the rectangle is $100x(x)=100x^2=100(20^2+2^2)=100(400+4)=100(404)=\boxed{40400\ \mathrm{(E)}}$.

2006 AMC 10B Problems/Problem 21

Problem

For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6, on each die are in the ratio $1:2:3:4:5:6$. What is the probability of rolling a total of 7 on the two dice?

$\mathrm{(A)}\ \frac{4}{63}\qquad \mathrm{(B)}\ \frac{1}{8}\qquad \mathrm{(C)}\ \frac{8}{63}\qquad \mathrm{(D)}\ \frac{1}{6}\qquad \mathrm{(E)}\ \frac{2}{7}$

Solution

Let x be the probability of rolling a 1. The probabilities of rolling a 2, 3, 4, 5, and 6 are $2x$, $3x$, $4x$, $5x$, and $6x$, respectively.

The sum of the probabilities of rolling each number must equal 1, so

$$x+2x+3x+4x+5x+6x=1$$

$$21x=1$$

$$x=\frac{1}{21}$$

So the probabilities of rolling a 1, 2, 3, 4, 5, and 6 are respectively $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}$, and $\frac{6}{21}$.

The possible combinations of two rolls that total 7 are: $(1,6) ; (2,5) ; (3,4) ; (4,3) ; (5,2) ; (6,1)$

The probability of rolling a total of 7 on the two dice is equal to the sum of the probabilities of rolling each combination.

$$P=\frac{1}{21}\cdot\frac{6}{21}+\frac{2}{21}\cdot\frac{5}{21}+\frac{3}{21}\cdot\frac{4}{21}+\frac{4}{21}\cdot\frac{3}{21}+\frac{5}{21}\cdot\frac{2}{21}+\frac{6}{21}\cdot\frac{1}{21}=\frac{8}{63} \Rightarrow C$$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Combinatorics Problems

2006 AMC 10B Problems/Problem 22

Problem

Elmo makes N sandwiches for a fundraiser. For each sandwich he uses B globs of peanut butter at $4¢$ per glob and J blobs of jam at $5¢$ per blob. The cost of the peanut butter and jam to make all the sandwiches is $\$2.53$. Assume that B, J, and N are positive integers with $N > 1$. What is the cost of the jam Elmo uses to make the sandwiches?

$\mathrm{(A)}\ \$1.05 \qquad \mathrm{(B)}\ \$1.25 \qquad \mathrm{(C)}\ \$1.45 \qquad \mathrm{(D)}\ \$1.65 \qquad \mathrm{(E)}\ \$1.85$

Solution

The peanut butter and jam for each sandwich costs $4B + 5J¢$, so the peanut butter and jam for N sandwiches costs $N(4B + 5J)¢$.

Setting this equal to $253¢$:

$$N(4B + 5J) = 253 = 11 \cdot 23$$

The only possible positive integer pairs $(N, 4B + 5J)$ whose product is 253 are:
$(1, 253); (11, 23); (23, 11); (253, 1)$

The first pair violates $N > 1$ and the third and fourth pair have no positive integer solutions for B and J.

So, $N = 11$ and $4B + 5J = 23$

The only integer solutions for B and J are $B = 2$ and $J = 3$

Therefore the cost of the jam Elmo uses to make the sandwiches is $3 \cdot 5 \cdot 11 = 165¢ = \$1.65 \Rightarrow D$

See Also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

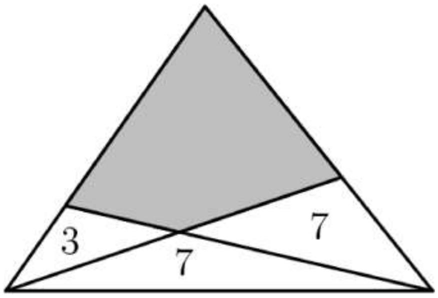
Category: Introductory Number Theory Problems

2006 AMC 10B Problems/Problem 23

Contents

Problem

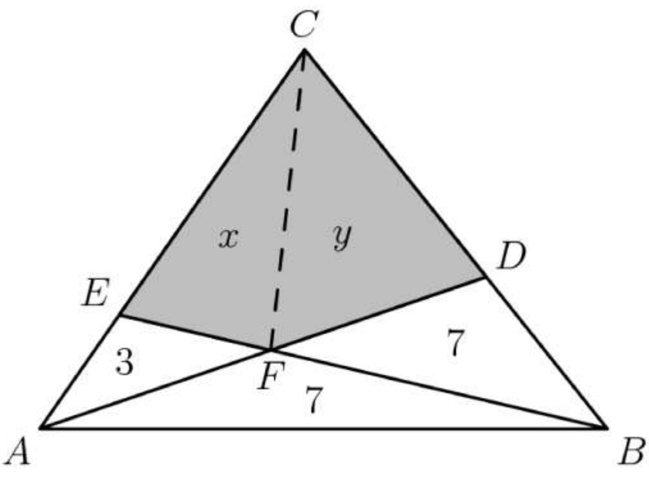
A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7 as shown. What is the area of the shaded quadrilateral?



$\mathrm{(A)}\ 15 \qquad \mathrm{(B)}\ 17 \qquad \mathrm{(C)}\ \frac{35}{2} \qquad \mathrm{(D)}\ 18 \qquad \mathrm{(E)}\ \frac{55}{3}$

Solution 1

Label the points in the figure as shown below, and draw the segment CF. This segment divides the quadrilateral into two triangles, let their areas be x and y.



Since triangles AFB and DFB share an altitude from B and have equal area, their bases must be equal, hence $AF=DF$.

Since triangles AFC and DFC share an altitude from C and their respective bases are equal, their areas must be equal, hence $x+3=y$.

Since triangles EFA and BFA share an altitude from A and their respective areas are in the ratio $3:7$, their bases must be in the same ratio, hence $EF:FB=3:7$.

Since triangles EFC and BFC share an altitude from C and their respective bases are in the ratio $3:7$, their areas must be in the same ratio, hence $x:(y+7)=3:7$, which gives us $7x=3(y+7)$.

Substituting $y = x + 3$ into the second equation we get $7x = 3(x + 10)$, which solves to $x = \frac{15}{2}$. Then $y = x + 3 = \frac{15}{2} + 3 = \frac{21}{2}$, and the total area of the quadrilateral is $x + y = \frac{15}{2} + \frac{21}{2} = \boxed{\textbf{(D) } 18}$.

Solution 2 (mass points)

We see that $EF : FB = 3 : 7$ and $AF = FD$. We assign a mass of 7 to E and 3 to B, making F have mass 10 and A and D each have mass 5. Now, C has mass 2. Therefore, the area of triangle CEB is $10 \cdot 2.5 = 25$, so the area of $CEFD$ is $\boxed{(D), 18}$

See also

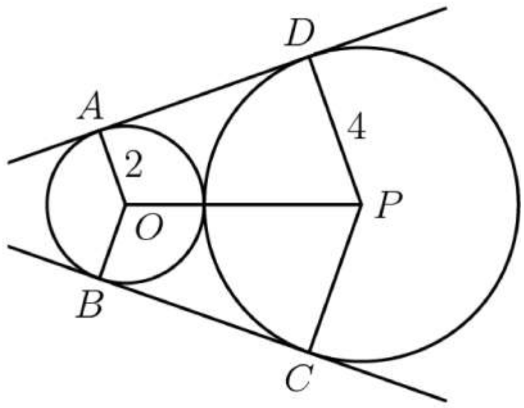
2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Categories: Introductory Geometry Problems | Area Problems

2006 AMC 10B Problems/Problem 24

Problem

Circles with centers O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B on the circle with center O and points C and D on the circle with center P are such that AD and BC are common external tangents to the circles. What is the area of the concave hexagon $AOBCPD$?



$\text{(A) } 18\sqrt{3} \qquad \text{(B) } 24\sqrt{2} \qquad \text{(C) } 36 \qquad \text{(D) } 24\sqrt{3} \qquad \text{(E) } 32\sqrt{2}$

Solution

Since a tangent line is perpendicular to the radius containing the point of tangency, $\angle OAD = \angle PDA = 90^\circ$.

Construct a perpendicular to DP that goes through point O. Label the point of intersection X.

Clearly $OADX$ is a rectangle, so $DX = 2$ and $PX = 2$. By the Pythagorean Theorem, $OX = \sqrt{6^2 - 2^2} = 4\sqrt{2}$.

The area of $OADX$ is $2 \cdot 4\sqrt{2} = 8\sqrt{2}$. The area of OXP is $\frac{1}{2} \cdot 2 \cdot 4\sqrt{2} = 4\sqrt{2}$, so the area of quadrilateral $OADP$ is $8\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$. Using similar steps, the area of quadrilateral $OBCP$ is also $12\sqrt{2}$. Therefore, the area of hexagon $AOBCPD$ is $2 \cdot 12\sqrt{2} = 24\sqrt{2} \Longrightarrow \boxed{\text{(B)}}$.

See also

2006 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2006))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2006 AMC 10B Problems/Problem 25

Contents

Problem

Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is not the age of one of Mr. Jones's children?

$\mathrm{(A) \ } 4\qquad \mathrm{(B) \ } 5\qquad \mathrm{(C) \ } 6\qquad \mathrm{(D) \ } 7\qquad \mathrm{(E) \ } 8$

Solutions

Solution 1

Let S be the set of the ages of Mr. Jones' children (in other words $i \in S$ if Mr. Jones has a child who is i years old). Then $|S| = 8$ and $9 \in S$. Let m be the positive integer seen on the license plate. Since at least one of 4 or 8 is contained in S, we have $4|m$.

We would like to prove that $5 \notin S$, so for the sake of contradiction, let us suppose that $5 \in S$. Then $5 \cdot 4 = 20|m$ so the units digit of m is 0. Since the number has two distinct digits, each appearing twice, another digit of m must be 0. Since Mr. Jones can't be 00 years old, the last two digits can't be 00. Therefore m must be of the form $d0d0$, where d is a digit. Since m is divisible by 9, the sum of the digits of m must be divisible by 9 (see Divisibility rules for 9). Hence $9|2d$ which implies $d = 9$. But $m = 9090$ is not divisible by 4, contradiction. So $5 \notin S$ and 5 is not the age of one of Mr. Jones' kids. $\boxed{B}$

(We might like to check that there does, indeed, exist such a positive integer m. If 5 is not an age of one of Mr. Jones' kids, then the license plate number must be a multiple of $lcm(1,2,3,4,6,7,8,9) = 504$. Since $11 \cdot 504 = 5544$ and 5544 is the only 4 digit multiple of 504 that fits all the conditions of the license plate's number, the license plate's number is 5544.)

Solution 2

Alternatively, we can see that if one of Mr.Jones' children is of the age 5, then the license plate will have to end in the digit 0 . This means that the license plate would have to have two 0 digits, and would either be of the form $XX00$ or $X0X0$ (X being the other digit in the license plate) . The condition $XX00$ is impossible as Mr. Jones can't be 00 years old. If we separate the second condition, $X0X0$ into its prime factors, we get $X \cdot 11 \cdot 101$. 11 and 101 are prime, therefore can't account for allowing $X0X0$ being evenly divisible by the childrens' ages. This leaves X, but no single digit number contains all the prime factors of the childrens ages, thus 5 $\boxed{B}$ can't be one of the childrens ages.

Solution 3

Another way to do the problem is by the process of elimination. The only possible correct choices are the highest powers of each prime, $2^3 = 8$, $3^2 = 9$, $5^1 = 5$, and $7^1 = 7$, since indivisibility by any powers lower than these means indivisibility by a higher power of the prime (for example, indivisibility by $2^2 = 4$ means indivisibility by $(2^3 = 8)$. Since the number is divisible by 9, it is not the answer, and the digits have to add up to 9. Let (a, b) be the digits: since $2 \cdot (a+b)|9$, $(a+b)|9$. The only possible choices for (a, b) are $(0, 9)$, $(1, 8)$, $(2, 7)$, $(3, 6)$, and $(4, 5)$. Since the last 2 digits are divisible by 4, $(1, 8)$, $(4, 5)$, and $(0, 9)$ can be eliminated. Only $(3, 6)$ and $(2, 7)$ remain; since there is no 5 or 0 in either pair, the number cannot be divisible by 5. $\boxed{B}$