2008 AMC 12A Problems/Problem 1

The following problem is from both the 2008 AMC 12A #1 and 2008 AMC 10A #1, so both problems redirect to this page.

Problem

A bakery owner turns on his doughnut machine at $8:30\ \text{AM}$. At $11:10\ \text{AM}$ the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?

$\text{(A)}\ 1:50\ \text{PM} \qquad \text{(B)}\ 3:00\ \text{PM} \qquad \text{(C)}\ 3:30\ \text{PM} \qquad \text{(D)}\ 4:30\ \text{PM} \qquad \text{(E)}\ 5:50\ \text{PM}$

Solution

The machine completes one-third of the job in $11:10 - 8:30 = 2:40$ hours. Thus, the entire job is completed in $3 \cdot (2:40) = 8:00$ hours.

Since the machine was started at $8:30\ \text{AM}$, the job will be finished 8 hours later, at $4:30\ \text{PM}$. The answer is (D).

See Also

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2008 AMC 10A Problems/Problem 2

Problem

A square is drawn inside a rectangle. The ratio of the width of the rectangle to a side of the square is $2:1$. The ratio of the rectangle's length to its width is $2:1$. What percent of the rectangle's area is inside the square?

$(A)\ 12.5 \qquad (B)\ 25 \qquad (C)\ 50 \qquad (D)\ 75 \qquad (E)\ 87.5$

Solution 1

Since they are asking for the "ratio" of two things, we can say that the side of the square is anything that we want. So if we say that it is 1, then width of the rectangle is 2, and the length is 4, thus making the total area of the rectangle 8. The area of the square is just 1. So the answer is just $\frac{1}{8} \times 100 = 12.5 \longrightarrow \boxed{\text{A}}$

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 10A Problems/Problem 3

Problem

For the positive integer n, let $\langle n \rangle$ denote the sum of all the positive divisors of n with the exception of n itself. For example, $\langle 4 \rangle = 1 + 2 = 3$ and $\langle 12 \rangle = 1 + 2 + 3 + 4 + 6 = 16$. What is $\langle \langle \langle 6 \rangle \rangle \rangle$?

$\text{(A) } 6 \quad \text{(B) } 12 \quad \text{(C) } 24 \quad \text{(D) } 32 \quad \text{(E) } 36$

Solution

$\langle \langle \langle 6 \rangle \rangle \rangle = \langle \langle 6 \rangle \rangle = \langle 6 \rangle = 6 \implies \text{(A)}$

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 3

The following problem is from both the 2008 AMC 12A #3 and 2008 AMC 10A #4, so both problems redirect to this page.

Problem

Suppose that $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\frac{1}{2}$ of 5 bananas?

$(A)\ 2 \qquad (B)\ \frac{5}{2} \qquad (C)\ 3 \qquad (D)\ \frac{7}{2} \qquad (E)\ 4$

Solution

If $\frac{2}{3} \cdot 10 \text{ bananas} = 8 \text{ oranges}$, then

$$\frac{1}{2} \cdot 5 \text{ bananas} = \left(\frac{1}{2} \cdot 5 \text{ bananas}\right) \cdot \left(\frac{8 \text{ oranges}}{\frac{2}{3} \cdot 10 \text{ bananas}}\right) = 3 \text{ oranges} \Longrightarrow (C).$$

See Also

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2008 AMC 12A Problems/Problem 4

The following problem is from both the 2008 AMC 12A #4 and 2008 AMC 10A #5, so both problems redirect to this page.

Problem

Which of the following is equal to the product

$$\frac{8}{4}\cdot\frac{12}{8}\cdot\frac{16}{12}\cdot\cdots\cdot\frac{4n+4}{4n}\cdot\cdots\cdot\frac{2008}{2004}?$$

$\textbf{(A)}\ 251 \qquad \textbf{(B)}\ 502 \qquad \textbf{(C)}\ 1004 \qquad \textbf{(D)}\ 2008 \qquad \textbf{(E)}\ 4016$

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Solution

Solution 1

$$\frac{8}{4}\cdot\frac{12}{8}\cdot\frac{16}{12}\cdots\frac{4n+4}{4n}\cdots\frac{2008}{2004}=\frac{1}{4}\cdot\left(\frac{8}{8}\cdot\frac{12}{12}\cdots\frac{4n}{4n}\cdots\frac{2004}{2004}\right)\cdot 2008=\frac{2008}{4}=502\Rightarrow B.$$

Solution 2

Notice that everything cancels out except for 2008 in the numerator and 4 in the denominator.

Thus, the product is $\frac{2008}{4}=502$, and the answer is $\textbf{(B)}$.

See Also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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2008 AMC 10A Problems/Problem 6

Problem

A triathlete competes in a triathlon in which the swimming, biking, and running segments are all of the same length. The triathlete swims at a rate of 3 kilometers per hour, bikes at a rate of 20 kilometers per hour, and runs at a rate of 10 kilometers per hour. Which of the following is closest to the triathlete's average speed, in kilometers per hour, for the entire race?

$\text{(A) } 3 \quad \text{(B) } 4 \quad \text{(C) } 5 \quad \text{(D) } 6 \quad \text{(E) } 7$

Solution

Let d be the length of one segment of the race.

Average speed is total distance divided by total time. The total distance is $3d$, and the total time is $\frac{d}{3}+\frac{d}{20}+\frac{d}{10}=\frac{29d}{60}$.

Thus, the average speed is $3d \div \left(\frac{29d}{60}\right) = \frac{180}{29}$. This is closest to 6, so the answer is (D).

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 10A Problems/Problem 7

Problem

The fraction

$$\frac{(3^{2008})^2-(3^{2006})^2}{(3^{2007})^2-(3^{2005})^2}$$

simplifies to which of the following?

(A) 1 (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$ (E) 9

Solution

Notice that 9 can be factored out of the numerator:

$$\frac{(3^{2008})^2-(3^{2006})^2}{(3^{2007})^2-(3^{2005})^2}=\frac{9\,(3^{2007})^2-9\,(3^{2005})^2}{(3^{2007})^2-(3^{2005})^2}=9.\frac{(3^{2007})^2-(3^{2005})^2}{(3^{2007})^2-(3^{2005})^2}$$

Thus, the expression is equal to 9, and the answer is (E).

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 6

The following problem is from both the 2008 AMC 12A #6 and 2004 AMC 10A #8, so both problems redirect to this page.

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Problem

Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a $\$90$ rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves $\$15$ by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

$\text{(A) } 750 \quad \text{(B) } 900 \quad \text{(C) } 1000 \quad \text{(D) } 1050 \quad \text{(E) } 1500$

Solution

Solution 1

Let the sticker price be x.

The price of the computer is $0.85x - 90$ at store A, and $0.75x$ at store B.

Heather saves $\$15$ at store A, so $0.85x - 90 + 15 = 0.75x$.

Solving, we find $x = 750$, and the thus answer is (A).

Solution 2

The $\$90$ in store A is $\$15$ better than the additional 10% off at store B.

Thus the 10% off is equal to $\$90 - \$15 = \$75$, and therefore the sticker price is $\$750$.

See Also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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2008 AMC 12A Problems/Problem 5

The following problem is from both the 2008 AMC 12A #5 and 2008 AMC 10A #9, so both problems redirect to this page.

Problem

Suppose that

$$\frac{2x}{3}-\frac{x}{6}$$

is an integer. Which of the following statements must be true about x?

(A) It is negative.
(B) It is even, but not necessarily a multiple of 3.
(C) It is a multiple of 3, but not necessarily even.
(D) It is a multiple of 6, but not necessarily a multiple of 12.
(E) It is a multiple of 12.

Solution

$$\frac{2x}{3}-\frac{x}{6} \implies \frac{4x}{6}-\frac{x}{6} \implies \frac{3x}{6} \implies \frac{x}{2}$$

For $\frac{x}{2}$ to be an integer, x must be even, but not necessarily divisible by 3. Thus, the answer is (B).

See also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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2008 AMC 10A Problems/Problem 10

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Problem

Each of the sides of a square S_1 with area 16 is bisected, and a smaller square S_2 is constructed using the bisection points as vertices. The same process is carried out on S_2 to construct an even smaller square S_3. What is the area of S_3?

$(\mathrm{A})\ \frac{1}{2} \qquad (\mathrm{B})\ 1 \qquad (\mathrm{C})\ 2 \qquad (\mathrm{D})\ 3 \qquad (\mathrm{E})\ 4$

Solution 1

Since the area of the large square is 16, the side equals 4 and if you bisect all of the sides, you get a square of side length $2\sqrt{2}$ thus making the area 8. If we repeat this process again, we notice that the area is just half that of the previous square, so the area of $S_3 = 4 \longrightarrow \boxed{\mathrm{E}}$

Solution 2

Since the length ratio is $\frac{1}{\sqrt{2}}$, then the area ratio is $\frac{1}{2}$. This means that $S_2 = 8$ and $S_3 = \boxed{\mathbf{(E)}\ 4}$.

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 7

The following problem is from both the 2008 AMC 12A #7 and 2008 AMC 10A #11, so both problems redirect to this page.

Problem

While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

$(A)\ 2 \qquad (B)\ 4 \qquad (C)\ 6 \qquad (D)\ 8 \qquad (E)\ 10$

Solution

It will take $\frac{1}{4}$ of an hour or 15 minutes to get to shore.

Since only 30 gallons of water can enter the boat, only $\frac{30}{15} = 2$ net gallons can enter the boat per minute.

Since 10 gallons of water enter the boat each minute, LeRoy must bail $10 - 2 = 8$ gallons per minute $\Rightarrow (D)$.

See Also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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Category: Introductory Algebra Problems

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2008 AMC 10A Problems/Problem 12

Problem

In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are r red marbles. What is the total number of marbles in the collection?

$(A)\ 2.85r \qquad (B)\ 3r \qquad (C)\ 3.4r \qquad (D)\ 3.85r \qquad (E)\ 4.25r$

Solution

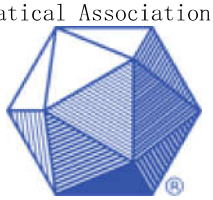
The number of blue marbles is $\frac{4}{5}r$, the number of green marbles is $\frac{8}{5}r$, and the number of red marbles is r.

Thus, the total number of marbles is $\frac{4}{5}r + \frac{8}{5}r + r = 3.4r$, and the answer is (C).

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 10

The following problem is from both the 2008 AMC 12A #10 and 2008 AMC 10A #13, so both problems redirect to this page.

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Problem

Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t?

$$\text{(A)}\ \left(\frac{1}{5}+\frac{1}{7}\right)(t+1)=1 \qquad \text{(B)}\ \left(\frac{1}{5}+\frac{1}{7}\right)t+1=1 \qquad \text{(C)}\ \left(\frac{1}{5}+\frac{1}{7}\right)t=1$$

$$\text{(D)}\ \left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1 \qquad \text{(E)}\ (5+7)\,t=1$$

Solution

Solution 1

Doug can paint $\frac{1}{5}$ of a room per hour, Dave can paint $\frac{1}{7}$ of a room in an hour, and the time they spend working together is $t-1$.

Since rate times time gives output, $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1 \Rightarrow \text{(D)}$

Solution 2

If one person does a job in a hours and another person does a job in b hours, the time it takes to do the job together is $\frac{ab}{a+b}$ hours.

Since Doug paints a room in 5 hours and Dave paints a room in 7 hours, they both paint in $\frac{5*7}{5+7}=\frac{35}{12}$ hours.

They also take 1 hour for lunch, so the total time $t=\frac{35}{12}+1$ hours.

Looking at the answer choices, (D) is the only one satisfied by $t=\frac{35}{12}+1$.

See Also

2008 AMC 12A Problems/Problem 9

The following problem is from both the 2008 AMC 12A #9 and 2008 AMC 10A #14, so both problems redirect to this page.

Problem

Older television screens have an aspect ratio of $4:3$. That is, the ratio of the width to the height is $4:3$. The aspect ratio of many movies is not $4:3$, so they are sometimes shown on a television screen by "letterboxing" - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of $2:1$ and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?



(A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3

Solution

Let the width and height of the screen be $4x$ and $3x$ respectively, and let the width and height of the movie be $2y$ and y respectively.

By the Pythagorean Theorem, the diagonal is $\sqrt{(3x)^2+(4x)^2}=5x=27$. So $x=\frac{27}{5}$.

Since the movie and the screen have the same width, $2y=4x \Rightarrow y=2x$.

Thus, the height of each strip is $\frac{3x-y}{2}=\frac{3x-2x}{2}=\frac{x}{2}=\frac{27}{10}=2.7 \Longrightarrow (D)$.

See Also

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2008 AMC 10A Problems/Problem 15

Problem

Yesterday Han drove 1 hour longer than Ian at an average speed 5 miles per hour faster than Ian. Jan drove 2 hours longer than Ian at an average speed 10 miles per hour faster than Ian. Han drove 70 miles more than Ian. How many more miles did Jan drive than Ian?

(A) 120 (B) 130 (C) 140 (D) 150 (E) 160

Solution

We let Ian's speed and time equal I_s and I_t, respectively. Similarly, let Han's and Jan's speed and time be H_s, H_t, J_s, J_t. The problem gives us 5 equations:

$$\begin{aligned} H_s &= I_s + 5 && (1)\\ H_t &= I_t + 1 && (2)\\ J_s &= I_s + 10 && (3)\\ J_t &= I_t + 2 && (4)\\ H_s \cdot H_t &= I_s \cdot I_t + 70 && (5) \end{aligned}$$

Substituting (1) and (2) equations into (5) gives:

$$(I_s+5)(I_t+1) = I_sI_t+70 \implies I_sI_t+I_s+5I_t+5 = I_sI_t+70 \implies I_s+5I_t = 65 \quad (*)$$

We are asked the difference between Jan's and Ian's distances, or

$$J_sJ_t - I_sI_t = x,$$

Where x is the difference between Jan's and Ian's distances and the answer to the problem. Substituting (3) and (4) equations into this equation gives:

$$(I_s+10)(I_t+2)-I_sI_t = x \implies I_sI_t+2I_s+10I_t+20-I_sI_t = x \implies$$

$$2I_s + 10I_t + 20 = x \implies 2(I_s + 5I_t) + 20 = x$$

Substituting $(*)$ into this equation gives:

$$2(65) + 20 = x \implies 130 + 20 = x \implies 150 = x$$

Therefore, the answer is 150 miles or $\boxed{(D)}$.

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 13

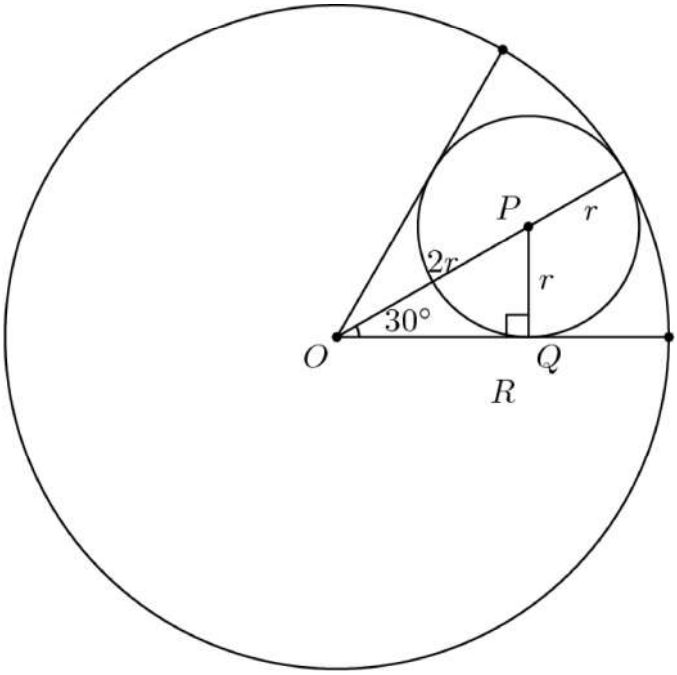
The following problem is from both the 2008 AMC 12A #13 and 2008 AMC 10A #16, so both problems redirect to this page.

Problem

Points A and B lie on a circle centered at O, and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

$\text{(A)}\ \frac{1}{16} \qquad \text{(B)}\ \frac{1}{9} \qquad \text{(C)}\ \frac{1}{8} \qquad \text{(D)}\ \frac{1}{6} \qquad \text{(E)}\ \frac{1}{4}$

Solution



Let P be the center of the small circle with radius r, and let Q be the point where the small circle is tangent to OA. Also, let C be the point where the small circle is tangent to the big circle with radius R.

Then PQO is a right triangle, and a $30-60-90$ triangle at that. Therefore, $OP = 2PQ$.

Since $OP = OC - PC = OC - r = R - r$, we have $R - r = 2PQ$, or $R - r = 2r$, or $\frac{1}{3} = \frac{r}{R}$.

Then the ratio of areas will be $\frac{1}{3}$ squared, or $\frac{1}{9} \Rightarrow \boxed{\text{B}}$.

See also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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2008 AMC 10A Problems/Problem 17

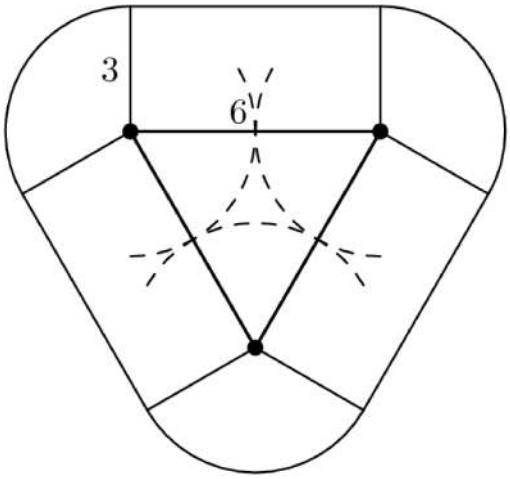
Problem

An equilateral triangle has side length 6. What is the area of the region containing all points that are outside the triangle but not more than 3 units from a point of the triangle?

$(A)\ 36+24\sqrt{3} \qquad (B)\ 54+9\pi \qquad (C)\ 54+18\sqrt{3}+6\pi \qquad (D)\ \left(2\sqrt{3}+3\right)^2\pi$

$(E)\ 9\left(\sqrt{3}+1\right)^2\pi$

Solution



The region described contains three rectangles of dimensions 3×6, and three 120° degree arcs of circles of radius 3. Thus the answer is

$$3(3\times 6)+3\left(\frac{120^\circ}{360^\circ}\times 3^2\pi\right)=54+9\pi \Longrightarrow (B).$$

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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Category: Introductory Geometry Problems

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2008 AMC 10A Problems/Problem 18

Problem

A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?

$(A)\ \frac{57}{4} \qquad (B)\ \frac{59}{4} \qquad (C)\ \frac{61}{4} \qquad (D)\ \frac{63}{4} \qquad (E)\ \frac{65}{4}$

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
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  - 2.3 Solution 3
  - 2.4 Solution 4
- 3 See also

Solution

Solution 1

Let the legs of the triangle have lengths a, b. Then, by the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{a^2+b^2}$, and the area of the triangle is $\frac{1}{2}ab$. So we have the two equations

$$a+b+\sqrt{a^2+b^2}=32$$

$$\frac{1}{2}ab=20$$

Re-arranging the first equation and squaring,

$$\sqrt{a^2+b^2}=32-(a+b)$$

$$a^2+b^2=32^2-64(a+b)+(a+b)^2$$

$$a^2+b^2+64(a+b)=a^2+b^2+2ab+32^2$$

$$a+b=\frac{2ab+32^2}{64}$$

From (2) we have $2ab=80$, so

$$a+b=\frac{80+32^2}{64}=\frac{69}{4}.$$

The length of the hypotenuse is $p-a-b=32-\frac{69}{4}=\frac{59}{4}$ $\boxed{(B)}$.

Solution 2

From the formula $A=rs$, where A is the area of a triangle, r is its inradius, and s is the semiperimeter, we can find that $r=\frac{20}{32/2}=\frac{5}{4}$. It is known that in a right triangle, $r=s-h$, where h is the hypotenuse, so $h=16-\frac{5}{4}=\frac{59}{4}$.

Solution 3

From the problem, we know that

$$a+b+c=32$$
$$2ab=80.$$

Subtracting c from both sides of the first equation and squaring both sides, we get

$$(a+b)^2=(32-c)^2$$
$$a^2+b^2+2ab=32^2+c^2-64c.$$

Now we substitute in $a^2+b^2=c^2$ as well as $2ab=80$ into the equation to get

$$80=1024-64c$$
$$c=\frac{944}{64}.$$

Further simplification yields the result of $\frac{59}{4}$.

Solution 4

Let a and b be the legs of the triangle, and c the hypotenuse.

Since the area is 20, we have $\frac{1}{2}ab=20 => ab=40.$

Since the perimeter is 32, we have $a+b+c=32.$

The Pythagorean Theorem gives $c^2=a^2+b^2.$

This gives us three equations with three variables:

$$ab=40$$
$$a+b+c=32$$
$$c^2=a^2+b^2$$

Rewrite equation 3 as $c^2=(a+b)^2-2ab.$ Substitute in equations 1 and 2 to get $c^2=(32-c)^2-80.$

$$c^2=(32-c)^2-80$$

$$c^2=1024-64c+c^2-80$$

$$64c=944$$

$$c=\frac{944}{64}=\frac{236}{16}=\frac{59}{4}$$.

The answer is choice (B).

See also

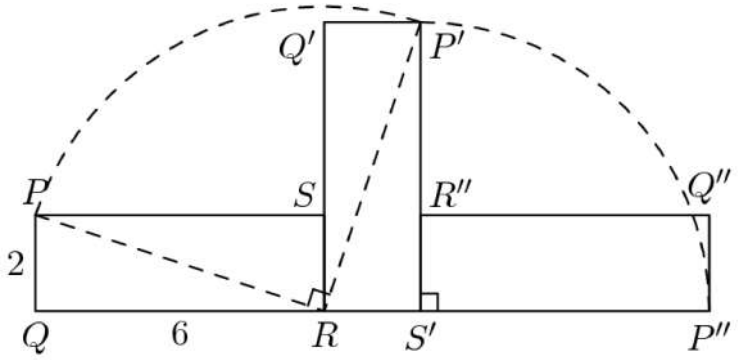
2008 AMC 10A Problems/Problem 19

Problem

Rectangle $PQRS$ lies in a plane with $PQ = RS = 2$ and $QR = SP = 6$. The rectangle is rotated 90° clockwise about R, then rotated 90° clockwise about the point S moved to after the first rotation. What is the length of the path traveled by point P?

(A) $\left(2\sqrt{3}+\sqrt{5}\right)\pi$ (B) 6π (C) $\left(3+\sqrt{10}\right)\pi$ (D) $\left(\sqrt{3}+2\sqrt{5}\right)\pi$
(E) $2\sqrt{10}\pi$

Solution



We let $P'Q'R'S'$ be the first rectangle after the rotation, and $P''Q''R''S''$ be the second rectangle after rotation. Point P pivots about R in an arc of a circle of radius $\sqrt{2^2+6^2} = 2\sqrt{10}$, and since $\angle PRS$, $\angle P'RQ'$ are complementary, it follows that the arc has a degree measure of 90° (or $1/4$ of the circumference). Thus, P travels $\frac{1}{4}\left(4\sqrt{10}\right)\pi = \sqrt{10}\pi$ in the first rotation.

Similarly, in the second rotation, P travels in a 90° arc about S', with the radius being 6. It travels $\frac{1}{4}(12)\pi = 3\pi$. Therefore, the total distance it travels is $\left(3+\sqrt{10}\right)\pi$ (C).

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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Category: Introductory Geometry Problems

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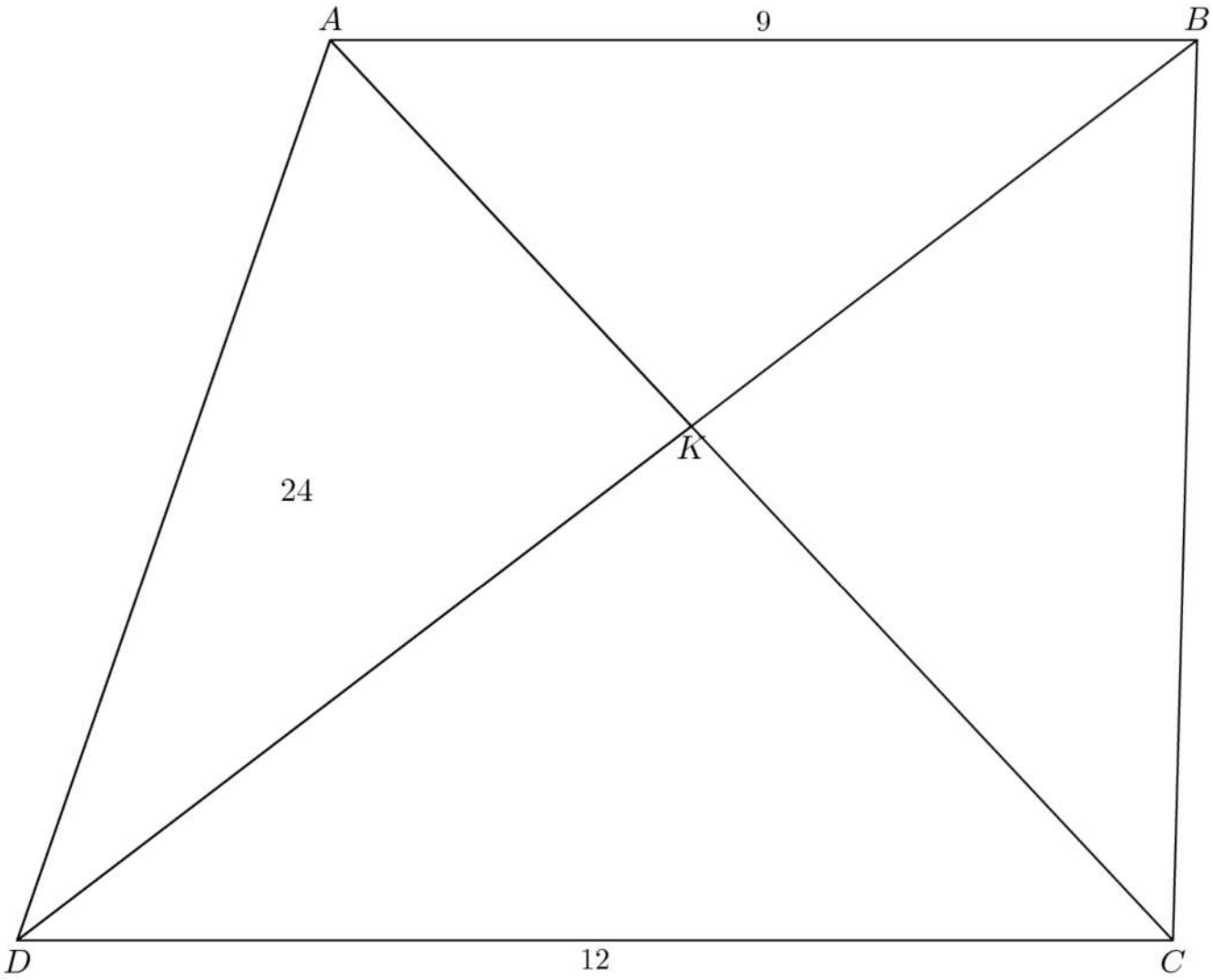
2008 AMC 10A Problems/Problem 20

Problem

Trapezoid $ABCD$ has bases $\overline{AB}$ and $\overline{CD}$ and diagonals intersecting at K. Suppose that $AB = 9$, $DC = 12$, and the area of $\triangle AKD$ is 24. What is the area of trapezoid $ABCD$?

$(A)\ 92 \qquad (B)\ 94 \qquad (C)\ 96 \qquad (D)\ 98 \qquad (E)\ 100$

Solution



Since $\overline{AB} \parallel \overline{DC}$ it follows that $\triangle ABK \sim \triangle CDK$. Thus $\frac{KA}{KC} = \frac{KB}{KD} = \frac{AB}{DC} = \frac{3}{4}$.

We now introduce the concept of area ratios: given two triangles that share the same height, the ratio of the areas is equal to the ratio of their bases. Since $\triangle AKB, \triangle AKD$ share a common altitude to $\overline{BD}$, it follows that (we let $[\triangle \ldots]$ denote the area of the triangle) $\frac{[\triangle AKB]}{[\triangle AKD]} = \frac{KB}{KD} = \frac{3}{4}$, so $[\triangle AKB] = \frac{3}{4}(24) = 18$.

Similarly, we find $[\triangle DKC] = \frac{4}{3}(24) = 32$ and $[\triangle BKC] = 24$.

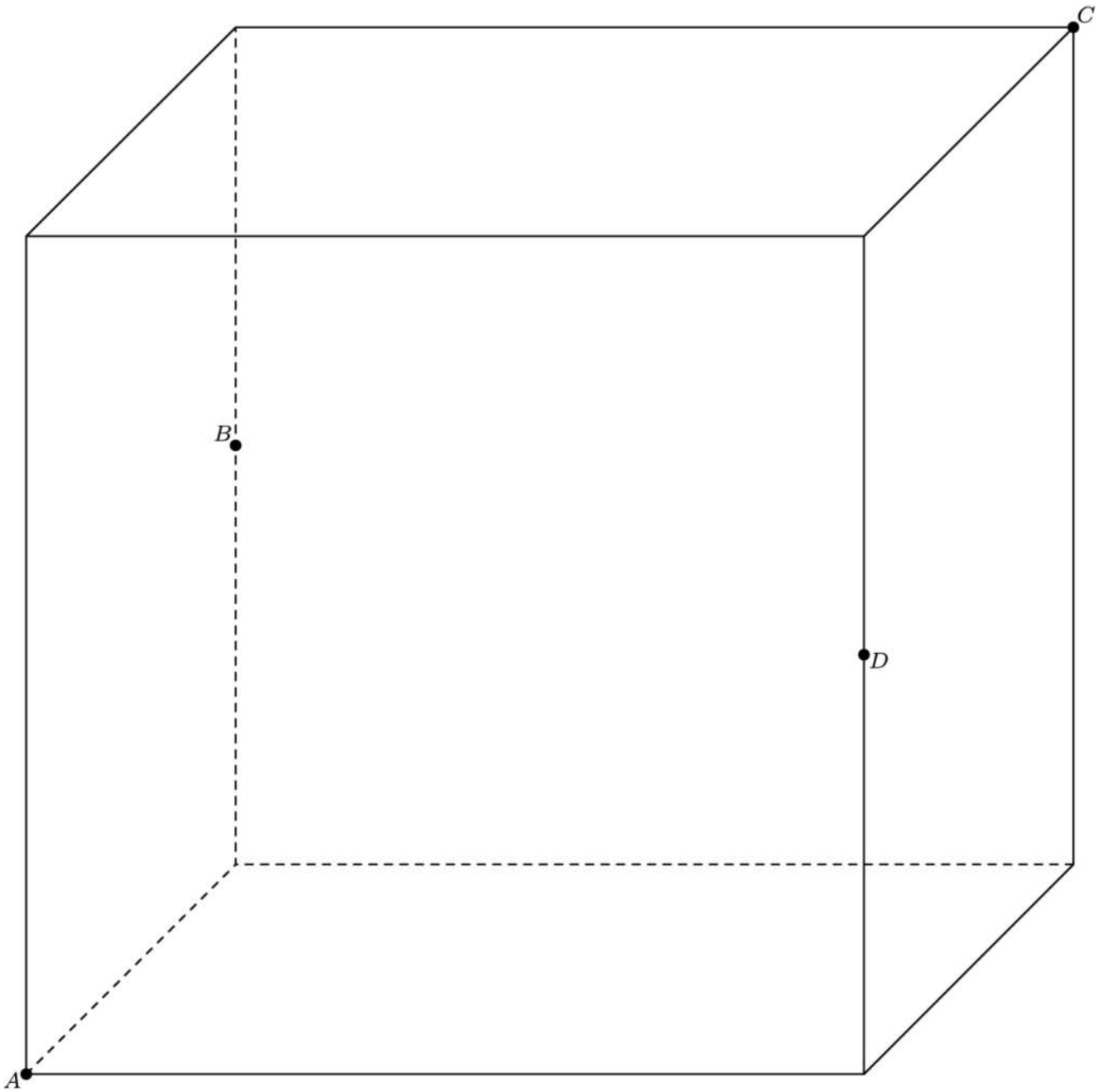
Therefore, the area of
$ABCD = [AKD] + [AKB] + [BKC] + [CKD] = 24 + 18 + 24 + 32 = 98\ (\mathrm{D})$.

See also

2008 AMC 10A Problems/Problem 21

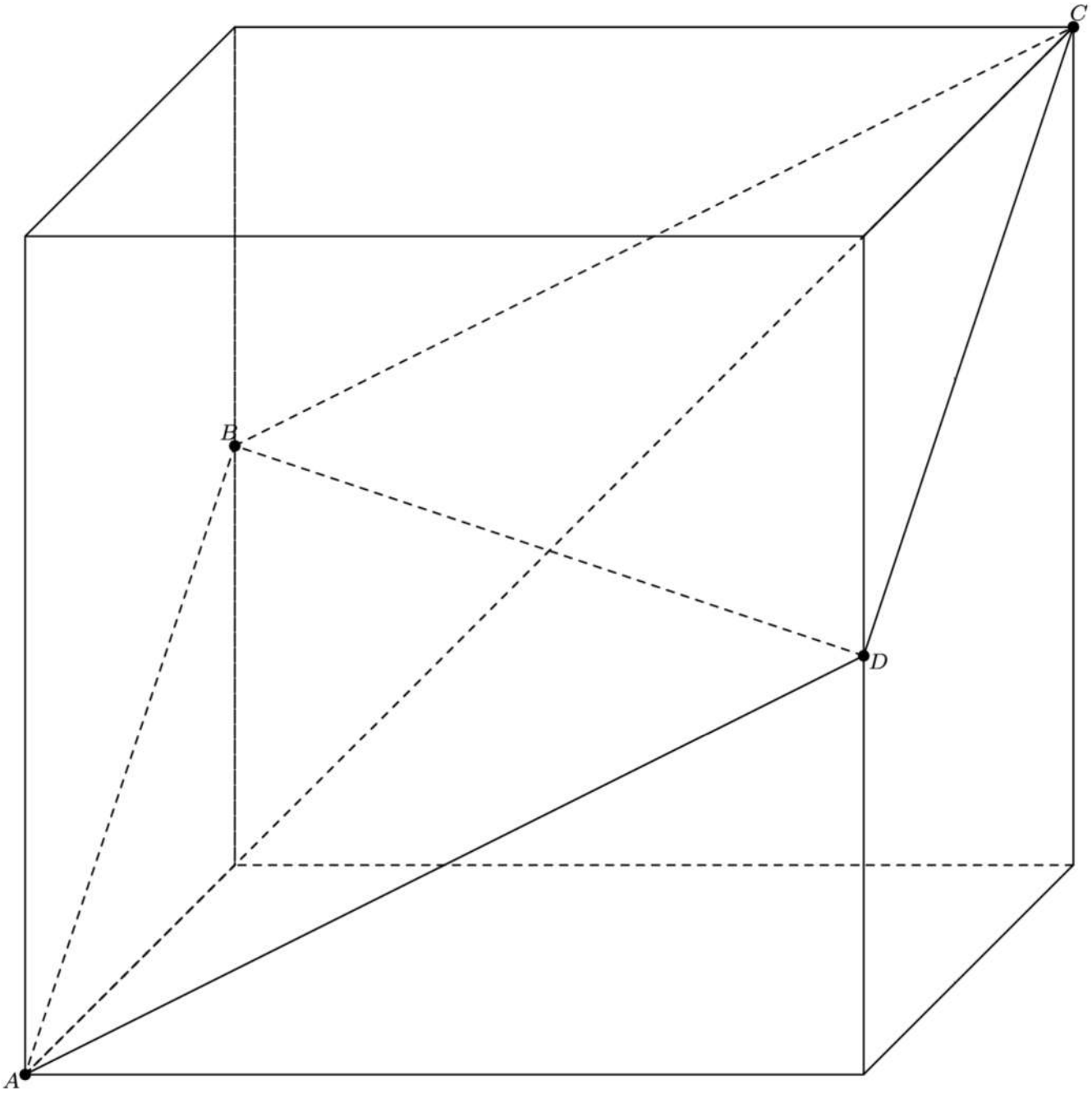
Problem

A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A or C, as shown. What is the area of quadrilateral $ABCD$?



(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

Solution



Since $AB = AD = CB = CD = \sqrt{\frac{1}{2}^2 + 1^2}$, it follows that $ABCD$ is a rhombus. The area of the rhombus can be computed by the formula $A = \frac{1}{2}d_1 d_2$, where d_1, d_2 are the diagonals of the rhombus (or of a kite in general). BD has the same length as a face diagonal, or $\sqrt{1^2 + 1^2} = \sqrt{2}$. AC is a space diagonal, with length $\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$. Thus $A = \frac{1}{2} \times \sqrt{2} \times \sqrt{3} = \frac{\sqrt{6}}{2}$ (A).

See also

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2008 AMC 10A Problems/Problem 22

Problem

Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

$(\mathrm{A})\ \frac{1}{6} \qquad (\mathrm{B})\ \frac{1}{3} \qquad (\mathrm{C})\ \frac{1}{2} \qquad (\mathrm{D})\ \frac{5}{8} \qquad (\mathrm{E})\ \frac{3}{4}$

Solution

We construct a tree showing all possible outcomes that Jacob may get after 3 flips; we can do this because there are only 8 possibilities:

$$6 \begin{cases} \mathrm{H}:11 \begin{cases} \mathrm{H}:21 \begin{cases} \mathrm{H}:\boxed{41} \\ \mathrm{T}:9.5 \end{cases} \\ \mathrm{T}:4.5 \begin{cases} \mathrm{H}:\boxed{8} \\ \mathrm{T}:1.25 \end{cases} \end{cases} \\ \mathrm{T}:2 \begin{cases} \mathrm{H}:3 \begin{cases} \mathrm{H}:\boxed{5} \\ \mathrm{T}:0.5 \end{cases} \\ \mathrm{T}:0 \begin{cases} \mathrm{H}:\boxed{-1} \\ \mathrm{T}:\boxed{-1} \end{cases} \end{cases} \end{cases}$$

There is a $\frac{5}{8}$ chance that Jacob ends with an integer, so the answer is (D).

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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Category: Introductory Combinatorics Problems

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2008 AMC 10A Problems/Problem 23

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
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- 3 See also

Problem

Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

$\text{(A) } 20 \quad \text{(B) } 40 \quad \text{(C) } 60 \quad \text{(D) } 160 \quad \text{(E) } 320$

Solution

Solution 1

First choose the two letters to be repeated in each set. $\binom{5}{2} = 10$. Now we have three remaining elements that we wish to place into two separate subsets. There are $2^3 = 8$ ways to do so (Do you see why? It's because each of the three remaining letters can be placed either into the first or second subset. Both of those subsets contain the two chosen elements, so their intersection is the two chosen elements). Unfortunately, we have over-counted (Take for example $S_1 = \{a, b, c, d\}$ and $S_2 = \{a, b, e\}$). Notice how S_1 and S_2 are interchangeable. A simple division by two will fix this problem. Thus we have:

$$\frac{10 \times 8}{2} = 40 \implies \boxed{\text{B}}$$

Alternatively, after picking the two elements in both sets in $\binom{5}{2} = 10$ ways, we can use stars and bars to assign the remaining 3 elements to the sets. There are 3 stars, and 1 bar, so there are 4 total ways of assigning the elements. Then there are $10 \cdot 4 = 40$ ways to create the sets.

Solution 2

Another way of looking at this problem is to break it down into cases.

First, our two subsets can have 2 and 5 elements. The 5-element subset (aka the set) will contain the 2-element subset. There are $\binom{5}{2} = 10$ ways to choose the 2-element subset. Thus, there are 10 ways to create these sets.

Second, the subsets can have 3 and 4 elements. $3 + 4 = 7$ non-distinct elements. $7 - 5 = 2$ elements in the intersection. There are $\binom{5}{3} = 10$ ways to choose the 3-element subset. For the 4-element subset, two of the

elements must be the remaining elements (not in the 3-element subset). The other two have to be a subset of the 3-element subset. There are $\binom{3}{2} = 3$ ways to choose these two elements, which means there are 3 ways to choose the 4-element subset. Therefore, there are $10 \cdot 3 = 30$ ways to choose these sets.

This leads us to the answer:

$$10 + 30 = 40 \implies \boxed{\textbf{(B) } 40}$$

Solution 3

We label the subsets subset 1 and subset 2. Suppose the first subset has k elements where $k < 5$. The second element has $5 - k$ elements which the first subset does not contain (in order for the union to be the whole set). Additionally, the second set has 2 elements in common with the first subset. Therefore the number of ways to choose these sets is $\binom{5}{k} \cdot \binom{k}{2}$. Computing for $k < 5$ we have $10 + 30 + 30 + 10 = 80$. Divide by 2 for order to get 40.

See also

2008 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2008))	
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2008 AMC 12A Problems/Problem 15

The following problem is from both the 2008 AMC 12A #15 and 2008 AMC 10A #24, so both problems redirect to this page.

Problem

Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

$(A)\ 0 \qquad (B)\ 2 \qquad (C)\ 4 \qquad (D)\ 6 \qquad (E)\ 8$

Solution

$k \equiv 2008^2 + 2^{2008} \equiv 8^2 + 2^4 \equiv 4 + 6 \equiv 0 \pmod{10}$.

So, $k^2 \equiv 0 \pmod{10}$. Since $k = 2008^2 + 2^{2008}$ is a multiple of four and the units digit of powers of two repeat in cycles of four, $2^k \equiv 2^4 \equiv 6 \pmod{10}$.

Therefore, $k^2 + 2^k \equiv 0 + 6 \equiv 6 \pmod{10}$. So the units digit is $6 \Rightarrow D$.

See Also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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2008 AMC 12A Problems/Problem 22

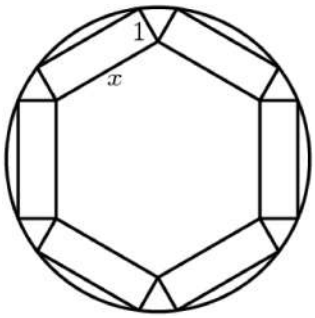
The following problem is from both the 2008 AMC 12A #22 and 2008 AMC 10A #25, so both problems redirect to this page.

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- 1 Problem
- 2 Solution
  - 2.1 Solution 1 (trigonometry)
  - 2.2 Solution 2 (without trigonometry)
- 3 See Also

Problem

A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?

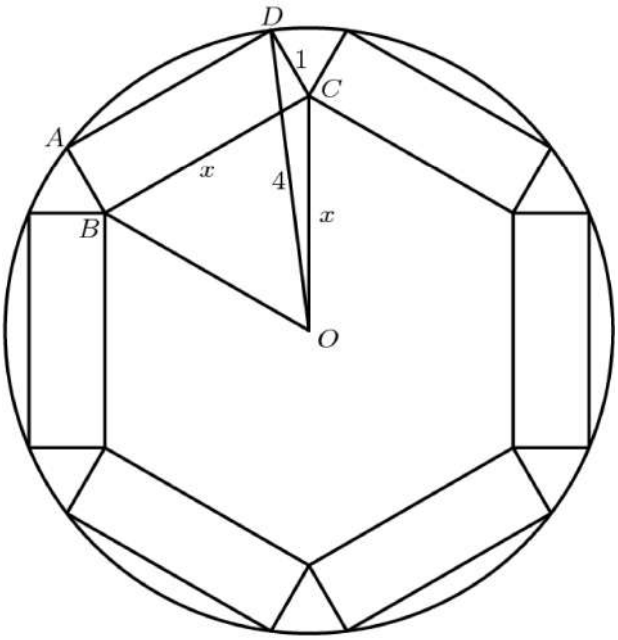


$\text{(A) } 2\sqrt{5}-\sqrt{3} \quad \text{(B) } 3 \quad \text{(C) } \frac{3\sqrt{7}-\sqrt{3}}{2} \quad \text{(D) } 2\sqrt{3} \quad \text{(E) } \frac{5+2\sqrt{3}}{2}$

Solution

Solution 1 (trigonometry)

Let one of the mats be $ABCD$, and the center be O as shown:



Since there are 6 mats, ΔBOC is equilateral. So, $BC = CO = x$. Also, $\angle OCD = \angle OCB + \angle BCD = 60^\circ + 90^\circ = 150^\circ$.

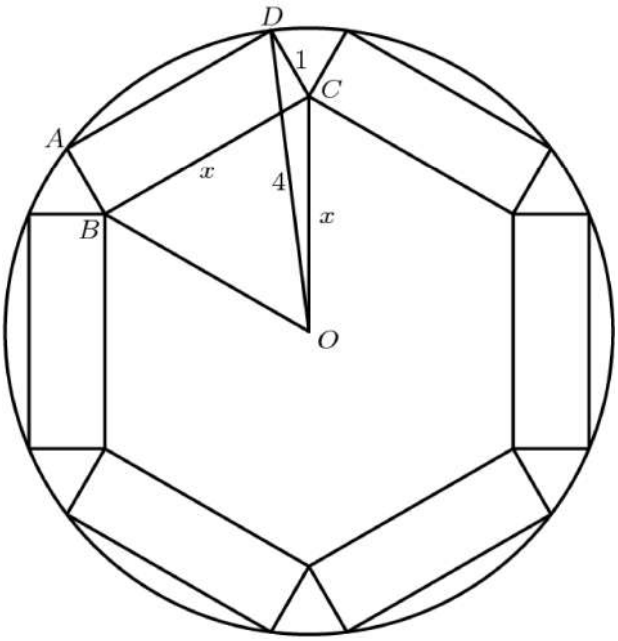
By the Law of Cosines:

$$4^2 = 1^2 + x^2 - 2 \cdot 1 \cdot x \cdot \cos(150^\circ) \Rightarrow x^2 + x\sqrt{3} - 15 = 0 \Rightarrow x = \frac{-\sqrt{3} \pm 3\sqrt{7}}{2}.$$

Since x must be positive, $x = \frac{3\sqrt{7} - \sqrt{3}}{2} \Rightarrow C$.

Solution 2 (without trigonometry)

Draw OD and OC as in the diagram. Draw the altitude from O to DC and call the intersection E



As proved in the first solution, $\angle OCD = 150^\circ$. That makes ΔOCE a $30 - 60 - 90$ triangle, so $OE = \frac{x}{2}$ and $CE = \frac{x\sqrt{3}}{2}$

Since ΔOED is a right triangle, $\left(\frac{x}{2}\right)^2 + \left(\frac{x\sqrt{3}}{2} + 1\right)^2 = 4^2 \Rightarrow x^2 + x\sqrt{3} - 15 = 0$

Solving for x gives $x = \frac{3\sqrt{7} - \sqrt{3}}{2} \Rightarrow C$

See Also

2008 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2008))	
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