2009 AMC 10A Problems/Problem 1

Problem

One can holds 12 ounces of soda. What is the minimum number of cans needed to provide a gallon (128 ounces) of soda?

$\textbf{(A)}\ 7\qquad \textbf{(B)}\ 8\qquad \textbf{(C)}\ 9\qquad \textbf{(D)}\ 10\qquad \textbf{(E)}\ 11$

Solution

10 cans would hold 120 ounces, but $128 > 120$, so 11 cans are required. Thus, the answer is $\boxed{\textbf{(E)}}$.

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 12A Problems/Problem 4

The following problem is from both the 2009 AMC 12A #4 and 2009 AMC 10A #2, so both problems redirect to this page.

Problem

Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could not be the total value of the four coins, in cents?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 45 \qquad \textbf{(E)}\ 55$

Solution

As all five options are divisible by 5, we may not use any pennies. (This is because a penny is the only coin that is not divisible by 5, and if we used between 1 and 4 pennies, the sum would not be divisible by 5.)

Hence the smallest coin we can use is a nickel, and thus the smallest amount we can get is $4 \cdot 5 = 20$. Therefore the option that is not reachable is $\boxed{15} \Rightarrow (A)$.

We can verify that we can indeed get the other ones:

- $25 = 10 + 5 + 5 + 5$
- $35 = 10 + 10 + 10 + 5$
- $45 = 25 + 10 + 5 + 5$
- $55 = 25 + 10 + 10 + 10$

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 12A Problems/Problem 2

The following problem is from both the 2009 AMC 12A #2 and 2009 AMC 10A #3, so both problems redirect to this page.

Problem

Which of the following is equal to $1+\frac{1}{1+\frac{1}{1+1}}$?

$\textbf{(A)}\ \frac{5}{4} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{5}{3} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Solution

We compute:

$$\begin{aligned} 1+\frac{1}{1+\frac{1}{1+1}} &= 1+\frac{1}{1+\frac{1}{1+1}} \\ &= 1+\frac{1}{1+\frac{1}{2}} \\ &= 1+\frac{1}{\frac{3}{2}} \\ &= 1+\frac{2}{3} \\ &= \frac{5}{3} \end{aligned}$$

This is choice $\boxed{\text{C}}$.

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 10A Problems/Problem 4

Problem

Eric plans to compete in a triathlon. He can average 2 miles per hour in the $\frac{1}{4}$-mile swim and 6 miles per hour in the 3-mile run. His goal is to finish the triathlon in 2 hours. To accomplish his goal what must his average speed in miles per hour, be for the 15-mile bicycle ride?

$\mathrm{(A)}\ \frac{120}{11} \qquad \mathrm{(B)}\ 11 \qquad \mathrm{(C)}\ \frac{56}{5} \qquad \mathrm{(D)}\ \frac{45}{4} \qquad \mathrm{(E)}\ 12$

Solution

Since $d = rt$, Eric takes $\frac{\frac{1}{4}}{2} = \frac{1}{8}$ hours for the swim. Then, he takes $\frac{3}{6} = \frac{1}{2}$ hours for the run. So he needs to take $2 - \frac{5}{8} = \frac{11}{8}$ hours for the 15 mile run. This is $\frac{15}{\frac{11}{8}} = \frac{120}{11} \frac{\text{miles}}{\text{hour}}$

$\longrightarrow \boxed{\text{A}}$

See also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 5

Contents

Problem

What is the sum of the digits of the square of $111,111,111$?

$\mathrm{(A)}\ 18\qquad \mathrm{(B)}\ 27\qquad \mathrm{(C)}\ 45\qquad \mathrm{(D)}\ 63\qquad \mathrm{(E)}\ 81$

Solution

Using the standard multiplication algorithm, $111,111,111^2=12,345,678,987,654,321,$ whose digit sum is $81 \longrightarrow \boxed{\text{E}}$.

Solution 2

Note that $11^2 = 121$, $111^2 = 12321$. We see a pattern and find that $111,111,111^2=12,345,678,987,654,321$ whose digit sum is $81 \longrightarrow \boxed{\text{E}}$.

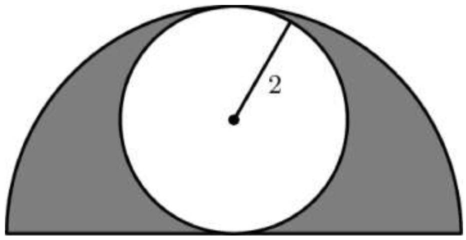
See also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 6

Problem

A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?



$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{\pi}{6} \qquad \textbf{(C)}\ \frac{2}{\pi} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{3}{\pi}$

Solution

Area of the circle inscribed inside the semicircle $= \pi r^2 \Rightarrow \pi(2^2) = 4\pi.$ Area of the larger circle (semicircle's area x 2)$= \pi r^2 \Rightarrow \pi(4^2) = 16\pi$ (4, or the diameter of the inscribed circle is the same thing as the radius of the semicircle). Thus, the area of the semicircle is $\frac{1}{2}(16\pi) \Rightarrow 8\pi.$ Part of the semicircle that is unshaded is $\frac{4\pi}{8\pi} = \frac{1}{2}$ Therefore, the shaded part is $1 - \frac{1}{2} = \frac{1}{2}$

Thus the answer is $\frac{1}{2} \Rightarrow \boxed{A}$

See also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 7

Problem

A carton contains milk that is 2% fat, an amount that is 40% less fat than the amount contained in a carton of whole milk. What is the percentage of fat in whole milk?

$\mathrm{(A)}\ \frac{12}{5} \qquad \mathrm{(B)}\ 3 \qquad \mathrm{(C)}\ \frac{10}{3} \qquad \mathrm{(D)}\ 38 \qquad \mathrm{(E)}\ 42$

Solution

Rewording the question, basically we are being asked "2 is 40% less than what number?" If x represents the number we are looking for, then 40% less than the number would be represented by $x-0.4x$ or $0.6x$. Thus $0.6x=2$; solving for x, we get $x=\frac{10}{3}$, $\longrightarrow \boxed{C}$

See also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 8

Problem 8

Three Generations of the Wen family are going to the movies, two from each generation. The two members of the youngest generation receive a 50% discount as children. The two members of the oldest generation receive a 25% discount as senior citizens. The two members of the middle generation receive no discount. Grandfather Wen, whose senior ticket costs <dollar/>6.00, is paying for everyone. How many dollars must he pay?

$\mathrm{(A)}\ 34 \qquad \mathrm{(B)}\ 36 \qquad \mathrm{(C)}\ 42 \qquad \mathrm{(D)}\ 46 \qquad \mathrm{(E)}\ 48$

Solution

A senior ticket costs <dollar/>6.00, so a regular ticket costs $6 \cdot \frac{1}{\frac{3}{4}} = 6 \cdot \frac{4}{3} = 8$ dollars. Therefore children's tickets cost half that, or <dollar/>4.00, so we have:

$$2(6+8+4) = 36$$

So Grandfather Wen pays <dollar/>36, or B.

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 9

Problem

Positive integers a, b, and 2009, with $a<b<2009$, form a geometric sequence with an integer ratio. What is a?

$\mathrm{(A)}\ 7 \qquad \mathrm{(B)}\ 41 \qquad \mathrm{(C)}\ 49 \qquad \mathrm{(D)}\ 289 \qquad \mathrm{(E)}\ 2009$

Solution

The prime factorization of 2009 is $2009 = 7\cdot 7\cdot 41$. As $a<b<2009$, the ratio must be positive and larger than 1, hence there is only one possibility: the ratio must be 7, and then $b=7\cdot 41$, and $a=41 \Rightarrow \mathrm{(B)}$.

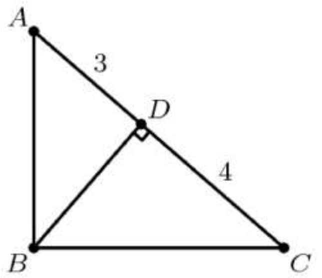
See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 10

Problem

Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?



$\mathrm{(A)}\ 4\sqrt3 \qquad \mathrm{(B)}\ 7\sqrt3 \qquad \mathrm{(C)}\ 21 \qquad \mathrm{(D)}\ 14\sqrt3 \qquad \mathrm{(E)}\ 42$

Solution

It is a well-known fact that in any right triangle ABC with the right angle at B and D the foot of the altitude from B onto AC we have $BD^2 = AD \cdot CD$. (See below for a proof.) Then $BD = \sqrt{3 \cdot 4} = 2\sqrt3$, and the area of the triangle ABC is $\dfrac{AC \cdot BD}{2} = 7\sqrt3 \Rightarrow \boxed{\mathrm{(B)}}$.

Proof: Consider the Pythagorean theorem for each of the triangles ABC, ABD, and CBD. We get:

1. $AB^2 + BC^2 = AC^2 = (AD + DC)^2 = AD^2 + DC^2 + 2 \cdot AD \cdot DC$.
2. $AB^2 = AD^2 + BD^2$
3. $BC^2 = BD^2 + CD^2$

Substituting equations 2 and 3 into the left hand side of equation 1, we get $BD^2 = AD \cdot DC$.

Alternatively, note that $\triangle ABD \sim \triangle BCD \Longrightarrow \dfrac{AD}{BD} = \dfrac{BD}{CD}$. $\blacksquare$

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2009_AMC_10A_Problems/Problem_10&oldid=62978"

Category: Introductory Geometry Problems

2009 AMC 12A Problems/Problem 5

The following problem is from both the 2009 AMC 12A #5 and 2009 AMC 10A #11, so both problems redirect to this page.

Problem

One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 125 \qquad \textbf{(E)}\ 216$

Solution

Let the original cube have edge length a. Then its volume is a^3. The new box has dimensions $a-1$, a, and $a+1$, hence its volume is $(a-1)a(a+1)=a^3-a$. The difference between the two volumes is a. As we are given that the difference is 5, we have $a=5$, and the volume of the original cube was $5^3=125\Rightarrow\boxed{\text{(D)}}$.

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

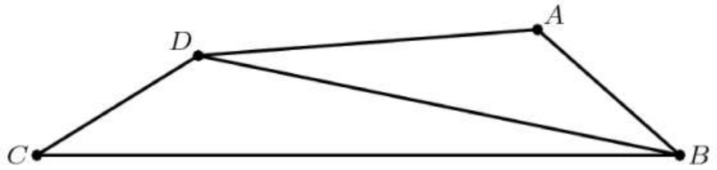
2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 12A Problems/Problem 10

The following problem is from both the 2009 AMC 12A #10 and 2009 AMC 10A #12, so both problems redirect to this page.

Problem

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?



$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Solution

By the triangle inequality we have $BD < DA + AB = 9 + 5 = 14$, and also $BD + CD > BC$, hence $BD > BC - CD = 17 - 5 = 12$.

We got that $12 < BD < 14$, and as we know that BD is an integer, we must have $BD = \boxed{13}$.

See Also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2009_AMC_12A_Problems/Problem_10&oldid=62977"

Category: Introductory Geometry Problems

2009 AMC 12A Problems/Problem 6

The following problem is from both the 2009 AMC 12A #6 and 2009 AMC 10A #13, so both problems redirect to this page.

Problem

Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m,n)?

$\textbf{(A)}\ P^2Q \qquad \textbf{(B)}\ P^nQ^m \qquad \textbf{(C)}\ P^nQ^{2m} \qquad \textbf{(D)}\ P^{2m}Q^n \qquad \textbf{(E)}\ P^{2n}Q^m$

Solution

We have $12^{mn} = (2\cdot 2\cdot 3)^{mn} = 2^{2mn}\cdot 3^{mn} = (2^m)^{2n}\cdot (3^n)^m = \boxed{\mathrm{E)}P^{2n}Q^m}$.

See Also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

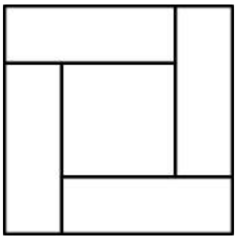
2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 12A Problems/Problem 8

The following problem is from both the 2009 AMC 12A #8 and 2009 AMC 10A #14, so both problems redirect to this page.

Problem

Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?



$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ \sqrt{10} \qquad \textbf{(C)}\ 2+\sqrt{2} \qquad \textbf{(D)}\ 2\sqrt{3} \qquad \textbf{(E)}\ 4$

Solution

$\boxed{(A)}$ The area of the outer square is 4 times that of the inner square. Therefore the side of the outer square is $\sqrt 4 = 2$ times that of the inner square.

Then the shorter side of the rectangle is $1/4$ of the side of the outer square, and the longer side of the rectangle is $3/4$ of the side of the outer square, hence their ratio is $\boxed{3}$.

See Also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2009_AMC_12A_Problems/Problem_8&oldid=72924"

Category: Introductory Geometry Problems

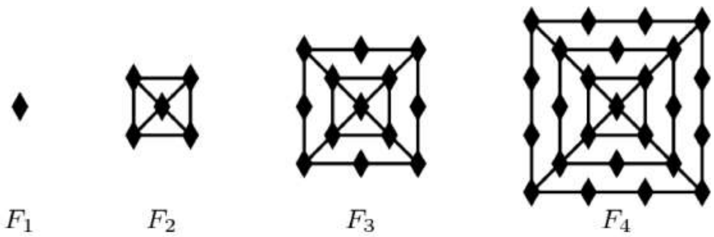
2009 AMC 12A Problems/Problem 11

The following problem is from both the 2009 AMC 12A #11 and 2009 AMC 10A #15, so both problems redirect to this page.

Contents

Problem

The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



$\textbf{(A)}\ 401 \qquad \textbf{(B)}\ 485 \qquad \textbf{(C)}\ 585 \qquad \textbf{(D)}\ 626 \qquad \textbf{(E)}\ 761$

Solution

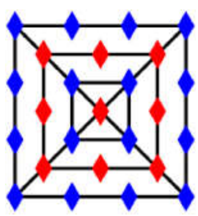
Solution 1

Split F_n into 4 congruent triangles by its diagonals (like in the pictures in the problem). This shows that the number of diamonds it contains is equal to 4 times the $(n-2)$th triangular number (i.e. the diamonds within the triangles or between the diagonals) and $4(n-1)+1$ (the diamonds on sides of the triangles or on the diagonals). The nth triangular number is $\frac{n(n+1)}{2}$. Putting this together for F_{20} this gives:

$$\frac{4(18)(19)}{2} + 4(19) + 1 = \boxed{761}$$

Solution 2

Color the diamond layers alternately blue and red, starting from the outside. You'll get the following pattern:



In the figure F_n, the blue diamonds form a $n \times n$ square, and the red diamonds form a $(n-1) \times (n-1)$ square. Hence the total number of diamonds in F_{20} is $20^2 + 19^2 = \boxed{761}$.

Solution 3

When constructing F_n from F_{n-1}, we add $4(n-1)$ new diamonds. Let d_n be the number of diamonds in F_n. We now know that $d_1 = 1$ and $\forall n > 1:\ d_n = d_{n-1} + 4(n-1)$.

Hence we get:

$$\begin{aligned} d_{20} &= d_{19} + 4\cdot 19 \\ &= d_{18} + 4\cdot 18 + 4\cdot 19 \\ &= \cdots \\ &= 1 + 4(1+2+\cdots+18+19) \\ &= 1 + 4\cdot\frac{19\cdot 20}{2} \\ &= \boxed{761} \end{aligned}$$

See Also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 16

Contents

Problem

Let a, b, c, and d be real numbers with $|a-b|=2$, $|b-c|=3$, and $|c-d|=4$. What is the sum of all possible values of $|a-d|$?

$\mathrm{(A)}\ 9 \qquad \mathrm{(B)}\ 12 \qquad \mathrm{(C)}\ 15 \qquad \mathrm{(D)}\ 18 \qquad \mathrm{(E)}\ 24$

Solution 1

From $|a-b|=2$ we get that $a=b\pm 2$

Similarly, $b=c\pm 3$ and $c=d\pm 4$.

Substitution gives $a=d\pm 4\pm 3\pm 2$. This gives $|a-d|=|\pm 4\pm 3\pm 2|$. There are $2^3=8$ possibilities for the value of $\pm 4\pm 3\pm 2$:

$4+3+2=\boxed{9}$,

$4+3-2=\boxed{5}$,

$4-3+2=\boxed{3}$,

$-4+3+2=\boxed{1}$,

$4-3-2=\boxed{-1}$,

$-4+3-2=\boxed{-3}$,

$-4-3+2=\boxed{-5}$,

$-4-3-2=\boxed{-9}$

Therefore, the only possible values of $|a-d|$ are 9, 5, 3, and 1. Their sum is $\boxed{18}$.

Solution 2

If we add the same constant to all of a, b, c, and d, we will not change any of the differences. Hence we can assume that $a=0$.

From $|a-b|=2$ we get that $|b|=2$, hence $b\in\{-2,2\}$.

If we multiply all four numbers by -1, we will not change any of the differences. Hence we can WLOG assume that $b=2$.

From $|b-c|=3$ we get that $c\in\{-1,5\}$.

From $|c-d|=4$ we get that $d\in\{-5,1,3,9\}$.

Hence $|a-d|=|d|\in\{1,3,5,9\}$, and the sum of possible values is $1+3+5+9=\boxed{18}$.

See Also

2009 AMC 10A Problems/Problem 17

Contents

Problem

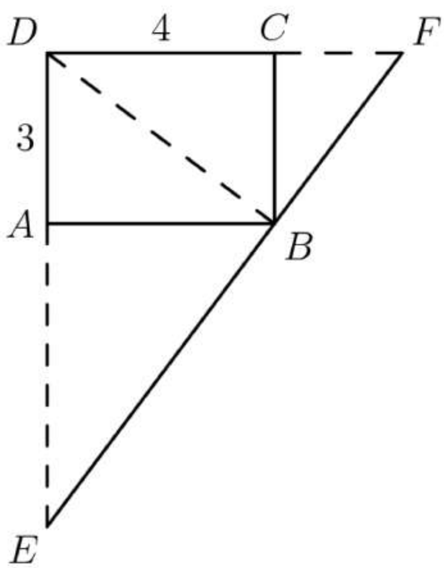
Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that EF is perpendicular to DB, and A and C lie on DE and DF, respectively. What is EF?

$\mathrm{(A)}\ 9 \qquad \mathrm{(B)}\ 10 \qquad \mathrm{(C)}\ \frac{125}{12} \qquad \mathrm{(D)}\ \frac{103}{9} \qquad \mathrm{(E)}\ 12$

Solutions

Solution 1

The situation is shown in the picture below.



From the Pythagorean theorem we have $BD = 5$.

Triangle EAB is similar to DAB, as they have the same angles. Segment BA is perpendicular to DA, meaning that angle DAB and BAE are right angles and congruent. Also, angle DBE is a right angle. Because it is a rectangle, angle BDC is congruent to DBA and angle ADC is also a right angle. By the transitive property:

$mADB + mBDC = mDBA + mABE$

$mBDC = mDBA$

$mADB + mBDC = mBDC + mABE$

$mADB = mABE$

Next, because every triangle has a degree measure of 180, angle E and angle DBA are congruent.

Hence $BE/AB = DB/AD$, and therefore $BE = AB \cdot DB/AD = 20/3$.

Also triangle CBF is similar to ABD. Hence $BF/BC = DB/AB$, and therefore $BF = BC \cdot DB/AB = 15/4$.

We then have $EF = EB + BF = \frac{20}{3} + \frac{15}{4} = \frac{80+45}{12} = \boxed{\frac{125}{12}}$.

Solution 2

Since BD is the altitude from B to EF, we can use the equation $BD^2 = EB \cdot BF$.

Looking at the angles, we see that triangle EAB is similar to DCB. Because of this, $\frac{AB}{CB} = \frac{EB}{DB}$. From the given information and the Pythagorean theorem, $AB = 4$, $CB = 3$, and $DB = 5$. Solving gives $EB = 20/3$.

We can use the above formula to solve for BF. $BD^2 = 20/3 \cdot BF$. Solve to obtain $BF = 15/4$.

We now know EB and BF. $EF = EB + BF = \frac{20}{3} + \frac{15}{4} = \frac{80+45}{12} = \boxed{\frac{125}{12}}$.

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2009 AMC 10A Problems/Problem 18

Contents

Problem

At Jefferson Summer Camp, 60% of the children play soccer, 30% of the children swim, and 40% of the soccer players swim. To the nearest whole percent, what percent of the non-swimmers play soccer?

$\mathrm{(A)}\ 30\% \qquad \mathrm{(B)}\ 40\% \qquad \mathrm{(C)}\ 49\% \qquad \mathrm{(D)}\ 51\% \qquad \mathrm{(E)}\ 70\%$

Solution 1

Out of the soccer players, 40% swim. As the soccer players are 60% of the whole, the swimming soccer players are $0.4 \cdot 0.6 = 0.24 = 24\%$ of all children.

The non-swimming soccer players then form $60\% - 24\% = 36\%$ of all the children.

Out of all the children, 30% swim. We know that 24% of all the children swim and play soccer, hence $30\% - 24\% = 6\%$ of all the children swim and don't play soccer.

Finally, we know that 70% of all the children are non-swimmers. And as 36% of all the children do not swim but play soccer, $70\% - 36\% = 34\%$ of all the children do not engage in any activity.

A quick summary of what we found out:

- 24%: swimming yes, soccer yes
- 36%: swimming no, soccer yes
- 6%: swimming yes, soccer no
- 34%: swimming no, soccer no

Now we can compute the answer. Out of all children, 70% are non-swimmers, and again out of all children 36% are non-swimmers that play soccer. Hence the part of non-swimmers that plays soccer is $\frac{36}{70} \approx \boxed{51\%}$.

Solution 2

Let us set that the total number of children is 100. So 60 children play soccer, 30 swim, and $0.4 \times 60 = 24$ play soccer and swim.

Thus, $60 - 24 = 36$ children only play soccer.

So our numerator is 36.

Our denominator is simply $100 - \text{Swimmers} = 100 - 30 = 70$

And so we get $\frac{36}{70}$ which is roughly $51.4 = \boxed{\text{D}}$

See Also

2009 AMC 10A Problems/Problem 19

Problem

Circle A has radius 100. Circle B has an integer radius $r < 100$ and remains internally tangent to circle A as it rolls once around the circumference of circle A. The two circles have the same points of tangency at the beginning and end of cirle B's trip. How many possible values can r have?

$\mathrm{(A)}\ 4\qquad \mathrm{(B)}\ 8\qquad \mathrm{(C)}\ 9\qquad \mathrm{(D)}\ 50\qquad \mathrm{(E)}\ 90$

Solution

The circumference of circle A is 200π, and the circumference of circle B with radius r is $2r\pi$. Since circle B makes a complete revolution and ends up on the same point, the circumference of A must be a multiple of the circumference of B, therefore the quotient must be an integer.

Thus, $\frac{200\pi}{2\pi \cdot r} = \frac{100}{r}$

Therefore r must then be a factor of 100, excluding 100 (because then circle B would be the same size as circle A). $100 = 2^2 \cdot 5^2$. Therefore 100 has $(2+1) \cdot (2+1)$ factors*. But you need to subtract 1 from 9, in order to exclude 100. Therefore the answer is $\boxed{8}$.

*The number of factors of $a^x \cdot b^y \cdot c^z$ ... and so on, where $a, b,$ and c are prime numbers, is $(x+1)(y+1)($

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 20

Problem

Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

$\mathrm{(A)}\ 20 \qquad\mathrm{(B)}\ 30 \qquad\mathrm{(C)}\ 55 \qquad\mathrm{(D)}\ 65 \qquad\mathrm{(E)}\ 80$

Solution

Let their speeds in kilometers per hour be v_A and v_L. We know that $v_A = 3v_L$ and that $v_A + v_L = 60$. (The second equation follows from the fact that $1\mathrm{km}/min = 60\mathrm{km}/h$.) This solves to $v_A = 45$ and $v_L = 15$.

As the distance decreases at a rate of 1 kilometer per minute, after 5 minutes the distance between them will be $20 - 5 = 15$ kilometers.

From this point on, only Lauren will be riding her bike. As there are 15 kilometers remaining and $v_L = 15$, she will need exactly an hour to get to Andrea. Therefore the total time in minutes is $5 + 60 = \boxed{65}$.

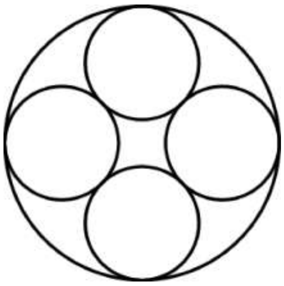
See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 10A Problems/Problem 21

Problem

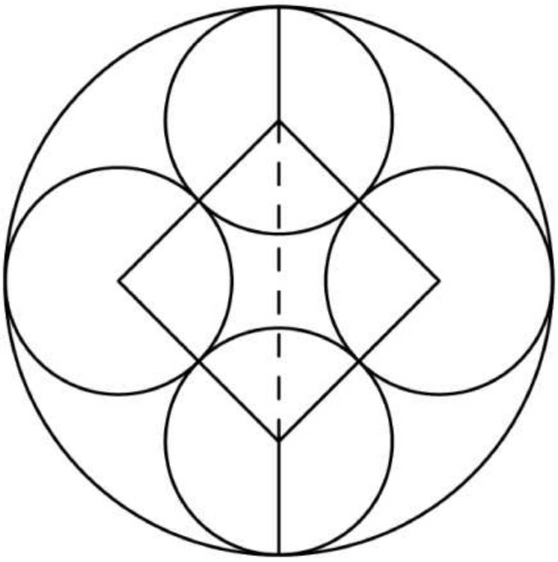
Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle, In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?



(A) $3-2\sqrt{2}$ (B) $2-\sqrt{2}$ (C) $4(3-2\sqrt{2})$ (D) $\frac{1}{2}(3-\sqrt{2})$ (E) $2\sqrt{2}-2$

Solution

Draw some of the radii of the small circles as in the picture below.



Out of symmetry, the quadrilateral in the center must be a square. Its side is obviously $2r$, and therefore its diagonal is $2r\sqrt{2}$. We can now compute the length of the vertical diameter of the large circle as $2r+2r\sqrt{2}$. Hence $2R=2r+2r\sqrt{2}$, and thus $R=r+r\sqrt{2}=r(1+\sqrt{2})$.

Then the area of the large circle is $L=\pi R^2=\pi r^2(1+\sqrt{2})^2=\pi r^2(3+2\sqrt{2})$. The area of four small circles is $S=4\pi r^2$. Hence their ratio is:

$$\begin{aligned}\frac{S}{L} &= \frac{4\pi r^2}{\pi r^2(3+2\sqrt{2})} \\ &= \frac{4}{3+2\sqrt{2}} \\ &= \frac{4}{3+2\sqrt{2}} \cdot \frac{3-2\sqrt{2}}{3-2\sqrt{2}} \\ &= \frac{4(3-2\sqrt{2})}{3^2-(2\sqrt{2})^2} \\ &= \frac{4(3-2\sqrt{2})}{1} \\ &= \boxed{4(3-2\sqrt{2})}\end{aligned}$$

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2009 AMC 10A Problems/Problem 22

Contents

Problem

Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

$\mathrm{(A)}\ \frac{1}{9} \qquad \mathrm{(B)}\ \frac{1}{8} \qquad \mathrm{(C)}\ \frac{1}{6} \qquad \mathrm{(D)}\ \frac{2}{11} \qquad \mathrm{(E)}\ \frac{1}{5}$

Solution

Solution 1

At the moment when the numbers are in the bag, imagine that each of them has a different color. Clearly the situation is symmetric at this moment. Hence after we draw them, attach them and throw the dice, the probability of getting some pair of colors is the same for any two colors.

There are $\binom{12}{2} = 66$ ways how to pick two of the colors. We now have to count the ways where the two chosen numbers will have sum 7.

Sum 7 can be obtained as $1+6$, $2+5$, or $3+4$. Each number in the bag has two different colors, hence each of these three options corresponds to four pairs of colors.

Out of the 66 pairs of colors we can get when throwing the dice, $3\cdot 4=12$ will give us the sum 7. Hence the probability that this will happen is $\frac{12}{66} = \boxed{\frac{2}{11}}$.

Solution 2

Ignoring the numbers that do not affect the probability of the desired outcome (the ones that are not on top of the dice), say that the number on top of the first die is n. For the sum of the 2 numbers to be 7, the second die must have the number $7-n$ on top. There are 11 remaining numbers that could be on top of the second die, 2 of which are $7-n$ (since $n \neq 7-n$ in all cases).

Thus, the probability of the sum of the 2 numbers being 7 is $\frac{2}{11}$, so the answer is $\mathrm{(D)}$.

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2009 AMC 12A Problems/Problem 20

The following problem is from both the 2009 AMC 12A #20 and 2009 AMC 10A #23, so both problems redirect to this page.

Problem

Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

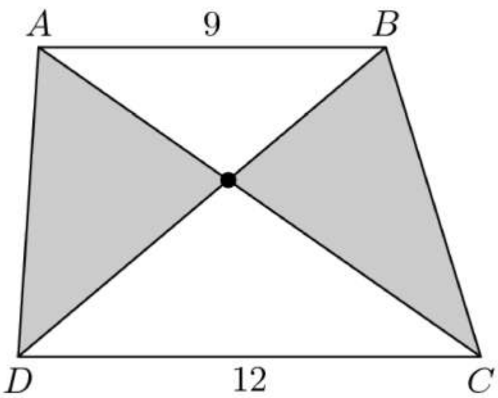
$\textbf{(A)}\ \frac{9}{2}\qquad \textbf{(B)}\ \frac{50}{11}\qquad \textbf{(C)}\ \frac{21}{4}\qquad \textbf{(D)}\ \frac{17}{3}\qquad \textbf{(E)}\ 6$

Contents

Solution

Solution 1

Let $[ABC]$ denote the area of triangle ABC. $[AED] = [BEC]$, so $[ABD] = [AED] + [AEB] = [BEC] + [AEB] = [ABC]$. Since triangles ABD and ABC share a base, they also have the same height and thus $\overline{AB}||\overline{CD}$ and $\triangle AEB \sim \triangle CED$ with a ratio of $3:4$. $AE = \frac{3}{7}AC = 6 \boxed{\textbf{(E)}}$.



Solution 2

Using the sine area formula on triangles AED and BEC, as $\angle AED = \angle BEC$, we see that

$$(AE)(ED) = (BE)(EC) \implies \frac{AE}{EC} = \frac{BE}{ED}.$$

Since $\angle AEB = \angle DEC$, triangles AEB and DEC are similar. Their ratio is $\frac{AB}{CD} = \frac{3}{4}$. Since $AE + EC = 14$, we must have $EC = 8$, so $AE = 6\ \textbf{(E)}$.

Solution 3 (which won't work when justification is required)

Consider an isosceles trapezoid with opposite bases of 9 and 12 and a diagonal of length 14. This trapezoid exists and satisfies all the conditions in the problem (the areas are congruent by symmetry). So, we can simply use this easier case to solve this problem, because the problem implies that the answer is invariant for all quadrilaterals satisfying the conditions.

Then, by similar triangles, the ratio of AE to EC is $3:4$, so $AE = \boxed{\textbf{(E)} 6}$.

See also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2009 AMC 10A Problems/Problem 24

Contents

Problem

Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

$\mathrm{(A)}\ \frac{1}{4} \qquad \mathrm{(B)}\ \frac{3}{8} \qquad \mathrm{(C)}\ \frac{4}{7} \qquad \mathrm{(D)}\ \frac{5}{7} \qquad \mathrm{(E)}\ \frac{3}{4}$

Solution

Solution 1

We will try to use symmetry as much as possible.

Pick the first vertex A, its choice clearly does not influence anything.

Pick the second vertex B. With probability $3/7$ vertices A and B have a common edge, with probability $3/7$ they are in opposite corners of the same face, and with probability $1/7$ they are in opposite corners of the cube. We will handle each of the cases separately.

In the first case, there are 2 faces that contain the edge AB. In each of these faces there are 2 other vertices. If one of these 4 vertices is the third vertex C, the entire triangle ABC will be on a face. On the other hand, if C is one of the two remaining vertices, the triangle will contain points inside the cube. Hence in this case the probability of choosing a good C is $2/6=1/3$.

In the second case, the triangle ABC will not intersect the cube if point C is one of the two points on the side that contains AB. Hence the probability of ABC intersecting the inside of the cube is $2/3$.

In the third case, already the diagonal AB contains points inside the cube, hence this case will be good regardless of the choice of C.

Summing up all cases, the resulting probability is:

$$\frac 37 \cdot \frac 13 + \frac 37 \cdot \frac 23 + \frac 17 \cdot 1 = \boxed{\frac 47}$$

Solution 2

There are $\binom{8}{3} = 56$ ways to pick three vertices from eight total vertices; this is our denominator. In order to have three points inside the cube, they cannot be on the surface. Thus, we can use complementary probability.

There are four ways to choose three points from the vertices of a single face. Since there are six faces, $4 \times 6 = 24$.

Thus, the probability of what we don't want is $\frac{24}{56} = \frac{3}{7}$. Using complementary,

$$1-\frac{3}{7}=\boxed{\frac{4}{7}}$$

See Also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2009_AMC_10A_Problems/Problem_24&oldid=75779"

Category: Introductory Combinatorics Problems

2009 AMC 12A Problems/Problem 18

The following problem is from both the 2009 AMC 12A #18 and 2009 AMC 10A #25, so both problems redirect to this page.

Contents

Problem

For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Solution

The number I_k can be written as $10^{k+2} + 64 = 5^{k+2} \cdot 2^{k+2} + 2^6$.

For $k \in \{1, 2, 3\}$ we have $I_k = 2^{k+2}\left(5^{k+2} + 2^{4-k}\right)$. The first value in the parentheses is odd, the second one is even, hence their sum is odd and we have $N(k) = k + 2 \le 5$.

For $k > 4$ we have $I_k = 2^6\left(5^{k+2} \cdot 2^{k-4} + 1\right)$. For $k > 4$ the value in the parentheses is odd, hence $N(k) = 6$.

This leaves the case $k = 4$. We have $I_4 = 2^6\left(5^6 + 1\right)$. The value $5^6 + 1$ is obviously even. And as $5 \equiv 1 \pmod 4$, we have $5^6 \equiv 1 \pmod 4$, and therefore $5^6 + 1 \equiv 2 \pmod 4$. Hence the largest power of 2 that divides $5^6 + 1$ is 2^1, and this gives us the desired maximum of the function N: $N(4) = \boxed{7}$.

Alternate Solution

Notice that 2 is a prime factor of an integer n if and only if n is even. Therefore, given any sufficiently high positive integral value of k, dividing I_k by 2^6 yields a terminal digit of zero, and dividing by 2 again leaves us with $2^7 \cdot a = I_k$ where a is an odd integer. Observe then that $\boxed{\textbf{(B)} 7}$ must be the maximum value for $N(k)$ because whatever value we choose for k, $N(k)$ must be less than or equal to 7.

EDIT: Isn't this solution incomplete because we need to show that $N(k) = 7$ can be reached?

See Also

2009 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	