

2009 AMC 10B Problems/Problem 1

The following problem is from both the 2009 AMC 10B #1 and 2009 AMC 12B #1, so both problems redirect to this page.

Problem

Each morning of her five-day workweek, Jane bought either a 50-cent muffin or a 75-cent bagel. Her total cost for the week was a whole number of dollars, How many bagels did she buy?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Solution

The only combination of five items with total cost a whole number of dollars is 3 muffins and

2

 bagels. The answer is **(B)**.

See also

2009 AMC 10B (Problems • Answer Key • Resources) (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009)	
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2009 AMC 10B Problems/Problem 2

Problem

Which of the following is equal to $\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{2} - \frac{1}{3}}$?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

Solution

Multiplying the numerator and the denominator by the same value does not change the value of the fraction. We can multiply both by **12**, getting $\frac{4 - 3}{6 - 4} = \boxed{\frac{1}{2}}$.

Alternately, we can directly compute that the numerator is $\frac{1}{12}$, the denominator is $\frac{1}{6}$, and hence their ratio is $\boxed{\frac{1}{2}}$.

See Also

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2009 AMC 10B Problems/Problem 3

The following problem is from both the 2009 AMC 10B #3 and 2009 AMC 12B #2, so both problems redirect to this page.

Problem

Paula the painter had just enough paint for 30 identically sized rooms. Unfortunately, on the way to work, three cans of paint fell off her truck, so she had only enough paint for 25 rooms. How many cans of paint did she use for the 25 rooms?

- (A) 10
- (B) 12
- (C) 15
- (D) 18
- (E) 25

Solution

Losing three cans of paint corresponds to being able to paint five fewer rooms. So $\frac{3}{5} \cdot 25 = \boxed{15}$. The answer is (C).

See also

2009 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
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2009 AMC 10B Problems/Problem 4

The following problem is from both the 2009 AMC 10B #4 and 2009 AMC 12B #4, so both problems redirect to this page.

Problem

A rectangular yard contains two flower beds in the shape of congruent isosceles right triangles. The remainder of the yard has a trapezoidal shape, as shown. The parallel sides of the trapezoid have lengths **15** and **25** meters. What fraction of the yard is occupied by the flower beds?



- (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

Solution

Each triangle has leg length $\frac{1}{2} \cdot (25 - 15) = 5$ meters and area $\frac{1}{2} \cdot 5^2 = \frac{25}{2}$ square meters. Thus the flower beds have a total area of 25 square meters. The entire yard has length 25 m and width 5 m, so its area is 125 square meters. The fraction of the yard occupied by the flower beds is $\frac{25}{125} = \boxed{\frac{1}{5}}$. The answer is (C).

See also

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2009 AMC 10B Problems/Problem 5

The following problem is from both the 2009 AMC 10B #5 and 2009 AMC 12B #3, so both problems redirect to this page.

Problem

Twenty percent off 60 is one-third more than what number?

- (A) 16
- (B) 30
- (C) 32
- (D) 36
- (E) 48

Solution

Twenty percent less than 60 is $\frac{4}{5} \cdot 60 = 48$. One-third more than a number n is $\frac{4}{3}n$. Therefore $\frac{4}{3}n = 48$ and the number is 36. The answer is (D).

See also

2009 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
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2009 AMC 10B Problems/Problem 6

The following problem is from both the 2009 AMC 10B #6 and 2009 AMC 12B #5, so both problems redirect to this page.

Problem

Kiana has two older twin brothers. The product of their three ages is 128. What is the sum of their three ages?
(A) 10 (B) 12 (C) 16 (D) 18 (E) 24

Solution

The age of each person is a factor of $128 = 2^7$. So the twins could be $2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8$ years of age and, consequently Kiana could be 128, 32, 8 or 2 years old, respectively. Because Kiana is younger than her brothers, she must be 2 years old. So the sum of their ages is $2 + 8 + 8 = \boxed{18}$. The answer is (D).

See also

2009 AMC 10B (Problems • Answer Key • Resources) (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009)	
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2009 AMC 10B Problems/Problem 7

The following problem is from both the 2009 AMC 10B #7 and 2009 AMC 12B #6, so both problems redirect to this page.

Problem

By inserting parentheses, it is possible to give the expression

$$2 \times 3 + 4 \times 5$$

several values. How many different values can be obtained?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Solution

The three operations can be performed on any of $3! = 6$ orders. However, if the addition is performed either first or last, then multiplying in either order produces the same result. So at most four distinct values can be obtained. It is easy to check that the values of the four expressions

$$(2 \times 3) + (4 \times 5) = 26,$$

$$(2 \times 3 + 4) \times 5 = 50,$$

$$2 \times (3 + (4 \times 5)) = 46,$$

$$2 \times (3 + 4) \times 5 = 70$$

are in fact all distinct. So the answer is 4, which is choice (C).

See also

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2009 AMC 10B Problems/Problem 8

The following problem is from both the 2009 AMC 10B #8 and 2009 AMC 12B #7, so both problems redirect to this page.

Problem

In a certain year the price of gasoline rose by **20%** during January, fell by **20%** during February, rose by **25%** during March, and fell by **$x\%$** during April. The price of gasoline at the end of April was the same as it had been at the beginning of January. To the nearest integer, what is **x**

- (A) 12 (B) 17 (C) 20 (D) 25 (E) 35

Solution

Let **p** be the price at the beginning of January. The price at the end of March was $(1.2)(0.8)(1.25)p = 1.2p$. Because the price at the of April was **p** , the price decreased by **$0.2p$** during April, and the percent decrease was

$$x = 100 \cdot \frac{0.2p}{1.2p} = \frac{100}{6} \approx 16.7.$$

So to the nearest integer **x** is 17. The answer is (B).

See also

2009 AMC 10B (Problems • Answer Key • Resources) (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009)	
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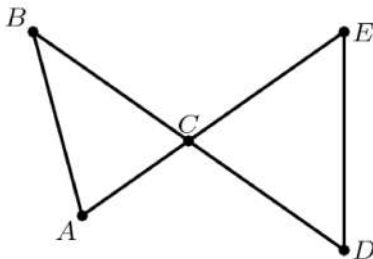
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2009 AMC 10B Problems/Problem 9

Problem

Segment BD and AE intersect at C , as shown, $AB = BC = CD = CE$, and $\angle A = \frac{5}{2}\angle B$. What is the degree measure of $\angle D$?



- (A) 52.5 (B) 55 (C) 57.7 (D) 60 (E) 62.5

Solution

$\triangle ABC$ is isosceles, hence $\angle ACB = \angle CAB$.

The sum of internal angles of $\triangle ABC$ can now be expressed as $\angle B + \frac{5}{2}\angle B + \frac{5}{2}\angle B = 6\angle B$, hence $\angle B = 30^\circ$, and each of the other two angles is 75° .

Now we know that $\angle DCE = \angle ACB = 75^\circ$.

Finally, $\triangle CDE$ is isosceles, hence each of the two remaining angles ($\angle D$ and $\angle E$) is equal to $\frac{180^\circ - 75^\circ}{2} = \frac{105^\circ}{2} = \boxed{52.5}$.

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Category: Introductory Geometry Problems

2009 AMC 10B Problems/Problem 10

Problem

A flagpole is originally **5** meters tall. A hurricane snaps the flagpole at a point **x** meters above the ground so that the upper part, still attached to the stump, touches the ground **1** meter away from the base. What is **x** ?

- (A) 2.0 (B) 2.1 (C) 2.2 (D) 2.3 (E) 2.4

Solution

The broken flagpole forms a right triangle with legs **1** and **x** , and hypotenuse **$5 - x$** . The Pythagorean theorem now states that **$1^2 + x^2 = (5 - x)^2$** , hence **$10x = 24$** , and **$x = \boxed{2.4}$** .

(Note that the resulting triangle is the well-known **5 - 12 - 13** right triangle, scaled by **$1/5$** .)

See Also

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2009 AMC 10B Problems/Problem 11

Problem

How many **7**-digit palindromes (numbers that read the same backward as forward) can be formed using the digits **2, 2, 3, 3, 5, 5, 5**?

- (A) 6 (B) 12 (C) 24 (D) 36 (E) 48

Solution

A seven-digit palindrome is a number of the form $\overline{abcdcba}$. Clearly, d must be **5**, as we have an odd number of fives. We are then left with $\{a, b, c\} = \{2, 3, 5\}$. Each of the 6 permutations of the set $\{2, 3, 5\}$ will give us one palindrome.

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2009 AMC 10B Problems/Problem 12

Problem

Distinct points A, B, C , and D lie on a line, with $AB = 1$, $BC = 2$, and $CD = 3$. Points E and F lie on a second line, parallel to the first, with $EF = 4$. A triangle with positive area has three of the six points as its vertices. How many possible values are there for the area of the triangle?

Solution

Consider the classical formula for triangle area: $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$. Each of the triangles that we can make has exactly one side lying on one of the two parallel lines. If we pick this side to be the base, the height will always be the same – it will be the distance between the two lines.

Hence each area is uniquely determined by the length of the base. And it can easily be seen, that the only possible base lengths are **1**, **2**, and **3**. Therefore there are only **3** possible values for the area.

(To be more precise in the last step, the possible base lengths are $AB = 1$, $BC = 2$, and $AD = 3$.)

See Also

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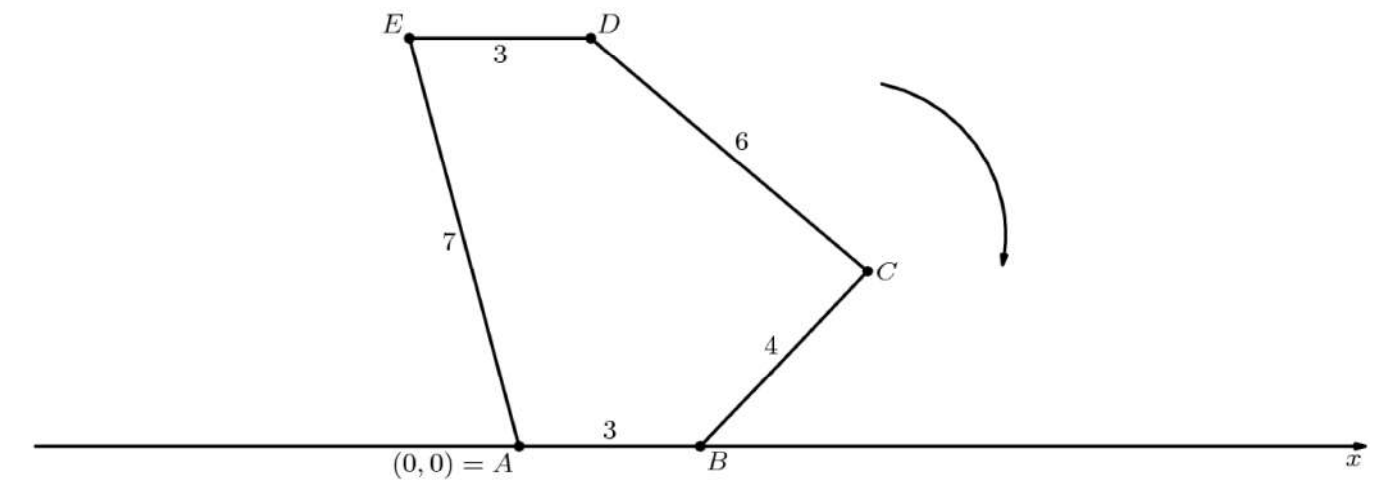
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Category: Introductory Geometry Problems

2009 AMC 10B Problems/Problem 13

Problem

As shown below, convex pentagon $ABCDE$ has sides $AB = 3$, $BC = 4$, $CD = 6$, $DE = 3$, and $EA = 7$. The pentagon is originally positioned in the plane with vertex A at the origin and vertex B on the positive x -axis. The pentagon is then rolled clockwise to the right along the x -axis. Which side will touch the point $x = 2009$ on the x -axis?



- (A) $\overline{AB}$
- (B) $\overline{BC}$
- (C) $\overline{CD}$
- (D) $\overline{DE}$
- (E) $\overline{EA}$

Solution

The perimeter of the polygon is $3 + 4 + 6 + 3 + 7 = 23$. Hence as we roll the polygon to the right, every 23 units the side $\overline{AB}$ will be the bottom side.

We have $2009 = 23 \times 87 + 8$. Thus at some point in time we will get the situation when $A = (2001, 0)$ and $\overline{AB}$ is the bottom side. Obviously, at this moment $B = (2004, 0)$.

After that, the polygon rotates around B until point C hits the x axis at $(2008, 0)$.

And finally, the polygon rotates around C until point D hits the x axis at $(2014, 0)$. At this point the side $\overline{CD}$ touches the point $(2009, 0)$. So the answer is $\boxed{C}$

See Also

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Category: Introductory Geometry Problems

2009 AMC 10B Problems/Problem 14

The following problem is from both the 2009 AMC 10B #14 and 2009 AMC 12B #11, so both problems redirect to this page.

Problem

On Monday, Millie puts a quart of seeds, **25%** of which are millet, into a bird feeder. On each successive day she adds another quart of the same mix of seeds without removing any seeds that are left. Each day the birds eat only **25%** of the millet in the feeder, but they eat all of the other seeds. On which day, just after Millie has placed the seeds, will the birds find that more than half the seeds in the feeder are millet?

(A) Tuesday (B) Wednesday (C) Thursday (D) Friday (E) Saturday

Solution

On Monday, day 1, the birds find $\frac{1}{4}$ quart of millet in the feeder. On Tuesday they find

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4}$$

quarts of millet. On Wednesday, day 3, they find

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4}$$

quarts of millet. The number of quarts of millet they find on day n is

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \cdots + \left(\frac{3}{4}\right)^{n-1} \cdot \frac{1}{4} = \frac{\left(\frac{1}{4}\right)\left(1 - \left(\frac{3}{4}\right)^n\right)}{1 - \frac{3}{4}} = 1 - \left(\frac{3}{4}\right)^n.$$

The birds always find $\frac{3}{4}$ quart of other seeds, so more than half the seeds are millet if $1 - \left(\frac{3}{4}\right)^n > \frac{3}{4}$, that

is, when $\left(\frac{3}{4}\right)^n < \frac{1}{4}$. Because $\left(\frac{3}{4}\right)^4 = \frac{81}{256} > \frac{1}{4}$ and $\left(\frac{3}{4}\right)^5 = \frac{243}{1024} < \frac{1}{4}$, this will first occur on day **5** which is **Friday**. The answer is **(D)**.

See also

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2009 AMC 10B Problems/Problem 15

The following problem is from both the 2009 AMC 10B #15 and 2009 AMC 12B #8, so both problems redirect to this page.

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Problem

When a bucket is two-thirds full of water, the bucket and water weigh a kilograms. When the bucket is one-half full of water the total weight is b kilograms. In terms of a and b , what is the total weight in kilograms when the bucket is full of water?

- (A) $\frac{2}{3}a + \frac{1}{3}b$ (B) $\frac{3}{2}a - \frac{1}{2}b$ (C) $\frac{3}{2}a + b$ (D) $\frac{3}{2}a + 2b$ (E) $3a - 2b$

Solution

Solution 1

Let x be the weight of the bucket and let y be the weight of the water in a full bucket. Then we are given that $x + \frac{2}{3}y = a$ and $x + \frac{1}{2}y = b$. Hence $\frac{1}{6}y = a - b$, so $y = 6a - 6b$. Thus $x = b - \frac{1}{2}(6a - 6b) = -3a + 4b$. Finally $x + y = \boxed{3a - 2b}$. The answer is (E).

Solution 2

Imagine that we take three buckets of the first type, to get rid of the fraction. We will have three buckets and two buckets' worth of water.

On the other hand, if we take two buckets of the second type, we will have two buckets and enough water to fill one bucket.

The difference between these is exactly one bucket full of water, hence the answer is $3a - 2b$.

Solution 3

We are looking for an expression of the form $xa + yb$.

We must have $x + y = 1$, as the desired result contains exactly one bucket. Also, we must have $\frac{2}{3}x + \frac{1}{2}y = 1$, as the desired result contains exactly one bucket of water.

At this moment, it is easiest to check that only the options (A), (B), and (E) satisfy $x + y = 1$, and out of these only (E) satisfies the second equation.

Alternately, we can directly solve the system, getting $x = 3$ and $y = -2$.

See also

2009 AMC 10B Problems/Problem 16

Contents

- 1 Problem
- 2 Solution
 - 2.1 Solution 1
 - 2.2 Solution 2
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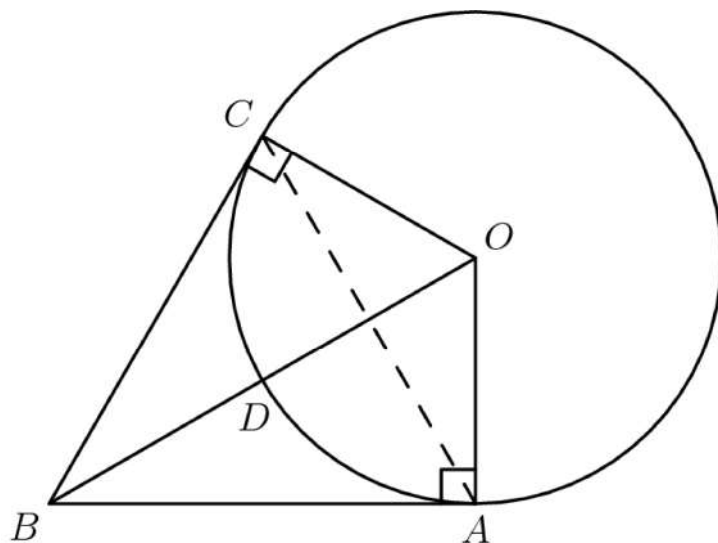
Problem

Points A and C lie on a circle centered at O , each of $\overline{BA}$ and $\overline{BC}$ are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects $\overline{BO}$ at D . What is $\frac{BD}{BO}$?

- (A) $\frac{\sqrt{2}}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{\sqrt{3}}{2}$

Solution

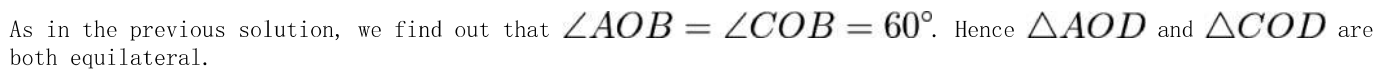
Solution 1



As $\triangle ABC$ is equilateral, we have $\angle BAC = \angle BCA = 60^\circ$, hence $\angle OAC = \angle OCA = 30^\circ$. Then $\angle AOC = 120^\circ$, and from symmetry we have $\angle AOB = \angle COB = 60^\circ$. Thus, this gives us $\angle ABO = \angle CBO = 30^\circ$.

We know that $DO = AO$, as D lies on the circle. From $\triangle ABO$ we also have $AO = BO \sin 30^\circ = \frac{BO}{2}$. Hence $DO = \frac{BO}{2}$, therefore $BD = BO - DO = \frac{BO}{2}$, and $\frac{BD}{BO} = \boxed{\frac{1}{2}} \Rightarrow B$.

Solution 2



equilateral, $\triangle BDE$ is also 180-centrifugal, hence $\angle BDE = \angle BDE$, and as $SE = SO = SO$, we have $2 \cdot SD = SD + SO = OD$, therefore $BD = OD$, and as before we conclude that $\frac{BD}{BO} = \frac{1}{2}$.

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Category: Introductory Geometry Problems

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2009 AMC 12B Problems/Problem 14

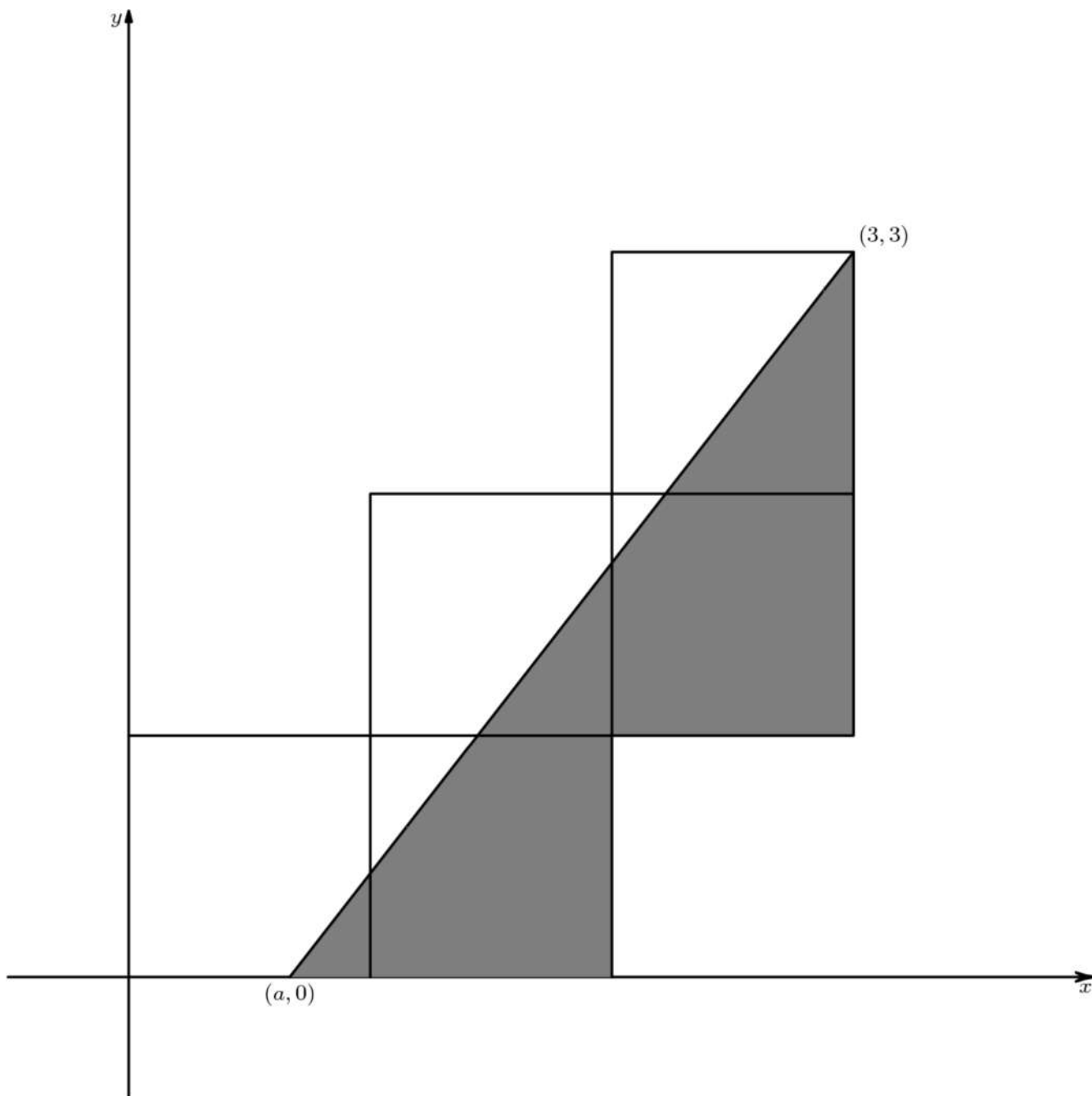
The following problem is from both the 2009 AMC 10B #17 and 2009 AMC 12B #14, so both problems redirect to this page.

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- 4 See Also

Problem

Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin. The slanted line, extending from $(a, 0)$ to $(3, 3)$, divides the entire region into two regions of equal area. What is a ?



(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

Solution

For $a \geq 1.5$ the shaded area is at most 1.5, which is too little. Hence $a < 1.5$, and therefore the point $(2, 1)$ is indeed inside the shaded part, as shown in the picture.

Then the area of the shaded part is one less than the area of the triangle with vertices $(a, 0)$, $(3, 0)$, and $(3, 3)$. Its area is obviously $\frac{3(3-a)}{2}$, therefore the area of the shaded part is $\frac{7-3a}{2}$.

The entire figure has area 5, hence we want the shaded part to have area $\frac{5}{2}$. Solving for a , we get $a = \boxed{\frac{2}{3}}$. The answer is (C).

Solution 2

The unit square is of area 1, so the five unit squares have area 5. Therefore the shaded space must occupy 2.5. The missing unit square is of area 1, and if reconstituted the original triangle would be of area 3.5. It can then be inferred: $(3-a) \cdot 3 = 7$.

$$3-a = \frac{7}{3}, \text{ so } 3 - \frac{7}{3} = a.$$

$$3 - \frac{7}{3} = \frac{9-7}{3} = \frac{2}{3}. \text{ (C).}$$

See Also

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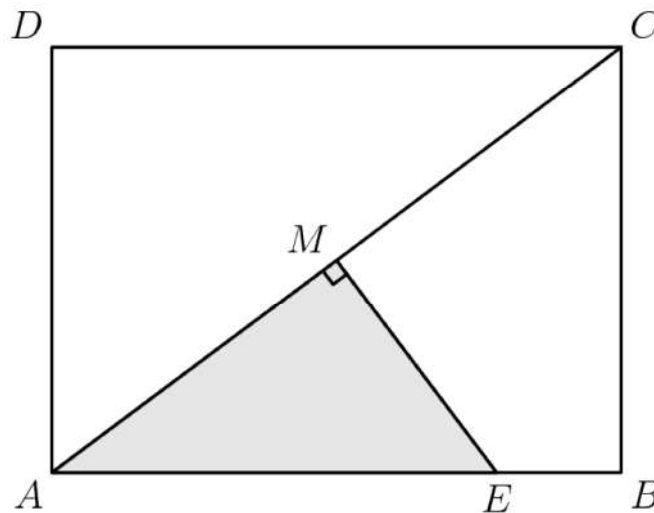
2009 AMC 10B Problems/Problem 18

Problem

Rectangle $ABCD$ has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal $\overline{AC}$, and E is on AB with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$?

- (A) $\frac{65}{8}$ (B) $\frac{25}{3}$ (C) 9 (D) $\frac{75}{8}$ (E) $\frac{85}{8}$

Solution



By the Pythagorean theorem we have $AC = 10$, hence $AM = 5$.

The triangles AME and ABC have the same angle at A and a right angle, thus all their angles are equal, and therefore these two triangles are similar.

The ratio of their sides is $\frac{AM}{AB} = \frac{5}{8}$, hence the ratio of their areas is $\left(\frac{5}{8}\right)^2 = \frac{25}{64}$.

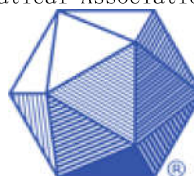
And as the area of triangle ABC is $\frac{6 \cdot 8}{2} = 24$, the area of triangle AME is $24 \cdot \frac{25}{64} = \boxed{\frac{75}{8}}$.

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2009 AMC 10B Problems/Problem 19

The following problem is from both the 2009 AMC 10B #19 and 2009 AMC 12B #10, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution
 - 2.1 Solution 1
 - 2.2 Solution 2
- 3 See also

Problem

A particular **12**-hour digital clock displays the hour and minute of a day. Unfortunately, whenever it is supposed to display a **1**, it mistakenly displays a **9**. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

- (A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$ (E) $\frac{9}{10}$

Solution

Solution 1

The clock will display the incorrect time for the entire hours of **1**, **10**, **11** and **12**. So the correct hour is displayed $\frac{2}{3}$ of the time. The minutes will not display correctly whenever either the tens digit or the ones digit is a **1**, so the minutes that will not display correctly are **10**, **11**, **12**, ..., **19** and **01**, **21**, **31**, **41**, and **51**. This amounts to fifteen of the sixty possible minutes for any given hour. Hence the fraction of the day that the clock shows the correct time is $\frac{2}{3} \cdot \left(1 - \frac{15}{60}\right) = \frac{2}{3} \cdot \frac{3}{4} = \boxed{\frac{1}{2}}$. The answer is (A).

Solution 2

The required fraction is the number of correct times divided by the total times. There are 60 minutes in an hour and 12 hours on a clock, so there are 720 total times.

We count the correct times directly; let a correct time be $\mathbf{x}:\mathbf{yz}$, where $\mathbf{x}$ is a number from 1 to 12 and $\mathbf{y}$ and $\mathbf{z}$ are digits, where $\mathbf{y} < 6$. There are 8 values of $\mathbf{x}$ that will display the correct time: 2, 3, 4, 5, 6, 7, 8, and 9. There are five values of $\mathbf{y}$ that will display the correct time: 0, 2, 3, 4, and 5. There are nine values of $\mathbf{z}$ that will display the correct time: 0, 2, 3, 4, 5, 6, 7, 8, and 9. Therefore there are $8 \cdot 5 \cdot 9 = 40 \cdot 9 = 360$ correct times.

Therefore the required fraction is $\frac{360}{720} = \frac{1}{2} \Rightarrow \boxed{(A)}$.

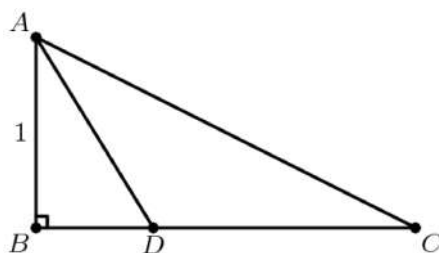
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2009 AMC 10B Problems/Problem 20

Problem

Triangle ABC has a right angle at B , $AB = 1$, and $BC = 2$. The bisector of $\angle BAC$ meets $\overline{BC}$ at D . What is BD ?



- (A) $\frac{\sqrt{3}-1}{2}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) $\frac{\sqrt{6}+\sqrt{2}}{2}$ (E) $2\sqrt{3}-1$

Solution

By the Pythagorean Theorem, $AC = \sqrt{5}$. Then, from the Angle Bisector Theorem, we have:

$$\frac{BD}{1} = \frac{2-BD}{\sqrt{5}}$$

$$BD \left(1 + \frac{1}{\sqrt{5}} \right) = \frac{2}{\sqrt{5}}$$

$$BD(\sqrt{5}+1) = 2$$

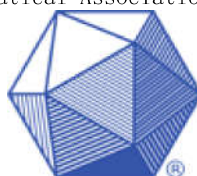
$$BD = \frac{2}{\sqrt{5}+1} = \boxed{\frac{\sqrt{5}-1}{2} \Rightarrow B}.$$

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2009 AMC 10B Problems/Problem 21

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Problem

What is the remainder when $3^0 + 3^1 + 3^2 + \cdots + 3^{2009}$ is divided by 8?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 6

Solution

Solution 1

The sum of any four consecutive powers of 3 is divisible by $3^0 + 3^1 + 3^2 + 3^3 = 40$ and hence is divisible by 8. Therefore

$$(3^2 + 3^3 + 3^4 + 3^5) + \cdots + (3^{2006} + 3^{2007} + 3^{2008} + 3^{2009})$$

is divisible by 8. So the required remainder is $3^0 + 3^1 = \boxed{4}$. The answer is (D).

Solution 2

We have $3^2 = 9 \equiv 1 \pmod{8}$. Hence for any k we have $3^{2k} \equiv 1^k = 1 \pmod{8}$, and then $3^{2k+1} = 3 \cdot 3^{2k} \equiv 3 \cdot 1 = 3 \pmod{8}$.

Therefore our sum gives the same remainder modulo 8 as $1 + 3 + 1 + 3 + 1 + \cdots + 1 + 3$. There are 2010 terms in the sum, hence there are $2010/2 = 1005$ pairs $1 + 3$, and thus the sum is $1005 \cdot 4 = 4020 \equiv 20 \equiv \boxed{4} \pmod{8}$.

See also

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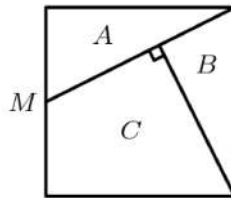
2009 AMC 10B Problems/Problem 22

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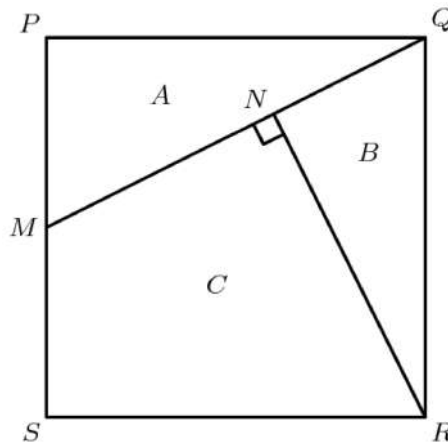
Problem

A cubical cake with edge length **2** inches is iced on the sides and the top. It is cut vertically into three pieces as shown in this top view, where **M** is the midpoint of a top edge. The piece whose top is triangle **B** contains **c** cubic inches of cake and **s** square inches of icing. What is **c + s**?



- (A) $\frac{24}{5}$ (B) $\frac{32}{5}$ (C) $8 + \sqrt{5}$ (D) $5 + \frac{16\sqrt{5}}{5}$ (E) $10 + 5\sqrt{5}$

Solution



Let's label the points as in the picture above. Let $[RNQ]$ be the area of $\triangle RNQ$. Then the volume of the corresponding piece is $c = 2[RNQ]$. This cake piece has icing on the top and on the vertical side that contains the edge QR . Hence the total area with icing is $[RNQ] + 2^2 = [RNQ] + 4$. Thus the answer to our problem is $3[RNQ] + 4$, and all we have to do now is to determine $[RNQ]$.

Solution 1

Introduce a coordinate system where $Q = (0, 0)$, $P = (2, 0)$ and $R = (0, 2)$.

In this coordinate system we have $M = (2, 1)$, and the line QM has the equation $2y - x = 0$.

As the line RN is orthogonal to QM , it must have the equation $y + 2x + q = 0$ for some suitable constant q . As this line contains the point $R = (0, 2)$, we have $q = -2$.

Substituting $x = 2y$ into $y + 2x - 2 = 0$, we get $y = \frac{2}{5}$, and then $x = \frac{4}{5}$.

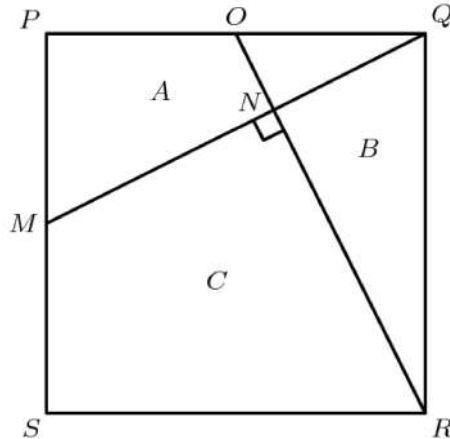
We can note that in $\triangle RNQ$ x is the height from N onto RQ , hence its area is

$$[RNQ] = \frac{x \cdot RQ}{2} = \frac{2x}{2} = x = \frac{4}{5}, \text{ and therefore the answer is}$$

$$3[RNQ] + 4 = 3 \cdot \frac{4}{5} + 4 = \boxed{\frac{32}{5} \Rightarrow B}.$$

Solution 2

Extend RN to intersect PQ at O :



It is now obvious that O is the midpoint of PQ . (Imagine rotating the square $PQRS$ by 90° clockwise around its center. This rotation will map the segment MN to a segment that is orthogonal to MN , contains R and contains the midpoint of PQ .)

From $\triangle PQM$ we can compute that $QM = \sqrt{1^2 + 2^2} = \sqrt{5}$.

Observe that $\triangle PQM$ and $\triangle NQO$ have the same angles and therefore they are similar. The ratio of their sides is $\frac{QM}{OQ} = \frac{\sqrt{5}}{1} = \sqrt{5}$.

Hence we have $ON = \frac{PM}{\sqrt{5}} = \frac{1}{\sqrt{5}}$, and $NQ = \frac{PQ}{\sqrt{5}} = \frac{2}{\sqrt{5}}$.

Knowing this, we can compute the area of $\triangle NQO$ as $[NQO] = \frac{ON \cdot NQ}{2} = \frac{1}{5}$.

Finally, we compute $[RNQ] = [ROQ] - [NQO] = 1 - \frac{1}{5} = \frac{4}{5}$, and conclude that the answer is

$$3[RNQ] + 4 = 3 \cdot \frac{4}{5} + 4 = \boxed{\frac{32}{5}}.$$

- You could also notice that the two triangles in the original figure are similar.

Solution 3

Use trigonometry.

The length of PM and PQ is 1 and 2 respectively. So $\angle PQM = \arctan \frac{1}{2}$, and $\angle NQR = 90 - \angle PQM = 90 - \arctan \frac{1}{2}$.

From the right-angled triangle $\triangle NQR$, the hypotenuse, $QR = 2$ So $NR = 2 \sin(90 - \arctan \frac{1}{2})$, and $NQ = 2 \cos(90 - \arctan \frac{1}{2})$

Knowing this, $[RNQ] = \frac{1}{2} \cdot NR \cdot NQ$. So we proceed as follows:

$$[RNQ] = \frac{1}{2} \cdot NR \cdot NQ$$

$$[RNQ] = \frac{1}{2} \cdot 2 \sin(90 - \arctan \frac{1}{2}) \cdot 2 \cos(90 - \arctan \frac{1}{2})$$

$$[RNQ] = 2 \sin(90 - \arctan \frac{1}{2}) \cos(90 - \arctan \frac{1}{2})$$

$$[RNQ] = \sin[2(90 - \arctan \frac{1}{2})] = \sin(2 \arctan \frac{1}{2})$$

$$[RNQ] = \frac{2 \cdot \frac{1}{2}}{1 + (\frac{1}{2})^2}$$

$$[RNQ] = \frac{4}{5}$$

So the answer is $3[RNQ] + 4 = 3 \cdot \frac{4}{5} + 4 = \boxed{\frac{32}{5}}$.

Note that we didn't use a calculator, but we used trigonometric identities

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2009 AMC 10B Problems/Problem 23

The following problem is from both the 2009 AMC 10B #23 and 2009 AMC 12B #18, so both problems redirect to this page.

Problem

Rachel and Robert run on a circular track. Rachel runs counterclockwise and completes a lap every 90 seconds, and Robert runs clockwise and completes a lap every 80 seconds. Both start from the same line at the same time. At some random time between 10 minutes and 11 minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Rachel and Robert are in the picture?

- (A) $\frac{1}{16}$
- (B) $\frac{1}{8}$
- (C) $\frac{3}{16}$
- (D) $\frac{1}{4}$
- (E) $\frac{5}{16}$

Solution

After 10 minutes (600 seconds), Rachel will have completed 6 laps and be 30 seconds from completing her seventh lap. Because Rachel runs one-fourth of a lap in 22.5 seconds, she will be in the picture between 18.75 seconds and 41.25 seconds of the tenth minute. After 10 minutes Robert will have completed 7 laps and will be 40 seconds past the starting line. Because Robert runs one-fourth of a lap in 20 seconds, he will be in the picture between 30 and 50 seconds of the tenth minute. Hence both Rachel and Robert will be in the picture if it is taken between 30 and 41.25 seconds of the tenth minute. So the probability that both runners are in the picture is

$$\frac{41.25 - 30}{60} = \boxed{\frac{3}{16}}.$$

The answer is (C).

See also

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Category: Introductory Combinatorics Problems

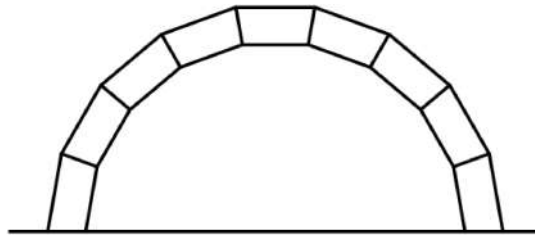
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Problem

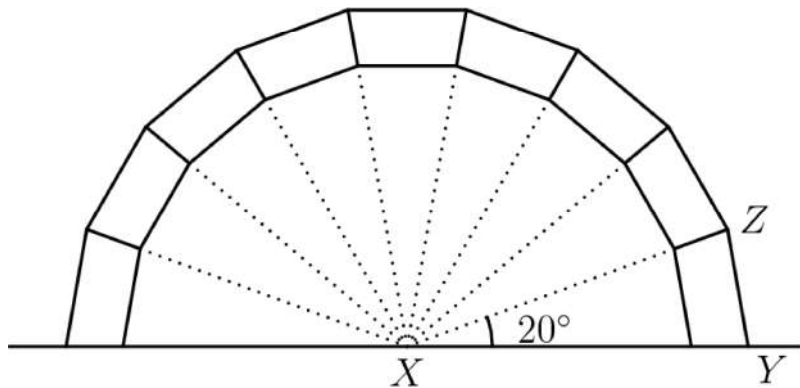
The keystone arch is an ancient architectural feature. It is composed of congruent isosceles trapezoids fitted together along the non-parallel sides, as shown. The bottom sides of the two end trapezoids are horizontal. In an arch made with **9** trapezoids, let ***x*** be the angle measure in degrees of the larger interior angle of the trapezoid. What is ***x***?



- (A) 100 (B) 102 (C) 104 (D) 106 (E) 108

Solution 1

Extend all the legs of the trapezoids. They will all intersect in the middle of the bottom side of the picture, forming the situation shown below.



Each of the angles at X has $\frac{180^\circ}{9} = 20^\circ$. From $\triangle XYZ$, the size of the smaller internal angle of the trapezoid is $\frac{180^\circ - 20^\circ}{2} = 80^\circ$, hence the size of the larger one is $180^\circ - 80^\circ = \boxed{100^\circ \Rightarrow A}$.

Proof that all the extended trapezoid legs intersect in the same point: It is sufficient to prove this for any pair of neighboring trapezoids. For two neighboring trapezoids, the situation is symmetric according to their common leg, therefore the extensions of both outside legs intersect the extension of the common leg in the same point, Q. E. D.

Knowing this, we can now easily see that the intersection point must be on the bottom side of our picture, as it lies on the bottom leg of the rightmost trapezoid. And by symmetry the point must be in the center of this side.

Solution 2

2016/11/19

Art of Problem Solving

A decagon can be formed from the trapezoids and the base. The sum of the decagon's angles is $180(10 - 2) = 1440^\circ$. Letting the larger angle in each trapezoid be x , the two angles formed by the line each measures $(180 - x)^\circ$. There are 8 congruent angles left. Each of those angles measures $(360 - 2x)^\circ$. Putting it all together: $8(360 - 2x) + 2(180 - x) = 1440 \implies x = \boxed{(A) 100^\circ}$.

Solution 3

If we reflect the arch across the line, we form an 18-gon. $\frac{180 * (18 - 2)}{18} = 160^\circ$ so each interior angle of the decagon is 160° . From the diagram, we see that $2x + 160 = 360$, so $2x = 200$ and $x = 100$, or $\boxed{A}$.

See Also

2009 AMC 10B (Problems • Answer Key • Resources) (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009)	
Preceded by Problem 23	Followed by Problem 25
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All AMC 10 Problems and Solutions	

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Category: Introductory Geometry Problems

2009 AMC 10B Problems/Problem 25

The following problem is from both the 2009 AMC 10B #25 and 2009 AMC 12B #17, so both problems redirect to this page.

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- 2 Solution
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Problem

Each face of a cube is given a single narrow stripe painted from the center of one edge to the center of the opposite edge. The choice of the edge pairing is made at random and independently for each face. What is the probability that there is a continuous stripe encircling the cube?

- (A) $\frac{1}{8}$ (B) $\frac{3}{16}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$ (E) $\frac{1}{2}$

Solution

Solution 1

There are two possible stripe orientations for each of the six faces of the cube, so there are $2^6 = 64$ possible stripe combinations. There are three pairs of parallel faces so, if there is an encircling stripe, then the pair of faces that do not contribute uniquely determine the stripe orientation for the remaining faces. In addition, the stripe on each face that does not contribute may be oriented in either of two different ways. So a total of $3 \cdot 2 \cdot 2 = 12$ stripe combinations on the cube result in a continuous stripe around the cube. The required

probability is $\frac{12}{64} = \boxed{\frac{3}{16}}$.

Solution 2

Without loss of generality, orient the cube so that the stripe on the top face goes from front to back. There are two mutually exclusive ways for there to be an encircling stripe: either the front, bottom and back faces are painted to complete an encircling stripe with the top face's stripe or the front, right, back and left faces are

painted to form an encircling stripe. The probability of the first case is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, and the probability of

the second case is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$. The cases are disjoint, so the probabilities sum $\frac{1}{8} + \frac{1}{16} = \boxed{\frac{3}{16}}$.

Solution 3

There are three possible orientations of an encircling stripe. For any one of these to appear, the stripes on the four faces through which the continuous stripe is to pass must be properly aligned. The probability of each such

stripe alignment is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$. Since there are three such possibilities and they are disjoint, the total

probability is $3 \cdot \frac{1}{16} = \boxed{\frac{3}{16}}$. The answer is (B).

Solution 4

Consider a vertex on the cube and the three faces that are adjacent to that vertex. If no two stripes on those three faces are aligned, then there is no stripe encircling the cube. The probability that the stripes aren't aligned is $\frac{1}{4}$, since for each alignment of one stripe, there is one and only one way to align the other two stripes out of four total possibilities. therefore there is a probability of $\frac{3}{4}$ that two stripes are aligned.

Now consider the opposing vertex and the three sides adjacent to it. Given the two connected stripes next to our first vertex, we have two more that must be connected to make a continuous stripe. There is a probability of $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ that they are aligned, so there is a probability of $\frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$ that there is a continuous stripe.

See also

2009 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
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2009 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2009))	
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