2010 AMC 10B

Problem 1

What is $100(100-3)-(100\cdot 100-3)$?

$100(100-3)-(100\cdot 100-3)$的值是多少？

$\textbf{(A)}\ -20,000 \qquad \textbf{(B)}\ -10,000 \qquad \textbf{(C)}\ -297 \qquad \textbf{(D)}\ -6 \qquad \textbf{(E)}\ 0$

Problem 2

Markala attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

Markala 在她工作的 9 小时里参加了 2 次会议。第一场会议耗时 45 分钟，第二场会议时长是第一场的 2 倍。问她工作总时间的百分之多少是花在了会议上？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$

Problem 3

A drawer contains red, green, blue, and white socks with at least 2 of each color. What is the minimum number of socks that must be pulled from the drawer to guarantee a matching pair?

一张抽屉里有红色，绿色，蓝色和白色的袜子，每种颜色至少 2 只。问从抽屉里至少需要抽出多少只袜子才能保证存在一双颜色匹配的袜子？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 4

For a real number x, define $\heartsuit(x)$ to be the average of x and x^2. What is $\heartsuit(1)+\heartsuit(2)+\heartsuit(3)$?

对于实数 x，定义$\heartsuit(x)$为x和x^2的平均值，$\heartsuit(1)+\heartsuit(2)+\heartsuit(3)$的值是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 20$

Problem 5

A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

某个月共有 31 天，其中周一的天数和周三的天数相同。一周 7 天有多少天可能成为这个月的第一天？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 6

A circle is centered at O, $\overline{AB}$ is a diameter and C is a point on the circle with $\angle COB = 50^\circ$. What is the degree measure of $\angle CAB$?

一个圆的圆心为 O，$\overline{AB}$为直径，C 是圆上一点满足$\angle COB = 50^\circ$。$\angle CAB$的度数是多少？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 65$

Problem 7

A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?

一个三角形的三条边长为 10，10 和 12。一个矩形的宽为 4，面积等于此三角形的面积。那么矩形的周长是多少？

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 36$

Problem 8

A ticket to a school play cost x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of $\$48$, and a group of 10th graders buys tickets costing a total of $\$64$. How many values for x are possible?

一张学校戏剧的票要花 x 美元，其中 x 是个整数。一组九年级学生买的票总共花了 48 美元，一组十年级学生买的票总共花了 64 美元。问 x 有多少个可能的值？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 9

Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?

幸运的 Larry 的老师叫他分别将值代入 a，b，c，d，和 e 以求得表达式$a-(b-(c-(d+e)))$的值。Larry 忽略了括号，但加减运算是正确的，结果碰巧得到了正确答案。已知 Larry 分别用 1，2，3，4 代替 a，b，c，d。则 Larry 用什么数代替了 e?

$\textbf{(A)}\ -5 \qquad \textbf{(B)}\ -3 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 5$

Problem 10

Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

Shelby 在不下雨的情况下以每小时 30 英里的速度驾驶她的滑板车，在下雨的情况下以每小时 20 英里的速度驾驶。今天，她早上驾驶时阳光明媚，晚上驾驶时飘着雨，结果总共用了 40 分钟驾驶了 16 英里。问她在雨中驾驶了多少分钟？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 30$

Problem 11

A shopper plans to purchase an item that has a listed price greater than $\$100$ and can use any one of the three coupons. Coupon A gives 15% off the listed price, Coupon B gives $\$30$ off the listed price, and Coupon C gives 25% off the amount by which the listed price exceeds $\$100$.
Let x and y be the smallest and largest prices, respectively, for which Coupon A saves at least as many dollars as Coupon B or Coupon C. What is $y-x$?

一个顾客计划买一件标价高于 100 美元的商品，并且可以使用下面 3 张优惠券中的任何一张。优惠券 A 将标价降价 15%，优惠券 B 将标价降低 30 美元，优惠券 C 可以将标价超出 100 美元的部分降低 25%。

令 x 和 y 分别表示使用优惠券 A 节省的钱至少和使用优惠券 B 或 C 一样多时，标价的最低价和最高价。那么$y-x$等于多少？

$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 75 \qquad \textbf{(D)}\ 80 \qquad \textbf{(E)}\ 100$

Problem 12

At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

在某学年的开始，Well 先生数学课上的 50%的学生对于“你喜欢数学吗”这个问题的回答为“是”，剩下 50%的学生回答“否”。这学年结束时，70%的学生回答“是”，剩下 30%的学生回答“否”。总的来说，x%的学生在学年开始和学年结束时给出了不同的回答。那么 x 的最大可能值和最小可能值之差是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 80$

Problem 13

What is the sum of all the solutions of $x = |2x - |60 - 2x||$?

$x = |2x - |60 - 2x||$的所有解之和是多少？

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 92 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 124$

Problem 14

The average of the numbers $1, 2, 3, \cdots, 98, 99,$ and x is $100x$. What is x?

数字 1，2，3，…，98，99，和 x 的平均值为 100x。则 x 是多少？

$\textbf{(A)}\ \frac{49}{101} \qquad \textbf{(B)}\ \frac{50}{101} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{51}{101} \qquad \textbf{(E)}\ \frac{50}{99}$

Problem 15

On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

在一次有 50 道选择题的数学竞赛中，学生如果答对一题得 4 分，空着不作答得 0 分，答错得 -1 分。Jesse 在这次竞赛的总分为 99 分。问 Jesse 最多可能答对多少题？

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 29 \qquad \textbf{(D)}\ 31 \qquad \textbf{(E)}\ 33$

Problem 16

A square of side length 1 and a circle of radius $\dfrac{\sqrt{3}}{3}$ share the same center. What is the area inside the circle, but outside the square?

一个边长为 1 的正方形和半径为 $\dfrac{\sqrt{3}}{3}$ 的圆，它们的中心重合。求位于圆内而在正方形之外的区域的面积是多少

$\textbf{(A)}\ \dfrac{\pi}{3}-1 \qquad \textbf{(B)}\ \dfrac{2\pi}{9}-\dfrac{\sqrt{3}}{3} \qquad \textbf{(C)}\ \dfrac{\pi}{18} \qquad \textbf{(D)}\ \dfrac{1}{4} \qquad \textbf{(E)}\ \dfrac{2\pi}{9}$

Problem 17

Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

欧几里得市的每一所高中都派出一支 3 名学生组成的队伍去参加某个数学竞赛。每个参赛选手都得到了不同的分数。Andrea 的分数在所有这些学生中是个中位数，并且她的分数是她所在的队的最高分。Andrea 的队友 Beth 和 Carla 分别排在第 37 名和第 64 名。问这个城市总共有多少所学校?

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 23 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 26$

Problem 18

Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

正整数 a，b 和 c 独立、随机且有放回的从集合$\{1, 2, 3, \ldots, 2010\}$中抽取。$abc + ab + a$能被 3 整除的概率是多少？

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{29}{81} \qquad \textbf{(C)}\ \frac{31}{81} \qquad \textbf{(D)}\ \frac{11}{27} \qquad \textbf{(E)}\ \frac{13}{27}$

Problem 19

A circle with center O has area 156π. Triangle ABC is equilateral, $\overline{BC}$ is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?

一个圆心为 O 的圆面积是156π。三角形 ABC 为等边三角形，$\overline{BC}$是圆的一条弦，$OA = 4\sqrt{3}$，且点O位于$\triangle ABC$之外。求$\triangle ABC$的边长是多少？

$\textbf{(A)}\ 2\sqrt{3} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 4\sqrt{3} \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 18$

Problem 20

Two circles lie outside regular hexagon $ABCDEF$. The first is tangent to $\overline{AB}$, and the second is tangent to $\overline{DE}$. Both are tangent to lines BC and FA. What is the ratio of the area of the second circle to that of the first circle?

2 个圆位于正六边形 $ABCDEF$ 之外。第一个圆和$\overline{AB}$相切，第二个圆和$\overline{DE}$相切。两个圆均与直线 BC 和 FA 相切。问第二个圆的面积和第一个圆的面积的比值是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 81 \qquad \textbf{(E)}\ 108$

Problem 21

A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

从所有位于 1000 到 10，000 之间的回环数中随机抽取 1 个，问这个回环数能被 7 整除的概率是多少？

$\textbf{(A)}\ \frac{1}{10} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{1}{7} \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{1}{5}$

Problem 22

Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

把 7 块不同的糖果分配到 3 个包中。要求红包和蓝包必须各分得至少一块糖果；白包可以为空。一共有多少种可能的分配方法？

$\textbf{(A)}\ 1930 \qquad \textbf{(B)}\ 1931 \qquad \textbf{(C)}\ 1932 \qquad \textbf{(D)}\ 1933 \qquad \textbf{(E)}\ 1934$

Problem 23

The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

一个 3 x 3 的阵列包含了 1 到 9 的所有数字，满足每行每列的数字都以升序排列。一共有多少个这样的阵列？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 60$

Problem 24

A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

Raiders 队和 Wildcats 队之间的一场高中篮球赛在第一节打成平局。已知 Raiders 队的 4 节比赛所得的 4 个分数成递增等比数列，而 Wildcats 队的 4 节比赛所得的 4 个分数成递增的等差数列。在第 4 节结束后，Raiders 队的总分比 Wildcats 的总分高 1 分。2 个队伍的总分均不超过 100 分。问这 2 支队伍在前 2 节比赛里的总得分是多少分？

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

Problem 25

Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$

$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

令 $a>0$，且多项式 P（x）的各项系数均为整数，满足

$$P(1) = P(3) = P(5) = P(7) = a,$$

$$P(2) = P(4) = P(6) = P(8) = -a$$

a 的最小可能值是多少？

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

2010 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	C	C	C	B	B	D	E	D	C	A	D	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	C	B	B	E	B	D	E	C	D	E	B	