2010 AMC 10A Problems/Problem 1

Problem 1

Mary's top book shelf holds five books with the following widths, in centimeters: 6, $\frac{1}{2}$, 1, 2.5, and 10. What is the average book width, in centimeters?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 3 \qquad \mathrm{(D)}\ 4 \qquad \mathrm{(E)}\ 5$

Solution

To find the average, we add up the widths 6, $\frac{1}{2}$, 1, 2.5, and 10, to get a total sum of 20. Since there are 5 books, the average book width is $\frac{20}{5}=4$ The answer is $\boxed{D}$.

See Also

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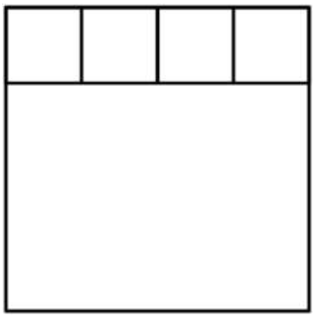
2010 AMC 10A Problems/Problem 2

Problem 2

Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?



$\mathrm{(A)}\ \frac{5}{4} \qquad \mathrm{(B)}\ \frac{4}{3} \qquad \mathrm{(C)}\ \frac{3}{2} \qquad \mathrm{(D)}\ 2 \qquad \mathrm{(E)}\ 3$

Solution

Let the length of the small square be x, intuitively, the length of the big square is $4x$. It can be seen that the width of the rectangle is $3x$. Thus, the length of the rectangle is $4x/3x = 4/3$ times large as the width. The answer is $\boxed{B}$.

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 3

Problem 3

Tyrone had 97 marbles and Eric had 11 marbles. Tyrone then gave some of his marbles to Eric so that Tyrone ended with twice as many marbles as Eric. How many marbles did Tyrone give to Eric?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 13 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 29$

Solution

Let x be the number of marbles Tyrone gave to Eric. Then, $97-x=2\cdot(11+x)$. Solving for x yields $75=3x$ and $x=25$. The answer is $\boxed{D}$.

See Also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 4

Problem 4

A book that is to be recorded onto compact discs takes 412 minutes to read aloud. Each disc can hold up to 56 minutes of reading. Assume that the smallest possible number of discs is used and that each disc contains the same length of reading. How many minutes of reading will each disc contain?

$\mathrm{(A)}\ 50.2 \qquad \mathrm{(B)}\ 51.5 \qquad \mathrm{(C)}\ 52.4 \qquad \mathrm{(D)}\ 53.8 \qquad \mathrm{(E)}\ 55.2$

Solution

Assuming that there were fractions of compact discs, it would take $412/56 \;= 7.357$ CDs to have equal reading time. However, since the number of discs can only be a whole number, there are at least 8 CDs, in which case it would have $412/8 = 51.5$ minutes on each of the 8 discs. The answer is $\boxed{B}$.

See Also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 5

Problem 5

The area of a circle whose circumference is 24π is $k\pi$. What is the value of k?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 144$

Solution

If the circumference of a circle is 24π, the radius would be 12. Since the area of a circle is πr^2, the area is 144π. The answer is $\boxed{E}$.

See Also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 6

Problem 6

For positive numbers x and y the operation $\spadesuit(x,y)$ is defined as

$$\spadesuit(x,y) = x - \frac{1}{y}$$

What is $\spadesuit(2,\spadesuit(2,2))$?

$\mathrm{(A)}\ \frac{2}{3} \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ \frac{4}{3} \qquad \mathrm{(D)}\ \frac{5}{3} \qquad \mathrm{(E)}\ 2$

Solution

$\spadesuit(2,2) = 2 - \frac{1}{2} = \frac{3}{2}$. Then, $\spadesuit(2,\frac{3}{2})$ is $2 - \frac{1}{\frac{3}{2}} = 2 - \frac{2}{3} = \frac{4}{3}$ The answer is $\boxed{C}$

See Also

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2010 AMC 10A Problems/Problem 7

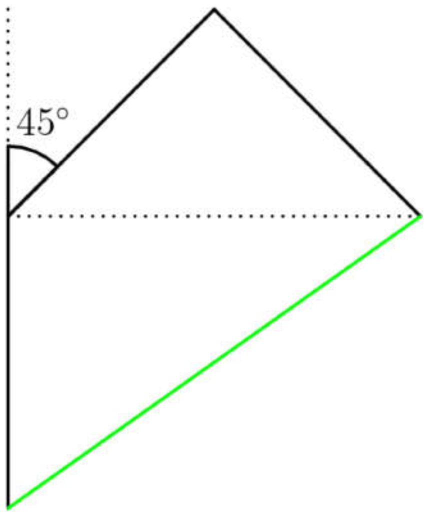
Problem 7

Crystal has a running course marked out for her daily run. She starts this run by heading due north for one mile. She then runs northeast for one mile, then southeast for one mile. The last portion of her run takes her on a straight line back to where she started. How far, in miles is this last portion of her run?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ \sqrt{2} \qquad \mathrm{(C)}\ \sqrt{3} \qquad \mathrm{(D)}\ 2 \qquad \mathrm{(E)}\ 2\sqrt{2}$

Solution

Crystal first runs North for one mile. Changing directions, she runs Northeast for another mile. The angle difference between North and Northeast is 45 degrees. She then switches directions to Southeast, meaning a 90 degree angle change. The distance now from travelling North for one mile, and her current destination is $\sqrt{2}$ miles, because it is the hypotenuse of a 45-45-90 triangle with side length one (mile). Therefore, Crystal's distance from her starting position, x, is equal to $\sqrt{((\sqrt{2})^2+1^2)}$, which is equal to $\sqrt{3}$. The answer is $\boxed{C}$



See Also

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2010 AMC 10A Problems/Problem 8

Contents

Problem 8

Tony works 2 hours a day and is paid $\$0.50$ per hour for each full year of his age. During a six month period Tony worked 50 days and earned $\$630$. How old was Tony at the end of the six month period?

$\mathrm{(A)}\ 9 \qquad \mathrm{(B)}\ 11 \qquad \mathrm{(C)}\ 12 \qquad \mathrm{(D)}\ 13 \qquad \mathrm{(E)}\ 14$

Solution

Solution 1

Tony worked 2 hours a day and is paid 0.50 dollars per hour for each full year of his age. This basically says that he gets a dollar for each year of his age. So if he is 12 years old, he gets 12 dollars a day. We also know that he worked 50 days and earned 630 dollars. If he was 12 years old at the beginning of his working period, he would have earned $12*50=600$ dollars. If he was 13 years old at the beginning of his working period, he would have earned $13*50=650$ dollars. Because he earned 630 dollars, we know that he was 13 for some period of time, but not the whole time, because then the money earned would be greater than or equal to 650. This is why he was 12 when he began, but turned 13 sometime in the middle and earned 630 dollars in total. So the answer is 13. The answer is $\boxed{D}$. We could find out for how long he was 12 and 13. $12\cdot x+13\cdot (50-x)=630$. Then x is 20 and we know that he was 12 for 20 days, and 13 for 30 days. Thus, the answer is 13.

Solution 2

Let x equal Tony's age at the end of the period. We know that his age changed during the time period (since 630 does not evenly divide 50). Thus, his age at the beginning of the time period is $x-1$.

Let d be the number of days Tony worked while his age was x. We know that his earnings every day equal his age (since $2\cdot 0.50=1$). Thus,

$$x\cdot d+(x-1)(50-d)=630$$

$$x\cdot d+50x-x\cdot d-50+d=630$$

$$50x+d=680$$

$$x=\frac{680-d}{50}$$

Since $0<d<50$, $d=30$. Then we know that $50x=650$ and $x=\boxed{(D) 13}$

See Also

2010 AMC 10A Problems/Problem 9

Problem

A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x+32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 24$

Solution

x is at most 999, so $x+32$ is at most 1031. The minimum value of $x+32$ is 1000. However, the only palindrome between 1000 and 1032 is 1001, which means that $x+32$ must be 1001.

It follows that x is 969, so the sum of the digits is $\boxed{\textbf{(E)}\ 24}$.

See also

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Category: Introductory Algebra Problems

2010 AMC 10A Problems/Problem 10

Problem 10

Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?

$\mathrm{(A)}\ 2011 \qquad \mathrm{(B)}\ 2012 \qquad \mathrm{(C)}\ 2013 \qquad \mathrm{(D)}\ 2015 \qquad \mathrm{(E)}\ 2017$

Solution

$\boxed{(E)}\ 2017$

There are 365 days in a non-leap year. There are 7 days in a week. Since $365 = 52 \cdot 7 + 1$ (or 365 is congruent to $1 \mod 7$), the same date (after February) moves "forward" one day in the subsequent year, if that year is not a leap year.

For example:

$5/27/08$ Tue

$5/27/09$ Wed

However, a leap year has 366 days, and $366 = 52 \cdot 7 + 2$. So the same date (after February) moves "forward" two days in the subsequent year, if that year is a leap year.

For example: $5/27/11$ Fri

$5/27/12$ Sun

You can keep count forward to find that the first time this date falls on a Saturday is in 2017:

$5/27/13$ Mon

$5/27/14$ Tue

$5/27/15$ Wed

$5/27/16$ Fri

$5/27/17$ Sat

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 11

Problem 11

The length of the interval of solutions of the inequality $a \le 2x+3 \le b$ is 10. What is $b-a$?

$\mathrm{(A)}\ 6 \qquad \mathrm{(B)}\ 10 \qquad \mathrm{(C)}\ 15 \qquad \mathrm{(D)}\ 20 \qquad \mathrm{(E)}\ 30$

Solution

Since we are given the range of the solutions, we must re-write the inequalities so that we have x in terms of a and b.

$$a \le 2x+3 \le b$$

Subtract 3 from all of the quantities:

$$a-3 \le 2x \le b-3$$

Divide all of the quantities by 2.

$$\frac{a-3}{2} \le x \le \frac{b-3}{2}$$

Since we have the range of the solutions, we can make them equal to 10.

$$\frac{b-3}{2} - \frac{a-3}{2} = 10$$

Multiply both sides by 2.

$$(b-3)-(a-3) = 20$$

Re-write without using parentheses.

$$b-3-a+3 = 20$$

Simplify.

$$b-a = 20$$

We need to find $b-a$ for the problem, so the answer is $\boxed{20\ \mathrm{(D)}}$

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 12A Problems/Problem 7

Problem

Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

$\textbf{(A)}\ 0.04 \qquad \textbf{(B)}\ \frac{0.4}{\pi} \qquad \textbf{(C)}\ 0.4 \qquad \textbf{(D)}\ \frac{4}{\pi} \qquad \textbf{(E)}\ 4$

Solution

The water tower holds $\frac{100000}{0.1} = 1000000$ times more water than Logan's miniature. Therefore, Logan should make his tower $\sqrt[3]{1000000} = 100$ times shorter than the actual tower. This is $\frac{40}{100} = \boxed{0.4}$ meters high, or choice $\textbf{(C)}$.

Also, the fact that $1 \text{ L} = 1 \text{ cm}^3$ doesn't matter since only the ratios are important.

See also

2010 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2010))	
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Categories: Introductory Geometry Problems | 3D Geometry Problems

2010 AMC 10A Problems/Problem 13

Problem

Angelina drove at an average rate of 80 kph and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 kph. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?

$\textbf{(A)}\ 80t + 100(\frac{8}{3} - t) = 250 \qquad \textbf{(B)}\ 80t = 250 \qquad \textbf{(C)}\ 100t = 250 \qquad \textbf{(D)}\ 90t = 250 \qquad \textbf{(E)}\ 80(\frac{8}{3} - t) + 100t = 250$

Solution

The answer is A because she drove at 80 kmh for t hours (the amount of time before the stop), and 100 kmh for $\frac{8}{3} - t$ because she wasn't driving for 20 minutes, or $\frac{1}{3}$ hours. Multiplying by t gives the total distance, which is 250 kms. Therefore, the answer is $80t + 100(\frac{8}{3} - t) = 250$ $\Rightarrow \boxed{(A)}$

See Also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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2010 AMC 10A Problems/Problem 14

Problem

Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

$\textbf{(A)}\ 60^\circ \qquad \textbf{(B)}\ 75^\circ \qquad \textbf{(C)}\ 90^\circ \qquad \textbf{(D)}\ 105^\circ \qquad \textbf{(E)}\ 120^\circ$

Solution

Let $\angle BAE = \angle ACD = x$.

$$\begin{aligned} \angle BCD &= \angle AEC = 60^\circ \\ \angle EAC + \angle FCA + \angle ECF + \angle AEC &= \angle EAC + x + 60^\circ + 60^\circ = 180^\circ \\ \angle EAC &= 60^\circ - x \\ \angle BAC &= \angle EAC + \angle BAE = 60^\circ - x + x = 60^\circ \end{aligned}$$

Since $\frac{AC}{AB} = \frac{1}{2}$, $\angle BCA = \boxed{90^\circ\ \textbf{(C)}}$

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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Category: Introductory Geometry Problems

2010 AMC 10A Problems/Problem 15

Contents

Problem

In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution 1

We can begin by first looking at Chris and LeRoy.

Suppose Chris and LeRoy are the same species. If Chris is a toad, then what he says is true, so LeRoy is a frog. However, if LeRoy is a frog, then he is lying, but clearly Chris is not a frog, and we have a contradiction. The same applies if Chris is a frog.

Clearly, Chris and LeRoy are different species, and so we have at least 1 frog out of the two of them.

Now suppose Mike is a toad. Then what he says is true because we already have 2 toads. However, if Brian is a frog, then he is lying, yet his statement is true, a contradiction. If Brian is a toad, then what he says is true, but once again it conflicts with his statement, resulting in contradiction.

Therefore, Mike must be a frog. His statement must be false, which means that there is at most 1 toad. Since either Chris or LeRoy is already a toad, Brian must be a frog. We can also verify that his statement is indeed false.

Both Mike and Brian are frogs, and one of either Chris or LeRoy is a frog, so we have 3 frogs total. $\boxed{\textbf{(D)}}$

Solution 2

Start with Brian. If he is a toad, he tells the truth, hence Mike is a frog. If Brian is a frog, he lies, hence Mike is a frog, too. Thus Mike must be a frog.

As Mike is a frog, his statement is false, hence there is at most one toad.

As there is at most one toad, at least one of Chris and LeRoy is a frog. But then the other one tells the truth, and therefore is a toad.

Hence we must have one toad and three frogs. $\boxed{\textbf{(D)}}$

See also

2010 AMC 10A Problems/Problem 16

Problem

Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 33 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 37$

Solution

By the Angle Bisector Theorem, we know that $\frac{AB}{3} = \frac{BC}{8}$. If we use the lowest possible integer values for AB and BC (the measures of AD and DC, respectively), then $AB + BC = AD + DC = AC$, contradicting the Triangle Inequality. If we use the next lowest values ($AB = 6$ and $BC = 16$), the Triangle Inequality is satisfied. Therefore, our answer is $6 + 16 + 3 + 8 = \boxed{33}$, or choice $\textbf{(B)}$.

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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Category: Introductory Geometry Problems

2010 AMC 10A Problems/Problem 17

Contents

Problem

A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Solutions

Solution 1

Imagine making the cuts one at a time. The first cut removes a box $2\times 2\times 3$. The second cut removes two boxes, each of dimensions $2\times 2\times 0.5$, and the third cut does the same as the second cut, on the last two faces. Hence the total volume of all cuts is $12 + 4 + 4 = 20$.

Therefore the volume of the rest of the cube is $3^3 - 20 = 27 - 20 = \boxed{7\ \textbf{(A)}}$.

Solution 2

We can use Principle of Inclusion-Exclusion to find the final volume of the cube.

There are 3 "cuts" through the cube that go from one end to the other. Each of these "cuts" has $2 \times 2 \times 3=12$ cubic inches. However, we can not just sum their volumes, as the central $2\times 2\times 2$ cube is included in each of these three cuts. To get the correct result, we can take the sum of the volumes of the three cuts, and subtract the volume of the central cube twice.

Hence the total volume of the cuts is $3(2\times 2\times 3) - 2(2\times2\times2) = 36 - 16 = 20$.

Therefore the volume of the rest of the cube is $3^3 - 20 = 27 - 20 = \boxed{7\ \textbf{(A)}}$.

Solution 3

We can visualize the final figure and see a cubic frame. We can find the volume of the figure by adding up the volumes of the edges and corners.

Each edge can be seen as a $2\times 0.5\times 0.5$ box, and each corner can be seen as a $0.5\times 0.5\times 0.5$ box.

$$12 \cdot \frac{1}{2} + 8 \cdot \frac{1}{8} = 6 + 1 = \boxed{7\ \textbf{(A)}}.$$

See also

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2010 AMC 10A Problems/Problem 18

Problem

Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

$\textbf{(A)}\ \frac{47}{72} \qquad \textbf{(B)}\ \frac{37}{56} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{49}{72} \qquad \textbf{(E)}\ \frac{39}{56}$

Solution

We can solve this by breaking the problem down into 2 cases and adding up the probabilities.

Case 1: Bernardo picks 9. If Bernardo picks a 9 then it is guaranteed that his number will be larger than Silvia's. The probability that he will pick a 9 is $\frac{1 \cdot \binom{8}{2}}{\binom{9}{3}} = \frac{1}{3}$.

Case 2: Bernardo does not pick 9. Since the chance of Bernardo picking 9 is $\frac{1}{3}$, the probability of not picking 9 is $\frac{2}{3}$.

If Bernardo does not pick 9, then he can pick any number from 1 to 8. Since Bernardo is picking from the same set of numbers as Silvia, the probability that Bernardo's number is larger is equal to the probability that Silvia's number is larger.

Ignoring the 9 for now, the probability that they will pick the same number is the number of ways to pick Bernardo's 3 numbers divided by the number of ways to pick any 3 numbers.

We get this probability to be $\frac{3!}{8 \cdot 7 \cdot 6} = \frac{1}{56}$

Probability of Bernardo's number being greater is

$$\frac{1 - \frac{1}{56}}{2} = \frac{55}{112}$$

Factoring the fact that Bernardo could've picked a 9 but didn't:

$$\frac{2}{3} \cdot \frac{55}{112} = \frac{55}{168}$$

Adding up the two cases we get $\frac{1}{3} + \frac{55}{168} = \boxed{\frac{37}{56}\ \textbf{(B)}}$

See also

2010 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2010))	
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Problem

Equiangular hexagon $ABCDEF$ has side lengths $AB=CD=EF=1$ and $BC=DE=FA=r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

$\textbf{(A)}\ \frac{4\sqrt{3}}{3} \qquad \textbf{(B)} \frac{10}{3} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ \frac{17}{4} \qquad \textbf{(E)}\ 6$

Solution

Solution 1

It is clear that $\triangle ACE$ is an equilateral triangle. From the Law of Cosines, we get that $AC^2 = r^2+1^2-2r\cos{\frac{2\pi}{3}} = r^2+r+1$. Therefore, the area of $\triangle ACE$ is $\frac{\sqrt{3}}{4}(r^2+r+1)$.

If we extend BC, DE and FA so that FA and BC meet at X, BC and DE meet at Y, and DE and FA meet at Z, we find that hexagon $ABCDEF$ is formed by taking equilateral triangle XYZ of side length $r+2$ and removing three equilateral triangles, ABX, CDY and EFZ, of side length 1. The area of $ABCDEF$ is therefore

$$\frac{\sqrt{3}}{4}(r+2)^2-\frac{3\sqrt{3}}{4} = \frac{\sqrt{3}}{4}(r^2+4r+1).$$

Based on the initial conditions,

$$\frac{\sqrt{3}}{4}(r^2+r+1) = \frac{7}{10}\left(\frac{\sqrt{3}}{4}\right)(r^2+4r+1)$$

Simplifying this gives us $r^2-6r+1 = 0$. By Vieta's Formulas we know that the sum of the possible value of r is $\boxed{\textbf{(E)}\ 6}$.

Solution 2

As above, we find that the area of $\triangle ACE$ is $\frac{\sqrt{3}}{4}(r^2+r+1)$.

We also find by the sine triangle area formula that $ABC = CDE = EFA = \frac{1}{2} \cdot 1 \cdot r \cdot \frac{\sqrt{3}}{2} = \frac{r\sqrt{3}}{4}$, and thus

$$\frac{\frac{\sqrt{3}}{4}(r^2+r+1)}{\frac{\sqrt{3}}{4}(r^2+r+1)+3\left(\frac{r\sqrt{3}}{4}\right)} = \frac{r^2+r+1}{r^2+4r+1} = \frac{7}{10}$$

This simplifies to $r^2 - 6r + 1 = 0 \Rightarrow \boxed{\textbf{(E)}\ 6}$.

See also

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Category: Introductory Geometry Problems

2010 AMC 10A Problems/Problem 20

Contents

Problem

A fly trapped inside a cubical box with side length 1 meter decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?

$\textbf{(A)}\ 4+4\sqrt{2} \qquad \textbf{(B)}\ 2+4\sqrt{2}+2\sqrt{3} \qquad \textbf{(C)}\ 2+3\sqrt{2}+3\sqrt{3} \qquad \textbf{(D)}\ 4\sqrt{2}+4\sqrt{3} \qquad \textbf{(E)}\ 3\sqrt{2}+5\sqrt{3}$

Solution 1

The distance of an interior diagonal in this cube is $\sqrt{3}$ and the distance of a diagonal on one of the square faces is $\sqrt{2}$. It is not possible for the fly to travel any interior diagonal twice, as then it would visit a corner more than once. So, the final sum can have at most 4 as the coefficient of $\sqrt{3}$. The other 4 paths taken can be across a diagonal on one of the faces (try to find a such path!), so the maximum distance traveled is $\textbf{(D)}\ 4\sqrt{2}+4\sqrt{3}$.

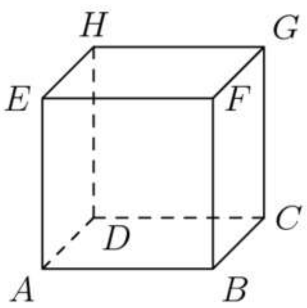
Solution 2

The path of the fly consists of eight line segments, where each line segment goes from one corner to another corner. The distance of each such line segment is 1, $\sqrt{2}$, or $\sqrt{3}$.

The only way to obtain a line segment of length $\sqrt{3}$ is to go from one corner of the cube to the opposite corner. Since the fly visits each corner exactly once, it cannot traverse such a line segment twice. Also, the cube has exactly four such diagonals, so the path of the fly can contain at most four segments of length $\sqrt{3}$. Hence, the length of the fly's path can be at most $\boxed{4\sqrt{3}+4\sqrt{2}}$. This length can be achieved by taking the path

$$A \to G \to B \to H \to C \to E \to D \to F \to A.$$

Hence, the answer is $\text{(D)} 4\sqrt{3}+4\sqrt{2}$.



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2010 AMC 10A Problems/Problem 21

Contents

Problem

The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a?

$\textbf{(A)}\ 78 \qquad \textbf{(B)}\ 88 \qquad \textbf{(C)}\ 98 \qquad \textbf{(D)}\ 108 \qquad \textbf{(E)}\ 118$

Solution

Solution 1

By Vieta's Formulas, we know that a is the sum of the three roots of the polynomial $x^3-ax^2+bx-2010$. Again Vieta's Formulas tells us that 2010 is the product of the three integer roots. Also, 2010 factors into $2\cdot3\cdot5\cdot67$. But, since there are only three roots to the polynomial, two of the four prime factors must be multiplied so that we are left with three roots. To minimize a, 2 and 3 should be multiplied, which means a will be $6+5+67=78$ and the answer is $\boxed{\textbf{(A)}}$.

Solution 2

We can expand $(x+a)(x+b)(x+c)$ as $(x^2+ax+bx+ab)(x+c)$

$$(x^2+ax+bx+ab)(x+c)=x^3+abx+acx+bcx+abx^2+acx^2+bcx^2+abc=x^3+x^2(a+b+c)+x(ab+ac+bc)+abc$$

We do not care about $+bx$ in this case, because we are only looking for a. We know that the constant term is $-2010=-(2*3*5*67)$ We are trying to minimize a, such that we have $-ax^2$ Since we have three positive solutions, we have $(x-a)(x-b)(x-c)$ as our factors. We have to combine two of the factors of $2*3*5*67$, and then sum up the 3 resulting factors. Since we are minimizing, we choose 2 and 3 to combine together. We get $(x-6)(x-5)(x-67)$ which gives us a coefficient of x^2 of $-6-5-67=-78$ Therefore $-a=-78$ or $a=\boxed{\textbf{(A)}78}$

See Also

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2010 AMC 10A Problems/Problem 22

Problem

Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 84 \qquad \textbf{(E)}\ 140$

Solution

To choose a chord, we know that two points must be chosen. This implies that for three chords to create a triangle and not intersect at a single point, six points need to be chosen. We also know that for any six points we pick, there is only 1 way to connect the chords such that a triangle is formed in the circle's interior. Therefore, the answer is ${8 \choose 6}$, which is equivalent to $\boxed{\textbf{(A)}\ 28}$.

See Also

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Problem

Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

$\textbf{(A)}\ 45 \qquad \textbf{(B)}\ 63 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 201 \qquad \textbf{(E)}\ 1005$

Solution

Solution 1

The probability of drawing a white marble from box k is $\frac{k}{k+1}$. The probability of drawing a red marble from box n is $\frac{1}{n+1}$.

The probability of drawing a red marble at box n is therefore

$$\frac{1}{n+1}\left(\prod_{k=1}^{n-1}\frac{k}{k+1}\right) < \frac{1}{2010}$$

$$\frac{1}{n+1}\left(\frac{1}{n}\right) < \frac{1}{2010}$$

$$(n+1)n > 2010$$

It is then easy to see that the lowest integer value of n that satisfies the inequality is $\boxed{45\ \textbf{(A)}}$.

Solution 2

Using the first few values of n, it is easy to derive a formula for $P(n)$. The chance that she stops on the second box ($n = 2$) is the chance of drawing a white marble then a red marble: $\frac{1}{2} * \frac{1}{3}$. The chance that she stops on the third box ($n = 3$) is the chance of drawing two white marbles then a red marble: $\frac{1}{2} * \frac{2}{3} * \frac{1}{4}$. If $n = 4$, $P(n) = \frac{1}{2} * \frac{2}{3} * \frac{3}{4} * \frac{1}{5}$.

Cross-cancelling in the fractions gives $P(2) = \frac{1}{2*3}$, $P(3) = \frac{1}{3*4}$, and $P(4) = \frac{1}{4*5}$. From this, it is clear that $P(n) = \frac{1}{(n)(n+1)}$. (Alternatively, $P(n) = \frac{(n-1)!}{(n+1)!}$.)

$$\frac{1}{(n+1)(n)} < \frac{1}{2010}$$

The lowest integer that satisfies the above inequality is $\boxed{(A) 45}$.

See also

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Category: Introductory Combinatorics Problems

2010 AMC 10A Problems/Problem 24

Problem

The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 52 \qquad \textbf{(E)}\ 68$

Solution

We will use the fact that for any integer n,

$$\begin{aligned}(5n+1)(5n+2)(5n+3)(5n+4) &= [(5n+4)(5n+1)][(5n+2)(5n+3)] \\ &= (25n^2+25n+4)(25n^2+25n+6) \equiv 4\cdot 6 \\ &= 24 \pmod{25} \equiv -1 \pmod{25}.\end{aligned}$$

First, we find that the number of factors of 10 in $90!$ is equal to $\left\lfloor \frac{90}{5}\right\rfloor + \left\lfloor\frac{90}{25}\right\rfloor = 18+3=21$. Let $N=\frac{90!}{10^{21}}$. The n we want is therefore the last two digits of N, or $N \pmod{100}$. Since there is clearly an excess of factors of 2, we know that $N \equiv 0 \pmod 4$, so it remains to find $N \pmod{25}$.

We can write N as $\frac{M}{2^{21}}$ where

$$M = 1\cdot 2\cdot 3\cdot 4\cdot 1\cdot 6\cdot 7\cdot 8\cdot 9\cdot 2\cdots 89\cdot 18 = \frac{90!}{5^{21}},$$

where every number in the form $5n$ is replaced by n.

The number M can be grouped as follows:

$$\begin{aligned}M= &(1\cdot 2\cdot 3\cdot 4)(6\cdot 7\cdot 8\cdot 9)\cdots(86\cdot 87\cdot 88\cdot 89) \\ &\cdot (1\cdot 2\cdot 3\cdot 4)(6\cdot 7\cdot 8\cdot 9)\cdots(16\cdot 17\cdot 18) \\ &\cdot (1\cdot 2\cdot 3).\end{aligned}$$

Hence, we can reduce M to

$$\begin{aligned}M &\equiv (-1)^{18}\cdot(-1)^3(16\cdot 17\cdot 18)\cdot(1\cdot 2\cdot 3) \\ &= 1\cdot -21\cdot 6 \\ &= -1 \pmod{25} = 24 \pmod{25}.\end{aligned}$$

Using the fact that $2^{10}=1024\equiv -1 \pmod{25}$, we can deduce that $2^{21}\equiv 2 \pmod{25}$. Therefore $N=\frac{M}{2^{21}} \equiv \frac{24}{2} \pmod{25} = 12 \pmod{25}$.

Finally, combining with the fact that $N\equiv 0 \pmod 4$ yields $n=\boxed{\textbf{(A)}\ 12}$.

See also

2010 AMC 10A Problems/Problem 25

Problem

Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{array}{ccccc} & & & & 55 \\ 55 & - & 7^2 & = & 6 \\ 6 & - & 2^2 & = & 2 \\ 2 & - & 1^2 & = & 1 \\ 1 & - & 1^2 & = & 0 \end{array}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 3 \qquad \mathrm{(C)}\ 5 \qquad \mathrm{(D)}\ 7 \qquad \mathrm{(E)}\ 9$

Solution

We can find the answer by working backwards. We begin with $1 - 1^2 = 0$ on the bottom row, then the 1 goes to the right of the equal's sign in the row above. We find the smallest value x for which $x - 1^2 = 1$ and $x > 1^2$, which is $x = 2$.

We repeat the same procedure except with $x - 1^2 = 1$ for the next row and $x - 1^2 = 2$ for the row after that. However, at the fourth row, we see that solving $x - 1^2 = 3$ yields $x = 4$, in which case it would be incorrect since $1^2 = 1$ is not the greatest perfect square less than or equal to x . So we make it a 2^2 and solve $x - 2^2 = 3$. We continue on using this same method where we increase the perfect square until x can be made bigger than it. When we repeat this until we have 8 rows, we get:

$$\begin{array}{ccccc} & & & & 7223 \\ 7223 & - & 84^2 & = & 167 \\ 167 & - & 12^2 & = & 23 \\ 23 & - & 4^2 & = & 7 \\ 7 & - & 2^2 & = & 3 \\ 3 & - & 1^2 & = & 2 \\ 2 & - & 1^2 & = & 1 \\ 1 & - & 1^2 & = & 0 \end{array}$$

Hence the solution is the last digit of 7223, which is $\boxed{\textbf{(B)}\ 3}$.

See also

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