2011 AMC 10A

Problem 1

A cell phone plan costs $\$20$ each month, plus $5¢$ per text message sent, plus $10¢$ for each minute used over 30 hours. In January Juan sent 100 text messages and talked for 30.5 hours. How much did he have to pay?

一种手机套餐每个月有 20 美元的固定费用。此外，每条短信收取 5 美分，语音通话超过 30 小时的部分，每分钟收取 10 美分。1 月份 Michelle 发送了 100 条短信，通话 30.5 小时。她需要支付多少美元？

(A) $\$24.00$ **(B)** $\$24.50$ **(C)** $\$25.50$ **(D)** $\$28.00$ **(E)** $\$30.00$

Problem 2

A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

一小瓶可以装 35 毫升洗发水，而一大瓶可以装 500 毫升洗发水。Jasmine 想买最少数量的小瓶洗发水来装满一大瓶。她必须买多少小瓶？

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

Problem 3

Suppose $[a\ b]$ denotes the average of a and b, and $\{a\ b\ c\}$ denotes the average of a, b, and c. What is $\{\{1\ 1\ 0\}\ [0\ 1]\ 0\}$?

假设$[a\ b]$表示 a 和 b 的平均值，$\{a\ b\ c\}$表示 $a, b,$ 和 c的平均值。那么$\{\{1\ 1\ 0\}\ [0\ 1]\ 0\}$是多少

(A) $\frac{2}{9}$ **(B)** $\frac{5}{18}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{7}{18}$ **(E)** $\frac{2}{3}$

Problem 4

Let X and Y be the following sums of arithmetic sequences:

$$\begin{aligned} X &= 10 + 12 + 14 + \cdots + 100, \\ Y &= 12 + 14 + 16 + \cdots + 102. \end{aligned}$$

What is the value of $Y - X$?

X 和 Y 分别表示下面等差数列各项之和：

$$\begin{aligned} X &= 10 + 12 + 14 + \cdots + 100, \\ Y &= 12 + 14 + 16 + \cdots + 102. \end{aligned}$$

则$Y - X$的值是多少？

(A) 92 **(B)** 98 **(C)** 100 **(D)** 102 **(E)** 112

Problem 5

At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

在一所小学，三年级，四年级和五年级的学生每天跑步的平均时间分别为 12 分钟，15 分钟和 10 分钟。已知三年级学生数是四年级的 2 倍，四年级学生数是五年级学生数的 2 倍。那么这些学生平均每个人每天的跑步时间是多少分钟？

(A) 12 **(B)** $\frac{37}{3}$ **(C)** $\frac{88}{7}$ **(D)** 13 **(E)** 14

Problem 6

Set A has 20 elements, and set B has 15 elements. What is the smallest possible number of elements in $A \cup B$, the union of A and B?

集合 A 有 20 个元素，集合 B 有 15 个元素。那么集合 A 和集合 B 的并集$A \cup B$，最少可能有多少个元素？

(A) 5 **(B)** 15 **(C)** 20 **(D)** 35 **(E)** 300

Problem 7

Which of the following equations does NOT have a solution?

下面哪个方程无解？

$\textbf{(A)}\ (x+7)^2=0$

$\textbf{(B)}\ |-3x|+5=0$

$\textbf{(C)}\ \sqrt{-x}-2=0$

$\textbf{(D)}\ \sqrt{x}-8=0$

$\textbf{(E)}\ |-3x|-4=0$

Problem 8

Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

去年夏天栖居在镇湖的鸟中，有 30%是鹅，25%是天鹅，10%是苍鹭，35%是鸭子。不是天鹅的鸟中，有百分之多少是鹅？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 60$

Problem 9

A rectangular region is bounded by the graphs of the equations $y=a, y=-b, x=-c,$ and $x=d,$ where $a, b, c,$ and d are all positive numbers. Which of the following represents the area of this region?

一个矩形区域由方程$y=a, y=-b, x=-c, x=d$包围起来，其中 a，b，c 和 d 都是正数。下面哪个表示这个区域的面积？

$\textbf{(A)}\ ac+ad+bc+bd \qquad \textbf{(B)}\ ac-ad+bc-bd \qquad \textbf{(C)}\ ac+ad-bc-bd \qquad \textbf{(D)}\ -ac-ad+bc+bd \qquad \textbf{(E)}\ ac-ad-bc+bd$

Problem 10

A majority of the 30 students in Ms. Deameanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was $\$17.71$. What was the cost of a pencil in cents?

Demeanor 女士班级的 30 个学生中，大多数学生都在学校书店买了一些铅笔。每个学生所买的铅笔数都一样，且大于 1。每支铅笔的价格（单位：美分）大于每个学生所购买的铅笔数，所有铅笔的总价是 17.71 美元，那么一支铅笔多少美分？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 77$

Problem 11

Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on $\overline{AB}$ with $AE = 7 \cdot EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

正方形 EFGH 的顶点分别位于正方形 *ABCD* 的四边上。点 E 在边$\overline{AB}$上且$AE = 7 \cdot EB$。问 *EFGH* 的面积和 *ABCD* 的面积的比值是多少？

$\textbf{(A)}\ \frac{49}{64} \qquad \textbf{(B)}\ \frac{25}{32} \qquad \textbf{(C)}\ \frac{7}{8} \qquad \textbf{(D)}\ \frac{5\sqrt{2}}{8} \qquad \textbf{(E)}\ \frac{\sqrt{14}}{4}$

Problem 12

The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

某篮球队投进一些三分球、两分球及一分的罚球。他们三分球所得的分数与两分球所得的分数相同，且罚球投进的球数比两分球投进的球数多一球。若此球队总共得到 61 分，则此球队罚球共投进了多少球

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Problem 13

How many even integers are there between 200 and 700 whose digits are all different and come from the set {1, 2, 5, 7, 8, 9}?

200 和 700 之间有多少个偶数，满足其各个位上数字均不同，且都来自于集合｛1，2，5，7，8，9｝？

(A) 12 **(B)** 20 **(C)** 72 **(D)** 120 **(E)** 200

Problem 14

A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

对 6 面标准骰子掷了一次。所得到的两个数之和决定了一个圆的直径。那么这个圆的面积在数值上比它的周长小的概率是多少？

(A) $\frac{1}{36}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{18}$

Problem 15

Roy bought a new battery-gasoline hybrid car. On a trip the car ran exclusively on its battery for the first 40 miles, then ran exclusively on gasoline for the rest of the trip, using gasoline at a rate of 0.02 gallons per mile. On the whole trip he averaged 55 miles per gallon. How long was the trip in miles?

Roy 买了辆电池和汽油混合动力的新车。在某次旅途中，汽车在旅程的前 40 英里只使用电池，在旅程的剩余路程只使用汽油，每英里需要使用 0.02 加仑的汽油。所以整个旅程平均下来每加仑汽油行驶了 55 英里。问旅程总共多少英里？

(A) 140 **(B)** 240 **(C)** 440 **(D)** 640 **(E)** 840

Problem 16

Which of the following is equal to $\sqrt{9-6\sqrt{2}}+\sqrt{9+6\sqrt{2}}$?

$\sqrt{9-6\sqrt{2}}+\sqrt{9+6\sqrt{2}}$的值是多少？

$\textbf{(A)}\ 3\sqrt{2} \qquad \textbf{(B)}\ 2\sqrt{6} \qquad \textbf{(C)}\ \frac{7\sqrt{2}}{2} \qquad \textbf{(D)}\ 3\sqrt{3} \qquad \textbf{(E)}\ 6$

Problem 17

In the eight-term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

在这个 8 项数列A, B, C, D, E, F, G, H中，C 的值为 5，且任何 3 个连续项之和为 30。问 A＋H 是多少?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 43$

Problem 18

Circles $A, B,$ and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside Circle C but outside circle A and circle B ?

圆 *A*，*B* 和 *C* 的半径均为 1。圆 *A* 和圆 *B* 切于一点。圆 *C* 与线段$\overline{AB}$切于$\overline{AB}$的中点。那么位于圆 *C* 内但在圆 *A* 和圆 *B* 外的区域的面积是多少？

$\textbf{(A)}\ 3-\frac{\pi}{2} \qquad \textbf{(B)}\ \frac{\pi}{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{3\pi}{4} \qquad \textbf{(E)}\ 1+\frac{\pi}{2}$

Problem 19

In 1991 the population of a town was a perfect square. Ten years later, after an increase of 150 people, the population was 9 more than a perfect square. Now, in 2011, with an increase of another 150 people, the population is once again a perfect square. Which of the following is closest to the percent growth of the town's population during this twenty-year period?

某个镇的人口数在 1991 年是个完全平方数。10 年后，增加了 150 人，人口总数是个比某个完全平方数大 9 的数。现在是 2011 年，又增加了 150 人之后，人口总数又是个完全平方数。问在这 20 年间，这个镇的人口数增加的百分数最接近下面哪个数？

$\textbf{(A)}\ 42 \qquad \textbf{(B)}\ 47 \qquad \textbf{(C)}\ 52 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 62$

Problem 20

Two points on the circumference of a circle of radius r are selected independently and at random. From each point a chord of length r is drawn in a clockwise direction. What is the probability that the two chords intersect?

从半径为 r 的圆周上独立且随机地选择 2 个点。从每个点沿顺时针方向分别引出 2 条长度为 r 的弦。问这 2 条弦相交的概率是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 21

Two counterfeit coins of equal weight are mixed with 8 identical genuine coins. The weight of each of the counterfeit coins is different from the weight of each of the genuine coins. A pair of coins is selected at random without replacement from the 10 coins. A second pair is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 selected coins are genuine?

2 枚等重的假币混在 8 枚一模一样的真币里。每一枚假币的重量和每一枚真币的重量不同。首先从这 10 枚硬币里不放回地随机选择一对硬币。然后再从剩下的 8 枚硬币里不放回地随机选择一对硬币。第一对硬币的总重等于第二对硬币的总重。问选择的 4 枚硬币都是真币的概率是多少？

$\textbf{(A)}\ \frac{7}{11} \qquad \textbf{(B)}\ \frac{9}{13} \qquad \textbf{(C)}\ \frac{11}{15} \qquad \textbf{(D)}\ \frac{15}{19} \qquad \textbf{(E)}\ \frac{15}{16}$

Problem 22

Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

一个凸五边形 *ABCDE* 的每个顶点都被涂一种颜色。一共有 6 种颜色可以选择，且要求每条对角线的两个端点的颜色必须不同。问有多少种涂色方法？

$\textbf{(A)}\ 2520 \qquad \textbf{(B)}\ 2880 \qquad \textbf{(C)}\ 3120 \qquad \textbf{(D)}\ 3250 \qquad \textbf{(E)}\ 3750$

Problem 23

Seven students count from 1 to 1000 as follows:

•Alice says all the numbers, except she skips the middle number in each consecutive group of three numbers. That is, Alice says 1, 3, 4, 6, 7, 9, ..., 997, 999, 1000.

•Barbara says all of the numbers that Alice doesn't say, except she also skips the middle number in each consecutive group of three numbers.

•Candice says all of the numbers that neither Alice nor Barbara says, except she also skips the middle number in each consecutive group of three numbers.

•Debbie, Eliza, and Fatima say all of the numbers that none of the students with the first names beginning before theirs in the alphabet say, except each also skips the middle number in each of her consecutive groups of three numbers.

•Finally, George says the only number that no one else says.

What number does George say?

7 个学生以如下方式从 1 数到 1000：

Alice 跳过每 3 个数组成的一组的中间那个数，以这样的方式数数。也就是，Alice 数的数是 1，3，4，6，7，9，…，997，999，1000.

Barbara 数的是 Alice 没有数的数，不过她仍然是跳过每 3 个数组成的一组数的中间那个数。

Candice 数的是 Alice 和 Barbara 都没有数的数，不过她也是跳过每 3 个数组成的一组数的中间那个数。

Debbie，Eliza，和 Fatima 也是以同样的方式数数，数的都是那些名字首字母按照字母顺序表在她们前面的人没有数过的数。不过依然是跳过每 3 个数组成的一组数的中间那个数。

最后，George 数了所有人都没数的数。问 George 数的数是多少？

$\textbf{(A)}\ 37 \qquad \textbf{(B)}\ 242 \qquad \textbf{(C)}\ 365 \qquad \textbf{(D)}\ 728 \qquad \textbf{(E)}\ 998$

Problem 24

Two distinct regular tetrahedra have all their vertices among the vertices of the same unit cube. What is the volume of the region formed by the intersection of the tetrahedra?

2 个不重合的正四面体以同一个单位正方体的顶点为顶点。则这 2 个正四面体相交形成的区域的体积是多少？

$\textbf{(A)}\ \frac{1}{12} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{12} \qquad \textbf{(C)}\ \frac{\sqrt{3}}{12} \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{\sqrt{2}}{6}$

Problem 25

Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called $n\text{-}ray$ partitional if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

R 是一个正方形区域，*n*≥4 是个整数。*X* 是一个位于 *R* 内部的点，若从 *X* 引出的 *n* 条射线将 *R* 分成 *n* 个等面积的三角形，那么称 *X* 为 *n*-射线分割。问有多少个是 100-射线分割但不是 60-射线分割的点？

$\textbf{(A)}\ 1500 \qquad \textbf{(B)}\ 1560 \qquad \textbf{(C)}\ 2320 \qquad \textbf{(D)}\ 2480 \qquad \textbf{(E)}\ 2500$

2011 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	E	D	A	C	C	B	C	A	B	B	A	A
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	C	B	C	C	E	D	D	C	C	D	C	