2011 AMC12A

Problem 1

A cell phone plan costs $\$20$ dollars each month, plus 5 cents per text message sent, plus 10 cents for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

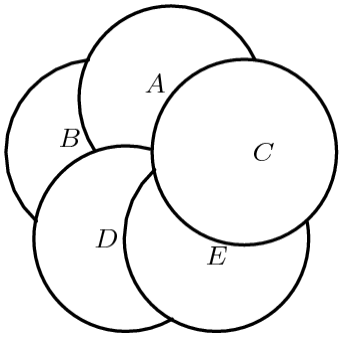
一种手机套餐每个月有 20 美元的固定费用。此外，每条短信收取 5 美分，语音通话超过 30 小时的部分，每分钟收取 10 美分。Michelle 在 1 月份共发送了 100 条短信，拨打了 30.5 小时语音通话，问她需要支付多少美元？

(A) 24.00 **(B)** 24.50 **(C)** 25.50 **(D)** 28.00 **(E)** 30.00

Problem 2

There are 5 coins placed flat on a table according to the figure. What is the order of the coins from top to bottom?

下图显示了平放在桌子上的 5 枚硬币。那么硬币从上到下的顺序分别为什么？



(A) (C, A, E, D, B) **(B)** (C, A, D, E, B) **(C)** (C, D, E, A, B) **(D)** (C, E, A, D, B) **(E)** (C, E, D, A, B)

Problem 3

A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

一个小瓶洗发水可以装 35 毫升洗发水，而一个大瓶洗发水可以装 500 毫升洗发水。Jasmine 想买最少数量的小瓶洗发水来装满一大瓶，她必领买多少小瓶？

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

Problem 4

At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

在一所小学里，三年级、四年级和五年级的学生每天跑步的平均时间分别为 12 分钟、15 分钟和 10 分钟。已知三年级学生数是四年级的 2 倍，四年级学生数是五年级学生数的 2 倍。那么这些学生平均每个人每天的跑步时间是多少分钟？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ \frac{37}{3} \qquad \textbf{(C)}\ \frac{88}{7} \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 14$

Problem 5

Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

去年夏天栖居在镇湖里的鸟中，有 30％是鹅，25％是天鹅，10％是苍鹭，35％是鸭子。不是天鹅的鸟中，有百分之多少是鹅？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 60$

Problem 6

The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

某篮球队投进了一些三分球、两分球及一分的罚球，他们三分球所得的分数与两分球所得的分数相同，且罚球投进的球数比两分球投进的球数多 1 球。若此球队总共得到 61 分，则此球队罚球共投进了多少球？

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Problem 7

A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was 17.71. What was the cost of a pencil in cents?

Demeanor 女士班级的 30 个学生中，大多数学生都在学校书店买了一些铅笔。每个学生所买的铅笔数都一样，且数量大于 1。每支铅笔的价格（单位：美分）大于每个学生所购买的铅笔数，所有铅笔的总价是 17.71 美元，那么一支铅笔多少美分？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 77$

Problem 8

In the eight term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

在共有 8 项的数列A, B, C, D, E, F, G, H中，C的值是 5，且任何三个连续项之和都为 30，那么$A + H$是多少?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 43$

Problem 9

At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her siblings and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

在一次双胞胎和三胞胎的集会上，有 9 对双胞胎和 6 组三胞胎，他们都来自不同的家庭。每个双胞胎都与除了他/她的兄弟姐妹之外的其他所有双胞胎握手，还和一半的三胞胎握手；每个三胞胎都和除了他/她的兄弟姐妹之外的其他所有的三胞胎握手，还和一半的双胞胎握手。那么集会上一共发生了多少次握手？

$\textbf{(A)}\ 324 \qquad \textbf{(B)}\ 441 \qquad \textbf{(C)}\ 630 \qquad \textbf{(D)}\ 648 \qquad \textbf{(E)}\ 882$

Problem 10

A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

一对 6 个面的标准骰子被掷了一次，所得到的两个数之和决定了一个圆的真径。那么这个圆的面积在数值上比它的周长小的概率是多少？

$\textbf{(A)}\ \frac{1}{36}\qquad \textbf{(B)}\ \frac{1}{12}\qquad \textbf{(C)}\ \frac{1}{6}\qquad \textbf{(D)}\ \frac{1}{4}\qquad \textbf{(E)}\ \frac{5}{18}$

Problem 11

Circles $A, B,$ and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?

圆 A，B 和 C 的半径均为 1，圆 A 和圆 B 切于一点，圆 C 与线段$\overline{AB}$切于$\overline{AB}$的中点。那么位于圆 C 内，但在圆 A 和圆 B 外的区域的面积是多少？

$\textbf{(A)}\ 3-\frac{\pi}{2}\qquad \textbf{(B)}\ \frac{\pi}{2}\qquad \textbf{(C)}\ 2\qquad \textbf{(D)}\ \frac{3\pi}{4}\qquad \textbf{(E)}\ 1+\frac{\pi}{2}$

Problem 12

A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock A. How many hours did it take the power boat to go from A to B?

一艘动力船和一艘木筏都离开了河上的 A 码头向下游驶去。木筏以河水的速度漂流，动力船以相对于水速恒定的速度行驶。动力船到达下游 B 码头后立即掉头返回上游，离开 A 码头 9 小时后，它最终在河上遇到了木筏。问动力船从 A 码头到 B 码头需要多少小时？

$\textbf{(A)}\ 3\qquad \textbf{(B)}\ 3.5\qquad \textbf{(C)}\ 4\qquad \textbf{(D)}\ 4.5\qquad \textbf{(E)}\ 5$

Problem 13

Triangle ABC has side-lengths $AB = 12, BC = 24,$ and $AC = 18.$ The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at $N.$ What is the perimeter of $\triangle AMN$?

三角形 A*BC* 的边长为 *AB*=12，*BC*=24，*AC*=18，通过$\triangle ABC$的内心且平行于$\overline{BC}$的直线交$\overline{AB}$于 *M*， 交$\overline{AC}$于 *N*。则$\triangle AMN$的周长是多少？

$\textbf{(A)}\ 27 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 33 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 42$

Problem 14

Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

假设a和b是随机且独立选择的 1 位正整数，则点(a, b)位于抛物线$y = ax^2 - bx$上方的概率是多少？

$\textbf{(A)}\ \frac{11}{81} \qquad \textbf{(B)}\ \frac{13}{81} \qquad \textbf{(C)}\ \frac{5}{27} \qquad \textbf{(D)}\ \frac{17}{81} \qquad \textbf{(E)}\ \frac{19}{81}$

Problem 15

The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

一个半径为 2 的半球的圆形底面放置在高度为 6 的正方锥（底面为正方形的四棱锥）的底面上。半球和椎体的其他四个面都相切，则椎底的边长是多少？

$\textbf{(A)}\ 3\sqrt{2} \qquad \textbf{(B)}\ \frac{13}{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Problem 16

Each vertex of convex polygon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

凸多边形$ABCDE$的每个顶点都被分配一种颜色，一共有 6 种颜色可选，且对角线的两个端点颜色必须不同，问一共有多少种不同的涂色方法？

$\textbf{(A)}\ 2520 \qquad \textbf{(B)}\ 2880 \qquad \textbf{(C)}\ 3120 \qquad \textbf{(D)}\ 3250 \qquad \textbf{(E)}\ 3750$

Problem 17

Circles with radii 1, 2, and 3 are mutually externally tangent. What is the area of the triangle determined by the points of tangency?

半径为 1，2 和 3 的圆两两相外切，问切点所确定的三角形的面积是多少？

$\textbf{(A)}\ \frac{3}{5} \qquad \textbf{(B)}\ \frac{4}{5} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{6}{5} \qquad \textbf{(E)}\ \frac{4}{3}$

Problem 18

Suppose that $|x+y|+|x-y|=2$. What is the maximum possible value of x^2-6x+y^2?

假设$|x+y|+|x-y|=2$，那么x^2-6x+y^2的最大可能值是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 19

At a competition with N players, the number of players given elite status is equal to $2^{1+\lfloor \log_2(N-1) \rfloor} - N$. Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N?

在一场有N个选手的比赛中，被授予精英地位的选手数量等于$2^{1+\lfloor \log_2(N-1) \rfloor} - N$，假设 19 位选手被授予精英地位，则$N$的两个最小可能值之和是多少？

$\textbf{(A)}\ 38 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 154 \qquad \textbf{(D)}\ 406 \qquad \textbf{(E)}\ 1024$

Problem 20

Let $f(x) = ax^2 + bx + c$, where a, b, and c are integers. Suppose that $f(1) = 0$, $50 < f(7) < 60$, $70 < f(8) < 80$, $5000k < f(100) < 5000(k+1)$ for some integer k. What is k?

令$f(x) = ax^2 + bx + c$，这里a，b和c都是整数。假设$f(1) = 0$，$50 < f(7) < 60$，$70 < f(8) < 80$，$5000k < f(100) < 5000(k+1)$，这里$k$是某个整数，问$k$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 21

Let $f_1(x) = \sqrt{1-x}$, and for integers $n \geq 2$, let $f_n(x) = f_{n-1}(\sqrt{n^2 - x})$. If N is the largest value of n for which the domain of f_n is nonempty, the domain of f_N is $\{c\}$. What is $N + c$?

令$f_1(x) = \sqrt{1-x}$，且对于整数$n \geq 2$，令$f_n(x) = f_{n-1}(\sqrt{n^2 - x})$。若使得$f_n$的定义域的非空的$n$的最大值为$N$，且此时$f_N$的定义域为$\{c\}$，那么$N + c$是多少？

$\textbf{(A)}\ -226 \qquad \textbf{(B)}\ -144 \qquad \textbf{(C)}\ -20 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 144$

Problem 22

Let R be a unit square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

R 是一个正方形区域，$n \geq 4$是个整数，X是一个位于 R 内部的点，若从X引出的 n 条射线将 R 分成 n 个等面积的三角形，那么称X为 n-射线分割。问有多少个是 100-射线分割但不是 60-射线分割的点？

$\textbf{(A)}\ 1500 \qquad \textbf{(B)}\ 1560 \qquad \textbf{(C)}\ 2320 \qquad \textbf{(D)}\ 2480 \qquad \textbf{(E)}\ 2500$

Problem 23

Let $f(z)=\frac{z+a}{z+b}$ and $g(z)=f(f(z))$, where a and b are complex numbers. Suppose that $|a|=1$ and $g(g(z))=z$ for all z for which $g(g(z))$ is defined. What is the difference between the largest and smallest possible values of $|b|$?

令$f(z)=\frac{z+a}{z+b}$，且$g(z)=f(f(z))$，这里a和b都是复数。假设$|a|=1$，且对任何z来说，只要$g(g(z))$有定义，那么都有$g(g(z))=z$，那么$|b|$的最大值和最小值的差是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \sqrt{2}-1 \qquad \textbf{(C)}\ \sqrt{3}-1 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2$

Problem 24

Consider all quadrilaterals $ABCD$ such that $AB=14$, $BC=9$, $CD=7$, and $DA=12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

考虑所有满足此条件的四边形 *ABCD*: *AB*=14，*BC*=9，*CD*=7，*DA*=12，问位于此四边形内部或者四边形的某几条边相切的最大的圆的半径是多少？

$\textbf{(A)}\ \sqrt{15} \qquad \textbf{(B)}\ \sqrt{21} \qquad \textbf{(C)}\ 2\sqrt{6} \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 2\sqrt{7}$

Problem 25

Triangle ABC has $\angle BAC=60^\circ$, $\angle CBA\le 90^\circ$, $BC=1$, and $AC\ge AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

在三角形 *ABC* 中，$\angle BAC=60^\circ$，$\angle CBA\le 90^\circ$，*BC*=1，$AC\ge AB$，令 *H*，*I*，和 *O* 分别为$\triangle ABC$的垂心、内心和外心。假设五边形 *BCOIH* 的面积取最大可能值，那么$\angle CBA$是多少度？

$\textbf{(A)}\ 60^\circ \qquad \textbf{(B)}\ 72^\circ \qquad \textbf{(C)}\ 75^\circ \qquad \textbf{(D)}\ 80^\circ \qquad \textbf{(E)}\ 90^\circ$

2011 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	E	E	C	C	A	B	C	B	B	C	D	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	A	C	D	D	C	C	A	C	C	C	D	