2011 AMC 10B Problems/Problem 1

Problem

What is

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}?$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{147}{60} \qquad \textbf{(E)}\ \frac{43}{3}$

Solution

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}=\frac{12}{9}-\frac{9}{12}=\frac{4}{3}-\frac{3}{4}=\boxed{\frac{7}{12}\ \textbf{(C)}}$$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
Preceded by First Problem	Followed by Problem 2
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2011 AMC 10B Problems/Problem 2

Problem

Josanna's test scores to date are $90, 80, 70, 60,$ and 85. Her goal is to raise here test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

$\textbf{(A)}\ 80 \qquad\textbf{(B)}\ 82 \qquad\textbf{(C)}\ 85 \qquad\textbf{(D)}\ 90 \qquad\textbf{(E)}\ 95$

Solution

The average of her current scores is 77. To raise it 3 points, she needs an average of 80, and so after her 6 tests, a sum of 480. Her current sum is 385, so she needs a $480 - 385 = \boxed{\textbf{(E)}\ 95}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 3

Problem

At a store, when a length is reported as x inches that means the length is at least $x - 0.5$ inches and at most $x + 0.5$ inches. Suppose the dimensions of a rectangular tile are reported as 2 inches by 3 inches. In square inches, what is the minimum area for the rectangle?

$\textbf{(A)}\ 3.75 \qquad \textbf{(B)}\ 4.5 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8.75$

Solution

The minimum dimensions of the rectangle are 1.5 inches by 2.5 inches. The minimum area is $1.5 \times 2.5 = \boxed{\textbf{(A)}\ 3.75}$ square inches.

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
Preceded by Problem 2	Followed by Problem 4
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All AMC 10 Problems and Solutions	

2011 AMC 10B Problems/Problem 4

Problem

LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

$\textbf{(A)}\ \frac{A+B}{2} \qquad \textbf{(B)}\ \frac{A-B}{2} \qquad \textbf{(C)}\ \frac{B-A}{2} \qquad \textbf{(D)}\ B-A \qquad \textbf{(E)}\ A+B$

Solution

The difference in how much LeRoy and Bernardo paid is $B-A$. To share the costs equally, LeRoy must give Bernardo half of the difference, which is $\boxed{(C) \frac{B-A}{2}}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 5

Problem

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

$\textbf{(A)}\ 116 \qquad \textbf{(B)}\ 161 \qquad \textbf{(C)}\ 204 \qquad \textbf{(D)}\ 214 \qquad \textbf{(E)}\ 224$

Solution

We have $161 = 7 \cdot 23.$ Since a has two digits, the factors must be 23 and $7,$ so $a = 32$ and $b = 7.$ Then, $ab = 7 \times 32 = \boxed{\textbf{(E)}\ 224}$.

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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All AMC 10 Problems and Solutions	

2011 AMC 10B Problems/Problem 6

Problem

On Halloween Casper ate $\frac{1}{3}$ of his candies and then gave 2 candies to his brother. The next day he ate $\frac{1}{3}$ of his remaining candies and then gave 4 candies to his sister. On the third day he ate his final 8 candies. How many candies did Casper have at the beginning?

$\textbf{(A)}\ 30 \qquad\textbf{(B)}\ 39 \qquad\textbf{(C)}\ 48 \qquad\textbf{(D)}\ 57 \qquad\textbf{(E)}\ 66$

Solution

Let x represent the amount of candies Casper had at the beginning.

$$\begin{aligned} \frac{2}{3}\left(\frac{2}{3}x-2\right)-4-8&=0\\ \frac{2}{3}x-2&=18\\ \frac{2}{3}x&=20\\ x&=\boxed{\textbf{(A)} 30} \end{aligned}$$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 7

Problem

The sum of two angles of a triangle is $\frac{6}{5}$ of a right angle, and one of these two angles is 30° larger than the other. What is the degree measure of the largest angle in the triangle?

$\textbf{(A)}\ 69 \qquad\textbf{(B)}\ 72 \qquad\textbf{(C)}\ 90 \qquad\textbf{(D)}\ 102 \qquad\textbf{(E)}\ 108$

Solution

The sum of two angles in a triangle is $\frac{6}{5}$ of a right angle $\longrightarrow \frac{6}{5} \times 90 = 108$

If x is the measure of the first angle, then the measure of the second angle is $x + 30$.

$$x + x + 30 = 108 \longrightarrow 2x = 78 \longrightarrow x = 39$$

Now we know the measure of two angles are 39° and 69°. By the Triangle Sum Theorem, the sum of all angles in a triangle is 180°, so the final angle is 72°. Therefore, the largest angle in the triangle is $\boxed{\textbf{(B)}\ 72}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 8

Problem

At a certain beach if it is at least $80^\circ F$ and sunny, then the beach will be crowded. On June 10 the beach was not crowded. What can be concluded about the weather conditions on June 10?

$\textbf{(A)}$ The temperature was cooler than 80°F and it was not sunny.

$\textbf{(B)}$ The temperature was cooler than 80°F or it was not sunny.

$\textbf{(C)}$ If the temperature was at least 80°F, then it was sunny.

$\textbf{(D)}$ If the temperature was cooler than 80°F, then it was sunny.

$\textbf{(E)}$ If the temperature was cooler than 80°F, then it was not sunny.

Solution

The beach not being crowded only means that it is not both hot and sunny. Equivalently, it is either cool or cloudy, which means $\boxed{\textbf{(B)}}$ is correct.

It could be in fact cool and sunny, which would violate (A) and (E). It could also be cool and cloudy, which would violate (D), or warm and sunny, which would violate (C).

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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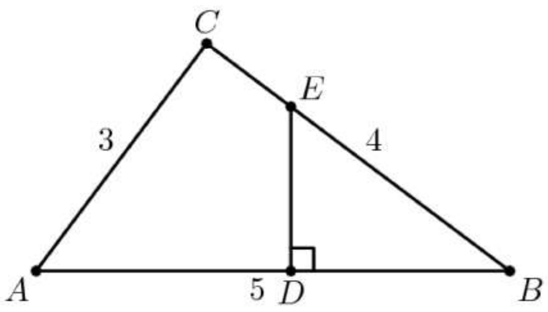
2011 AMC 10B Problems/Problem 9

Problem

The area of $\triangle EBD$ is one third of the area of $3-4-5$ $\triangle ABC$. Segment DE is perpendicular to segment AB. What is BD?



$\textbf{(A)}\ \frac{4}{3} \qquad \textbf{(B)}\ \sqrt{5} \qquad \textbf{(C)}\ \frac{9}{4} \qquad \textbf{(D)}\ \frac{4\sqrt{3}}{3} \qquad \textbf{(E)}\ \frac{5}{2}$

Solution

$\triangle ABC \sim \triangle EBD$ by AA Similarity. Therefore $DE = \frac{3}{4}BD$. Find the areas of the triangles.

$$\triangle ABC : 3 \times 4 \times \frac{1}{2} = 6 \triangle EBD : BD \times \frac{3}{4}BD \times \frac{1}{2} = \frac{3}{8}BD^2$$

The area of $\triangle EBD$ is one third of the area of $\triangle ABC$.

$$\frac{3}{8}BD^2 = 6 \times \frac{1}{3}$$
$$9BD^2 = 48$$
$$BD^2 = \frac{16}{3}$$
$$BD = \boxed{\textbf{(D)} \frac{4\sqrt{3}}{3}}$$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 10

Contents

Problem

Consider the set of numbers $\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

$\textbf{(A)}\ 1 \qquad\textbf{(B)}\ 9 \qquad\textbf{(C)}\ 10 \qquad\textbf{(D)}\ 11 \qquad\textbf{(E)} 101$

Solution 1

The requested ratio is

$$\frac{10^{10}}{10^9 + 10^8 + \ldots + 10 + 1}.$$

Using the formula for a geometric series, we have

$$10^9 + 10^8 + \ldots + 10 + 1 = \frac{10^{10} - 1}{10 - 1} = \frac{10^{10} - 1}{9},$$

which is very close to $\frac{10^{10}}{9}$, so the ratio is very close to $\boxed{\textbf{(B)}\ 9}$.

Solution 2

The problem asks for the value of

$$\frac{10^{10}}{10^9 + 10^8 + \ldots + 10 + 1}.$$

Written in base 10, we can find the value of $10^9 + 10^8 + \ldots + 10 + 1$ to be $1111111111.$ Long division gives us the answer to be $\boxed{\textbf{(B)}\ 9}$.

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 11

Problem

There are 52 people in a room. what is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 12$

Solution

Pretend you have 52 people you want to place in 12 boxes, because there are 12 months in a year. By the Pigeonhole Principle, one box must have at least $\left\lceil \frac{52}{12} \right\rceil$ people $\longrightarrow \boxed{\textbf{(D)} 5}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 12

Contents

Problem

Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has a width of 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

$\textbf{(A)}\ \frac{\pi}{3} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Solution

Solution 1

Let s be Keiko's speed in meters per second, a be the length of the straight parts of the track, b be the radius of the smaller circles, and $b+6$ be the radius of the larger circles. The length of the inner edge will be $2a+2b\pi$ and the length of the outer edge will be $2a+2\pi(b+6)$. Since it takes 36 seconds longer for Keiko to walk on the outer edge,

$$\begin{aligned} \frac{2a+2b\pi}{s}+36 &= \frac{2a+2\pi(b+6)}{s} \\ 2a+2b\pi+36s &= 2a+2b\pi+12\pi \\ 36s &= 12\pi \\ s &= \boxed{\textbf{(A)} \frac{\pi}{3}} \end{aligned}$$

Solution 2

It is basically the same as Solution 1 except you can completely disregard the straight edges of the track since it will take Keiko the same time to walk that length. Instead, think of it has just a circular track 6 meters in width. If the diameter of the smaller circle were r, then the length of the smaller circle would be $r\pi$ and the length of the larger circle would be $(r+12)\pi$. Since it still takes 36 seconds longer,

$$\begin{aligned} \frac{r\pi}{s}+36 &= \frac{(r+12)\pi}{s} \\ r\pi+36s &= r\pi+12\pi \\ s &= \boxed{\textbf{(A)} \frac{\pi}{3}} \end{aligned}$$

See Also

2011 AMC 10B Problems/Problem 13

Problem

Two real numbers are selected independently at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

Solution

For the product of two numbers to be greater than zero, they either have to both be negative or both be positive. The interval for a positive number is $\frac{1}{3}$ of the total interval, and the interval for a negative number is $\frac{2}{3}$. Therefore, the probability the product is greater than zero is

$$\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{1}{9} + \frac{4}{9} = \boxed{\textbf{(D)} \frac{5}{9}}$$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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All AMC 10 Problems and Solutions	

2011 AMC 10B Problems/Problem 14

Problem

A rectangular parking lot has a diagonal of 25 meters and an area of 168 square meters. In meters, what is the perimeter of the parking lot?

$\textbf{(A)}\ 52 \qquad\textbf{(B)}\ 58 \qquad\textbf{(C)}\ 62 \qquad\textbf{(D)}\ 68 \qquad\textbf{(E)}\ 70$

Solution

Let the sides of the rectangular parking lot be a and b. Then $a^2+b^2=625$ and $ab=168$. Add the two equations together, then factor.

$$a^2+2ab+b^2=625+168\times 2$$
$$(a+b)^2=961$$
$$a+b=31$$

The perimeter of a rectangle is $2(a+b)=2(31)=\boxed{\textbf{(C)} 62}$

See Also

<table>
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2011 AMC 10B Problems/Problem 15

Problem

Let $@$ denote the "averaged with" operation: $a@b = \frac{a+b}{2}$. Which of the following distributive laws hold for all numbers $x, y,$ and z?

$$\text{I. x @ (y + z) = (x @ y) + (x @ z)}$$

$$\text{II. x + (y @ z) = (x + y) @ (x + z)}$$

$$\text{III. x @ (y @ z) = (x @ y) @ (x @ z)}$$

$\textbf{(A)}\ \text{I only} \qquad \textbf{(B)}\ \text{II only} \qquad \textbf{(C)}\ \text{III only} \qquad \textbf{(D)}\ \text{I and III only} \qquad \textbf{(E)}\ \text{II and III only}$

Solution

Just simplify each operation and see which ones hold true.

$$\begin{aligned} \text{I.} \qquad x@(y+z) &= (x@y)+(x@z) \\ \frac{x+y+z}{2} &= \frac{x+y}{2}+\frac{x+z}{2} \\ \frac{x+y+z}{2} &\neq \frac{2x+y+z}{2} \end{aligned}$$

$$\begin{aligned} \text{II.} \qquad x+(y@z) &= (x+y)@(x+z) \\ x+\frac{y+z}{2} &= \frac{2x+y+z}{2} \\ \frac{2x+y+z}{2} &= \frac{2x+y+z}{2} \end{aligned}$$

$$\begin{aligned} \text{III.} \qquad x@(y@z) &= (x@y)@(x@z) \\ x@\frac{y+z}{2} &= \frac{x+y}{2}@\frac{x+z}{2} \\ \frac{2x+y+z}{4} &= \frac{2x+y+z}{4} \end{aligned}$$

$\boxed{\textbf{(E)} \text{II and III only}}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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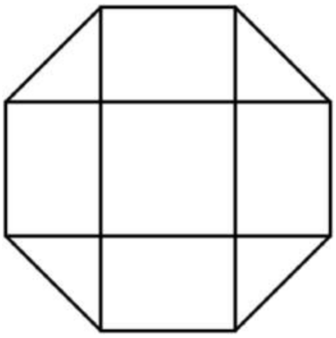
2011 AMC 10B Problems/Problem 16

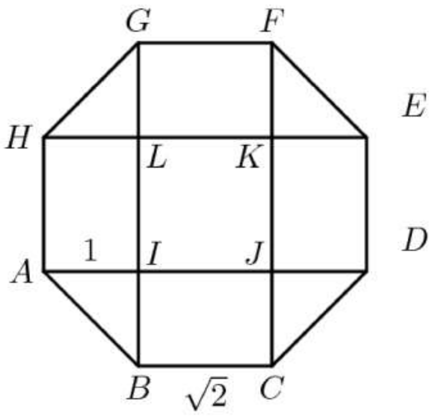
Problem

A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is probability that the dart lands within the center square?



$\textbf{(A)}\ \frac{\sqrt{2}-1}{2} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{2-\sqrt{2}}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(E)}\ 2-\sqrt{2}$

Solution



If the side lengths of the dart board and the side lengths of the center square are all $\sqrt{2}$, then the side length of the legs of the triangles are 1.

$$\text{area of center square}: \sqrt{2} \times \sqrt{2} = 2$$

$$\text{total area}: (\sqrt{2})^2 + 4(1 \times \sqrt{2}) + 4(1 \times 1 \times \frac{1}{2}) = 2 + 4\sqrt{2} + 2 = 4 + 4\sqrt{2}$$

Use Geometric probability by putting the area of the desired region over the area of the entire region.

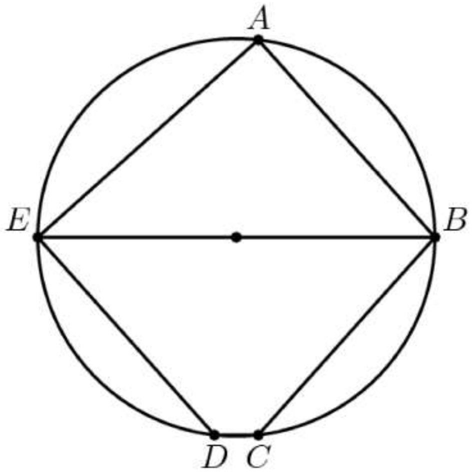
$$\frac{2}{4+4\sqrt{2}} = \frac{1}{2+2\sqrt{2}} \times \frac{2-2\sqrt{2}}{2-2\sqrt{2}} = \frac{2-2\sqrt{2}}{-4} = \boxed{\textbf{(A)} \frac{\sqrt{2}-1}{2}}$$

See Also

2011 AMC 10B Problems/Problem 17

Problem

In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4:5$. What is the degree measure of angle BCD?



$\textbf{(A)}\ 120 \qquad\textbf{(B)}\ 125 \qquad\textbf{(C)}\ 130 \qquad\textbf{(D)}\ 135 \qquad\textbf{(E)}\ 140$

Solution

We can let $\angle AEB$ be $4x$ and $\angle ABE$ be $5x$ because they are in the ratio $4:5$. When an inscribed angle contains the diameter, the inscribed angle is a right angle. Therefore by triangle sum theorem, $4x+5x+90=180 \longrightarrow x=10$ and $\angle ABE=50$.

$\angle ABE = \angle BED$ because they are alternate interior angles and $\overline{AB} \parallel \overline{ED}$. Opposite angles in a cyclic quadrilateral are supplementary, so $\angle BED + \angle BCD = 180$. Use substitution to get $\angle ABE + \angle BCD = 180 \longrightarrow 50 + \angle BCD = 180 \longrightarrow \angle BCD = \boxed{\textbf{(C)} 130}$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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Category: Introductory Geometry Problems

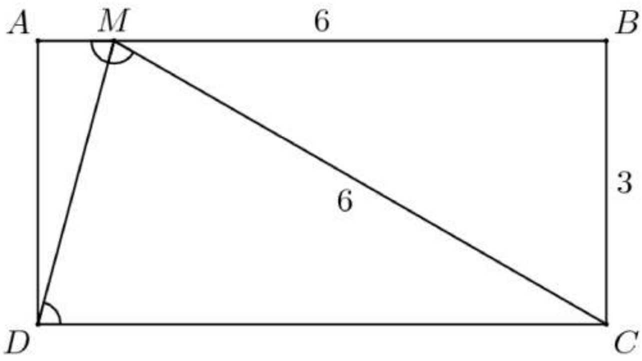
2011 AMC 10B Problems/Problem 18

Problem

Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Solution



It is given that $\angle AMD \cong \angle CMD$. Since $\angle AMD$ and $\angle CDM$ are alternate interior angles and $\overline{AB} \parallel \overline{DC}$, $\angle AMD \cong \angle CDM \longrightarrow \angle CMD \cong \angle CDM$. Use the Base Angle Theorem to show $\overline{DC} \cong \overline{MC}$. We know that $ABCD$ is a rectangle, so it follows that $\overline{MC} = 6$. We notice that $\triangle BMC$ is a $30-60-90$ triangle, and $\angle BMC = 30^\circ$. If we let x be the measure of $\angle AMD$, then

$$2x + 30 = 180$$
$$2x = 150$$
$$x = \boxed{\textbf{(E)}75}$$

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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Category: Introductory Geometry Problems

2011 AMC 10B Problems/Problem 19

Contents

Problem

What is the product of all the roots of the equation

$$\sqrt{5|x|+8}=\sqrt{x^2-16}.$$

$\textbf{(A)}\ -64 \qquad \textbf{(B)}\ -24 \qquad \textbf{(C)}\ -9 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 576$

Solution 1

First, square both sides, and isolate the absolute value.

$$\begin{aligned} 5|x|+8&=x^2-16 \\ 5|x|&=x^2-24 \\ |x|&=\frac{x^2-24}{5} \end{aligned}$$

Solve for the absolute value and factor.

$$x=\frac{x^2-24}{5} \longrightarrow 5x=x^2-24 \longrightarrow 0=x^2-5x-24 \longrightarrow (x-8)(x+3)=0 x=\frac{24-x^2}{5} \longrightarrow 5x=24-x^2 \longrightarrow 0=x^2+5x-24 \longrightarrow (x+8)(x-$$

$$x=-8,-3,3,8$$

However, this is not the final answer. Plug it back into the original equation to ensure it still works. Whether the number is positive or negative does not matter since the absolute value or square will cancel it out anyways.

$$\sqrt{5|3|+8}=\sqrt{3^2-16} \longrightarrow \sqrt{15+8}=\sqrt{9-16} \longrightarrow \sqrt{23}\neq\sqrt{-7}\sqrt{5|8|+8}=\sqrt{8^2-16} \longrightarrow \sqrt{40+8}=\sqrt{64-16} \longrightarrow \sqrt{48}=\sqrt{48}$$

The roots of this equation are -8 and 8 and product is $-8\times8=\boxed{\textbf{(A)}-64}$

Solution 2

Square both sides, to get $5|x|+8=x^2-16$. Rearrange to get $x^2-5|x|-24=0$. Seeing that $x^2=|x|^2$, substitute to get $|x|^2-5|x|-24=0$. We see that this is a quadratic in |x|. Factoring, we get $(|x|-8)(|x|+3)=0$, so $|x|=\{8,-3\}$. Since $|x|$ can't be negative, the sole solution is $|x|=8$. Therefore, the x can be 8 or -8. The product is then $-8\times8=\boxed{\textbf{(A)}-64}$.

See Also

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2011 AMC 10B Problems/Problem 20

The following problem is from both the 2011 AMC 12B #16 and 2011 AMC 10B #20, so both problems redirect to this page.

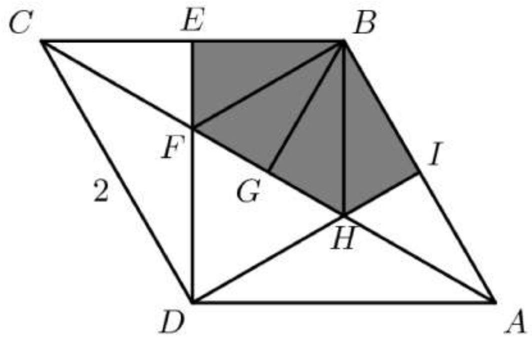
Problem

Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

$\textbf{(A)}\ \frac{\sqrt{3}}{3} \qquad\textbf{(B)}\ \frac{\sqrt{3}}{2} \qquad\textbf{(C)}\ \frac{2\sqrt{3}}{3} \qquad\textbf{(D)}\ 1+\frac{\sqrt{3}}{3} \qquad\textbf{(E)}\ 2$

Solution

Suppose that P is a point in the rhombus $ABCD$ and let ℓ_{BC} be the perpendicular bisector of $\overline{BC}$. Then $PB < PC$ if and only if P is on the same side of ℓ_{BC} as B. The line ℓ_{BC} divides the plane into two half-planes; let S_{BC} be the half-plane containing B. Let us define similarly ℓ_{BD}, S_{BD} and ℓ_{BA}, S_{BA}. Then R is equal to $ABCD \cap S_{BC} \cap S_{BD} \cap S_{BA}$. The region turns out to be an irregular pentagon. We can make it easier to find the area of this region by dividing it into four triangles:



Since $\triangle BCD$ and $\triangle BAD$ are equilateral, ℓ_{BC} contains D, ℓ_{BD} contains A and C, and ℓ_{BA} contains D. Then $\triangle BEF \cong \triangle BGF \cong \triangle BGH \cong \triangle BIH$ with $BE = 1$ and $EF = \frac{1}{\sqrt{3}}$ so $[BEF] = \frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{3}$. Multiply this by 4 and it turns out that the pentagon has area $\boxed{(C) \frac{2\sqrt{3}}{3}}$.

See Also

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2011 AMC 10B Problems/Problem 21

Contents

Problem

Brian writes down four integers whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

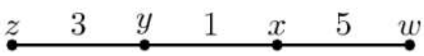
$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 93$

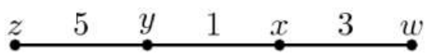
Solution

Solution 1

The largest difference, $9,$ must be between w and $z.$

The smallest difference, $1,$ must be directly between two integers. This also means the differences directly between the other two should add up to $8.$ The only remaining differences that would make this possible are 3 and $5.$ However, those two differences can't be right next to each other because they would make a difference of $8.$ This means 1 must be the difference between y and $x.$ We can express the possible configurations as the lines.





If we look at the first number line, you can express x as $w-5,$ y as $w-6,$ and z as $w-9.$ Since the sum of all these integers equal $44,$

$$\begin{aligned} w+w-5+w-6+w-9 &= 44 \\ 4w &= 64 \\ w &= 16 \end{aligned}$$

You can do something similar to this with the second number line to find the other possible value of $w.$

$$\begin{aligned} w+w-3+w-4+w-9 &= 44 \\ 4w &= 60 \\ w &= 15 \end{aligned}$$

The sum of the possible values of w is $16+15=\boxed{\textbf{(B)}\ 31}$

Solution 2

First, like Solution 1, we know that $w-z=9\ (1)$, because no sum could be smaller. Next, we find the sum of all the differences; since w is in the positive part of a difference 3 times, and has no differences where it contributes as the negative part, the sum of the differences includes $3w$. Continuing in this way, we find that

$$3w+x-y-3z=28\ (2)$$

. Now, we can subtract $3w-3z=27$ from (2) to get $x-y=1\ (3)$. Also, adding (2) with $w+x+y+z=44$ gives $4w+2x-2z=72$, or $2w+x-z=36$. Subtracting (1) from this gives $w+x=27$. Since we know $w-z$ and $x-y$, we find that

$$(w-z)+(x-y)=(w-y)+(x-z)=9+1=10$$

. This means that $w-y$ and $x-z$ must be 4 and 6, in some order. If $w-y=6$, then subtracting this from (3) gives $(w-y)-(x-y)=6-1=5$, so $w-x=5$. This means that $(w-x)+(w+x)=2w=27+5=32$, so $w=16$. Similarly, w can also equal 15.

Now if you are in a rush, you would have just answered $16+15=\boxed{\textbf{(B)}\ 31}$. But we do have to check if these work. In fact, they do, giving solutions $\boxed{16,11,10,7}$ and $\boxed{15,12,11,6}$.

See Also

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2011 AMC 10B Problems/Problem 22

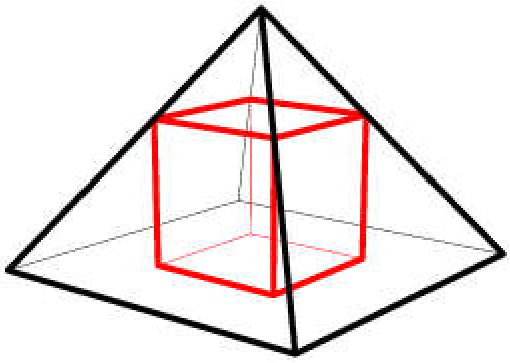
Problem

A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

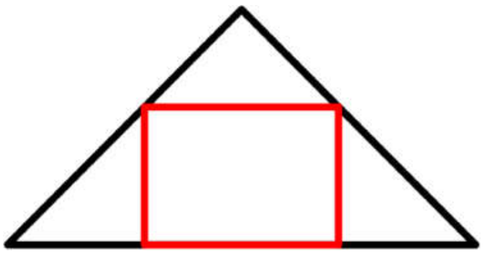
$\textbf{(A)}\ 5\sqrt{2} - 7 \qquad \textbf{(B)}\ 7 - 4\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{2}}{27} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{9} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{9}$

Solution

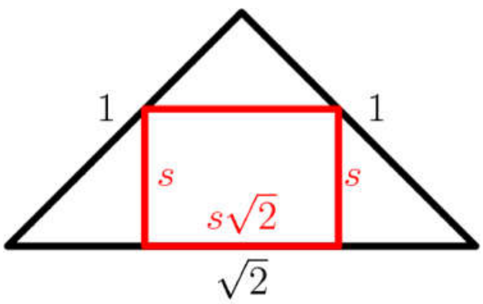
It is often easier to first draw a diagram for such a problem.



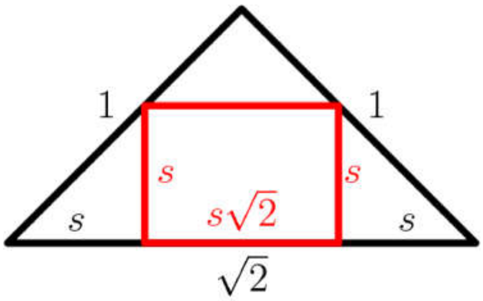
Sometimes, it may also be easier to think of the problem in 2D. Take a cross section of the pyramid through the apex and two points from the base that are opposite to each other. Place it in two dimensions.



The dimensions of this triangle are $1, 1,$ and $\sqrt{2}$ because the sidelengths of the pyramid are 1 and the base of the triangle is the diagonal of the pyramid's base. This is a $45-45-90$ triangle. Also, we can let the dimensions of the rectangle be s and $s\sqrt{2}$ because the longer side was the diagonal of the cube's base and the shorter cube was a side of the cube.



The two triangles on the right and left of the rectangle are also $45-45-90$ triangles because the rectangle is perpendicular to the base, and they share a 45° angle with the larger triangle. Therefore, the legs of the right triangles can be expressed as s.



Now we can just use segment addition to find the value of s.

$$\sqrt{2}=s+s\sqrt{2}+s=2s+s\sqrt{2}=(2+\sqrt{2})s$$

$$s=\frac{\sqrt{2}}{2+\sqrt{2}}=\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$$

The volume of the cube is

$$s^3=(\sqrt{2}-1)^3=(\sqrt{2}-1)(3-2\sqrt{2})=3\sqrt{2}-3-4+2\sqrt{2}=\boxed{\textbf{(A)} 5\sqrt{2}-7}$$

See Also

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2011 AMC 10B Problems/Problem 23

Contents

Problem

What is the hundreds digit of 2011^{2011}?

$\textbf{(A) } 1 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 5 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 9$

Solution 1

Since $2011 \equiv 11 \pmod{1000}$, we know that $2011^{2011} \equiv 11^{2011} \pmod{1000}$.

To compute this, we use a clever application of the binomial theorem.

$$\begin{aligned} 11^{2011} &= (1+10)^{2011} \\ &= 1 + \binom{2011}{1} \cdot 10 + \binom{2011}{2} \cdot 10^2 + \cdots \end{aligned}$$

In all of the other terms, the power of 10 is greater than 3 and so is equivalent to 0 modulo 1000, which means we can ignore it. We have:

$$\begin{aligned} 11^{2011} &\equiv 1 + 2011 \cdot 10 + \frac{2011 \cdot 2010}{2} \cdot 100 \\ &\equiv 1 + 20110 + \frac{11 \cdot 10}{2} \cdot 100 \\ &= 1 + 20110 + 5500 \\ &\equiv 1 + 110 + 500 \\ &= 611 \pmod{1000} \end{aligned}$$

Therefore, the hundreds digit is $\boxed{\textbf{(D) } 6}$.

Solution 2

We need to compute $2011^{2011} \pmod{1000}$. By the Chinese Remainder Theorem, it suffices to compute $2011^{2011} \pmod{8}$ and $2011^{2011} \pmod{125}$.

In modulo 8, we have $2011^4 \equiv 1 \pmod{8}$ by Euler's Theorem, and also $2011 \equiv 3 \pmod{8}$, so we have

$$2011^{2011} = (2011^4)^{502} \cdot 2011^3 \equiv 1^{502} \cdot 3^3 \equiv 3 \pmod{8}.$$

In modulo $125,$ we have $2011^{100} \equiv 1 \pmod{125}$ by Euler's Theorem, and also $2011 \equiv 11 \pmod{125}.$

Therefore, we have

$$\begin{aligned} 2011^{2011} &= (2011^{100})^{20} \cdot 2011^{11} \\ &\equiv 1^{20} \cdot 11^{11} \\ &= 121^5 \cdot 11 \\ &= (-4)^5 \cdot 11 = -1024 \cdot 11 \\ &\equiv -24 \cdot 11 = -264 \\ &\equiv 111 \pmod{125}. \end{aligned}$$

After finding the solution $2011^{2011} \equiv 611 \pmod{1000},$ we conclude it is the only one by the Chinese Remainder Theorem. Thus, the hundreds digit is $\boxed{\textbf{(D)}\ 6}$.

See Also

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2011 AMC 10B Problems/Problem 24

Problem

A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \le 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

$\textbf{(A)}\ \frac{51}{101} \qquad \textbf{(B)}\ \frac{50}{99} \qquad \textbf{(C)}\ \frac{51}{100} \qquad \textbf{(D)}\ \frac{52}{101} \qquad \textbf{(E)}\ \frac{13}{25}$

Solution

We see that for the graph of $y = mx + 2$ to not pass through any lattice points, the denominator of m must be greater than 100, or else it would be canceled by some $0 < x \le 100$ which would make y an integer. By using common denominators, we find that the order of the fractions from smallest to largest is $(A), (B), (C), (D), (E)$. We can see that when $x = \frac{50}{99}$, y would be an integer, so therefore any fraction greater than $\frac{50}{99}$ would not work, as substituting our fraction $\frac{50}{99}$ for m would produce an integer for y. So now we are left with only $\frac{51}{101}$ and $\frac{50}{99}$. But since $\frac{51}{101} = \frac{5049}{9999}$ and $\frac{50}{99} = \frac{5050}{9999}$, we can be absolutely certain that there isn't a number between $\frac{51}{101}$ and $\frac{50}{99}$ that can reduce to a fraction whose denominator is less than or equal to 100. Since we are looking for the maximum value of a, we take the larger of $\frac{51}{101}$ and $\frac{50}{99}$, which is $\boxed{\textbf{(B)}\ \frac{50}{99}}$.

See Also

2011 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2011))	
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2011 AMC 10B Problems/Problem 25

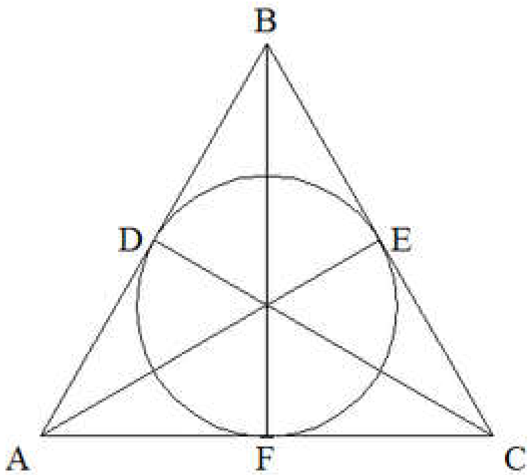
Problem

Let T_1 be a triangle with sides $2011, 2012,$ and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and $D, E,$ and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC and $AC,$ respectively, then T_{n+1} is a triangle with side lengths $AD, BE,$ and $CF,$ if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

$\textbf{(A)}\ \frac{1509}{8} \qquad \textbf{(B)}\ \frac{1509}{32} \qquad \textbf{(C)}\ \frac{1509}{64} \qquad \textbf{(D)}\ \frac{1509}{128} \qquad \textbf{(E)}\ \frac{1509}{256}$

Solution

By constructing the bisectors of each angle and the perpendicular radii of the incircle the triangle consists of 3 kites.



Hence $AD = AF$ and $BD = BE$ and $CE = CF$. Let $AD = x, BD = y$ and $CE = z$ gives three equations:

$x + y = a - 1$

$x + z = a$

$y + z = a + 1$

(where $a = 2012$ for the first triangle.)

Solving gives:

$x = \frac{a}{2} - 1$

$y = \frac{a}{2}$

$z = \frac{a}{2} + 1$

Subbing in gives that T_2 has sides of $1005, 1006, 1007$.

T_3 can easily be derivied from this as the sides still differ by 1 hence the above solutions still work (now with $a = 1006$). All additional triangles will differ by one as the solutions above differ by one so this process can be repeated indefinitely until the side lengths no longer form a triangle.

Subbing in gives T_3 with sides $502, 503, 504$.

T_4 has sides $\frac{501}{2}, \frac{503}{2}, \frac{505}{2}$.

T_5 has sides $\frac{499}{4}, \frac{503}{4}, \frac{507}{4}$.

T_6 has sides $\frac{495}{8}, \frac{503}{8}, \frac{511}{8}$.

T_7 has sides $\frac{487}{16}, \frac{503}{16}, \frac{519}{16}$.

T_8 has sides $\frac{471}{32}, \frac{503}{32}, \frac{535}{32}$.

T_9 has sides $\frac{439}{64}, \frac{503}{64}, \frac{567}{64}$.

T_{10} has sides $\frac{375}{128}, \frac{503}{128}, \frac{631}{128}$.

T_{11} would have sides $\frac{247}{256}, \frac{503}{256}, \frac{759}{256}$ but these lengths do not make a triangle as $\frac{247}{256} + \frac{503}{256} < \frac{759}{256}$.

Hence the perimeter is $\frac{375}{128} + \frac{503}{128} + \frac{631}{128} = \boxed{\textbf{(D)} \frac{1509}{128}}$

See Also

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