2012 AMC 10A

Problem 1

Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

Cagney 每 20 秒可以给一个蛋糕上霜，Lacey 每 30 秒可以给一个蛋糕上霜，他俩合作 5 分钟内可以给多少个蛋糕上霜？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 30$

Problem 2

A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles?

一个边长为 8 的正方形被分成两半，得到两个全等的矩形。问其中一个矩形的尺寸是多少?

$\textbf{(A)}\ 2 \text{ by } 4 \qquad \textbf{(B)}\ 2 \text{ by } 6 \qquad \textbf{(C)}\ 2 \text{ by } 8 \qquad \textbf{(D)}\ 4 \text{ by } 4 \qquad \textbf{(E)}\ 4 \text{ by } 8$

Problem 3

A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

一个虫子从-2 这个点开始沿着数轴爬行。它先爬到-6，然后掉头再爬到 5。那么这只虫子总共爬了多长?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Problem 4

Let $\angle ABC = 24^\circ$ and $\angle ABD = 20^\circ$. What is the smallest possible degree measure for angle CBD?

令$\angle ABC = 24^\circ$，$\angle ABD = 20^\circ$。问角 CBD 最少是多少度？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 12$

Problem 5

Last year 100 adult cats, half of whom were female, were brought into the Smallville Animal Shelter. Half of the adult female cats were accompanied by a litter of kittens. The average number of kittens per litter was 4. What was the total number of cats and kittens received by the shelter last year?

去年 100 只成年猫被带到 Smallville 动物收养所，其中一半是母猫，母猫中的一半各自带着一窝小猫，平均每窝有 4 只小猫，那么去年收养所收到的成年猫和小猫总共多少只？

$\textbf{(A)}\ 150 \qquad \textbf{(B)}\ 200 \qquad \textbf{(C)}\ 250 \qquad \textbf{(D)}\ 300 \qquad \textbf{(E)}\ 400$

Problem 6

The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

两个正数的乘积是 9。其中一个数的倒数是另一个数的倒数的 4 倍。问这两个数的和是多少?

$\textbf{(A)}\ \frac{10}{3} \qquad \textbf{(B)}\ \frac{20}{3} \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ \frac{15}{2} \qquad \textbf{(E)}\ 8$

Problem 7

In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

一包玻璃球有$\frac{3}{5}$是蓝色的，剩下的都是红色的。如果红色玻璃球的数量翻倍而蓝色的个数保持不变，那么最后总数的几分之几将是红色？

$\textbf{(A)}\ \frac{2}{5} \qquad \textbf{(B)}\ \frac{3}{7} \qquad \textbf{(C)}\ \frac{4}{7} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{4}{5}$

Problem 8

The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

三个整数两两求和结果分别为 12，17 和 19。问中间那个数是多少?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 9

A pair of six-sided dice are labeled so that one die has only even numbers (two each of 2, 4, and 6), and the other die has only odd numbers (two each of 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?

两个六面的骰子每个面都标了数字，满足其中一个骰子每个面都是偶数（数字是 2，4，6 的面各有 2 个），另一个骰子每个面都是奇数（数字是 1，3，5 的面各有 2 个）。掷了这一对骰子。问这两个骰子朝上的面的数字之和是 7 的概率是多少?

$\textbf{(A)}\ \frac{1}{6}\qquad \textbf{(B)}\ \frac{1}{5}\qquad \textbf{(C)}\ \frac{1}{4}\qquad \textbf{(D)}\ \frac{1}{3}\qquad \textbf{(E)}\ \frac{1}{2}$

Problem 10

Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

玛丽把一个圆分成了 12 个扇形。这些扇形的圆心角的度数都是整数且形成一个等差数列。问最小的那个扇形角度最小可能是多少度？

$\textbf{(A)}\ 5\qquad \textbf{(B)}\ 6\qquad \textbf{(C)}\ 8\qquad \textbf{(D)}\ 10\qquad \textbf{(E)}\ 12$

Problem 11

Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?

圆心在点 *A* 和 *B* 且半径分别是 5 和 3 的两个圆相外切，两圆的公切线和射线 *AB* 交于点 *C*。问 *BC* 长度是多少？

$\textbf{(A)}\ 4\qquad \textbf{(B)}\ 4.8\qquad \textbf{(C)}\ 10.2\qquad \textbf{(D)}\ 12\qquad \textbf{(E)}\ 14.4$

Problem 12

A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

如果某一年份能被 400 整除（例如 2000 年）或能被 4 整除但不能被 100 整除（例如 2012 年），那么这一年被称为闰年。小说家查尔斯狄更斯的 200 周年纪念日是 2012 年 2 月 7 日星期二。问狄更斯是星期几出生的?

$\textbf{(A)}$ Friday $\qquad \textbf{(B)}$ Saturday $\qquad \textbf{(C)}$ Sunday $\qquad \textbf{(D)}$ Monday $\qquad \textbf{(E)}$ Tuesday

Problem 13

An iterative average of the numbers 1, 2, 3, 4, and 5 is computed the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

数字 1，2，3，4，5 的选代平均值由以下方式计算：把这 5 个数字以某种顺序排列。先计算前 2 个数的平均值，然后算出这个平均值和第三个数的平均值，然后再算出新得到的平均值和第四个数的平均值，最后算出这个平均值和第五个数的平均值。问用这种方法最后得到的数的最大值和最小值之差为多少？

$\textbf{(A)}\ \frac{31}{16}\qquad \textbf{(B)}\ 2\qquad \textbf{(C)}\ \frac{17}{8}\qquad \textbf{(D)}\ 3\qquad \textbf{(E)}\ \frac{65}{16}$

Problem 14

Chubby makes nonstandard checkerboards that have 31 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?

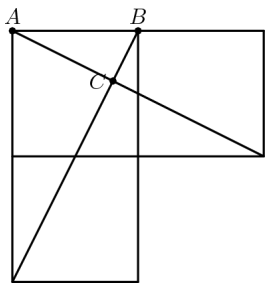
Chubby 做了一张非标准跳棋棋盘，棋盘的每条边有 31 个方块，每个角落都是黑色方块，并且每行每列都是红黑方块交替出现。问在这样一张跳棋棋盘上，总共有多少个黑色方块？

$\textbf{(A)}\ 480\qquad \textbf{(B)}\ 481\qquad \textbf{(C)}\ 482\qquad \textbf{(D)}\ 483\qquad \textbf{(E)}\ 484$

Problem 15

Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of $\triangle ABC$?

三个单位正方形和连接两对顶点的两条直线如下图所示，问$\triangle ABC$的面积是多少？



$\textbf{(A)}\ \frac{1}{6}\qquad \textbf{(B)}\ \frac{1}{5}\qquad \textbf{(C)}\ \frac{2}{9}\qquad \textbf{(D)}\ \frac{1}{3}\qquad \textbf{(E)}\ \frac{\sqrt{2}}{4}$

Problem 16

Three runners start running simultaneously from the same point on a 500-meter circular track. They each run clockwise around the course maintaining constant speeds of 4.4, 4.8, and 5.0 meters per second. The runners stop once they are all together again somewhere on the circular course. How many seconds do the runners run?

三个田径运动员在 500 米圆形跑道上的同一个起点同时开始跑。他们都沿着跑道顺时针方向分别以 4.4，4.8，5.0 米每秒的恒定速度跑。当这三个运动员在跑道的某一点又重新相遇时，就停止跑步，问他们跑了多少秒？

$\textbf{(A)}\ 1,000 \qquad \textbf{(B)}\ 1,250 \qquad \textbf{(C)}\ 2,500 \qquad \textbf{(D)}\ 5,000 \qquad \textbf{(E)}\ 10,000$

Problem 17

Let a and b be relatively prime positive integers with $a > b > 0$ and $\dfrac{a^3 - b^3}{(a-b)^3} = \dfrac{73}{3}$. What is $a - b$?

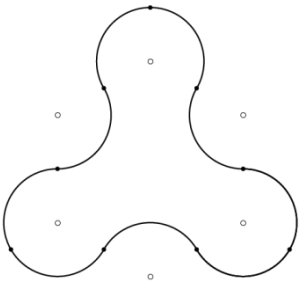
a，b 是互质的正整数，满足$a > b > 0$且$\dfrac{a^3 - b^3}{(a-b)^3} = \dfrac{73}{3}$，$a - b$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 18

The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?

图中所示的闭合曲线由 9 段全等的圆弧组成，每段圆弧长度均为 $\frac{2\pi}{3}$，且这 9 段圆弧所对应的圆心是一个边长为 2 的正六边形的顶点。问这段闭合曲线所包围的面积是多少?



$\textbf{(A)}\ 2\pi+6 \qquad \textbf{(B)}\ 2\pi+4\sqrt{3} \qquad \textbf{(C)}\ 3\pi+4 \qquad \textbf{(D)}\ 2\pi+3\sqrt{3}+2 \qquad \textbf{(E)}\ \pi+6\sqrt{3}$

Problem 19

Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ P.M. How long, in minutes, was each day's lunch break?

油漆匠 Paula 和她的两个助手每个人都以恒定且不同的速度粉刷油漆，他们总是在早上 8:00 开始工作，并且三个人中午吃饭的时长都相同，周一这一天这三人粉刷了房子的 50%，在下午 4:00 下班。周二 Paula 不在，两个助手总共仅粉刷了房子的 24%，下午 2:12 就下班了。周三 Paula 自己一个人工作，一直粉刷到晚上 7:12 把整个房子全部粉刷完。问每天的午餐时长是多少分钟？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 60$

Problem 20

A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability the grid is now entirely black?

一个 3x3 的正方形被分成 9 个单位正方形。每个单位正方形被涂成白色或者黑色，颜色的选择是等可能且是随机独立的。然后把整个大的正方形绕着它的中心顺时针旋转 90° 。如果白色正方形被转到了之前被黑色正方形占用的位置，那么这个白色正方形就被涂成黑色，所有其他正方形的颜色不变，问整个 3 x 3 的正方形现在是全黑的概率是多少？

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

Problem 21

Let points $A = (0,0,0)$, $B = (1,0,0)$, $C = (0,2,0)$, and $D = (0,0,3)$. Points E, F, G, and H are midpoints of line segments $\overline{BD}$, $\overline{AB}$, $\overline{AC}$, and $\overline{DC}$ respectively. What is the area of $EFGH$?

已知四个点： *A*=（0，0，0），*B*=（1，0，0），*C*=（0，2，0）， *D*=（0，0，3），点 *E*，*F*，*G*，*H* 分别是线段$\overline{BD}$, $\overline{AB}$, $\overline{AC}$,和$\overline{DC}$的中点，问四边形 *EFGH* 的面积是多少？

(A) $\sqrt{2}$ **(B)** $\frac{2\sqrt{5}}{3}$ **(C)** $\frac{3\sqrt{5}}{4}$ **(D)** $\sqrt{3}$ **(E)** $\frac{2\sqrt{7}}{3}$

Problem 22

The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n?

前 *m* 个正奇数之和比前 *n* 个正偶数之和多 212。问 *n* 的所有可能值之和是多少?

(A) 255 **(B)** 256 **(C)** 257 **(D)** 258 **(E)** 259

Problem 23

Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

Adam，Benin，Chiang，Deshawn，Esther 和 Fiona 都有网络账号。他们中的一些人（但不是所有）相互之间是网络朋友，并且他们都没有在这一组人之外的网络朋友。他们每个人都有相同数目的网络朋友，问他们一共可能有多少种不同的方式成为朋友？

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

Problem 24

Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

What is a?

a，b，c 是正整数，且 $a \geqslant b \geqslant c$，满足

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

求 a 的值。

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

Problem 25

Real numbers x, y, and z are chosen independently and at random from the interval $[0, n]$ for some positive integer n. The probability that no two of x, y, and z are within 1 unit of each other is greater than $\frac{1}{2}$. What is the smallest possible value of n?

实数 x，y，z 都是独立且随机的从区间［0，n］中选择，这里 n 是某个正整数，已知 x，y 和 z 相互之间的距离都大于 1 的概率大于 2。问 n 的最小可能值是多少?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

2012 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	E	E	C	B	D	C	D	D	C	D	A	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	B	C	C	E	D	A	C	A	B	E	D	