2012 AMC 10B Problems/Problem 1

Problem

Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

$\textbf{(A)}\ 48\qquad\textbf{(B)}\ 56\qquad\textbf{(C)}\ 64\qquad\textbf{(D)}\ 72\qquad\textbf{(E)}\ 80$

Solution

In each class, there are $18-2=16$ more students than rabbits. So for all classrooms, the difference between students and rabbits is $16 \times 4 = \boxed{\textbf{(C)}\ 64}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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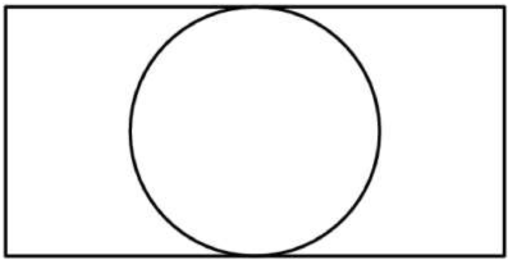
2012 AMC 10B Problems/Problem 2

Problem

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?



$\textbf{(A)}\ 50\qquad\textbf{(B)}\ 100\qquad\textbf{(C)}\ 125\qquad\textbf{(D)}\ 150\qquad\textbf{(E)}\ 200$

Solution

Note that the diameter of the circle is equal to the shorter side of the rectangle. Since the radius is 5, the diameter is $2 \cdot 5 = 10$. Since the sides of the rectangle are in a $2:1$ ratio, the longer side has length $2 \cdot 10 = 20$. Therefore the area is $20 \cdot 10 = 200$ or $\boxed{\textbf{(E)}\ 200}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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Category: Introductory Geometry Problems

2012 AMC 10B Problems/Problem 3

Problem

The point in the xy-plane with coordinates (1000, 2012) is reflected across the line $y = 2000$. What are the coordinates of the reflected point?

$\textbf{(A)}\ (998,2012)\qquad\textbf{(B)}\ (1000,1988)\qquad\textbf{(C)}\ (1000,2024)\qquad\textbf{(D)}\ (1000,4012)\qquad\textbf{(E)}\ (1012,2012)$

Solution

The line $y = 2000$ is a horizontal line located 12 units beneath the point $(1000, 2012)$. When a point is reflected about a horizontal line, only the y - coordinate will change. The x - coordinate remains the same. Since the y-coordinate of the point is 12 units above the line of reflection, the new y - coordinate will be $2000 - 12 = 1988$. Thus, the coordinates of the reflected point are $(1000, 1988)$. $\boxed{\textbf{(B)}}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 4

Problem 4

When Ringo places his marbles into bags with 6 marbles per bag, he has 4 marbles left over. When Paul does the same with his marbles, he has 3 marbles left over. Ringo and Paul pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be left over?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

In total, there were $3+4=7$ marbles left from both Ringo and Paul.We know that $7 \equiv 1 \pmod{6}$. This means that there would be 1 marble left over, or $\boxed{A}$.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 5

Problem 5

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of 27.50 dollars for dinner. What is the cost of her dinner without tax or tip in dollars?

$\textbf{(A)}\ 18 \qquad\textbf{(B)}\ 20 \qquad\textbf{(C)}\ 21 \qquad\textbf{(D)}\ 22 \qquad\textbf{(E)}\ 24$

Solutions

Let x be the cost of her dinner.

$$27.50 = x + \frac{1}{10} * x + \frac{3}{20} * x$$

$$27 + \frac{1}{2} = \frac{5}{4} * x$$

$$\frac{55}{2} = \frac{5}{4}x$$

$$\frac{55}{2} * \frac{4}{5} = x$$

$$\boxed{22 = x}$$

OR

$\textbf{(D)}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 6

Problem 6

In order to estimate the value of $x-y$ where x and y are real numbers with $x>y>0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

A) Her estimate is larger than $x-y$ B) Her estimate is smaller than $x-y$ C) Her estimate equals $x-y$ D) Her estimate equals $y-x$ E) Her estimate is 0

Solutions

Say Z=is the amount rounded up by and down by.

Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values.

Which translates to:

$(X+Z)-(Y-Z)$=$X+Z-Y+Z$=$X+2Z-Y$

This is $2Z$ bigger than the original amount of $X-Y$.

Therefore, her estimate is larger than $X-Y$

or

$\textbf{(A)}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 7

Problem 7

For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 54$

Solutions

Let x be the number of acorns that both animals had.

So by the info in the problem:

$$\frac{x}{3}=\left(\frac{x}{4}\right)+4$$

Subtracting $\frac{x}{4}$ from both sides leaves

$$\frac{x}{12}=4$$

$$\boxed{x=48}$$

This is answer choice $\textbf{(D)}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 8

Problem 8

What is the sum of all integer solutions to $1 < (x-2)^2 < 25$?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 25$

Solutions

$(x-2)^2$ = perfect square.

1< perfect square< 25

Perfect square can equal: 4, 9, or 16

Solve for x:

$(x-2)^2 = 4$

$x = 4, 0$

and

$(x-2)^2 = 9$

$x = 5, -1$

and

$(x-2)^2 = 16$

$x = 6, -2$

What is the sum of all integer solutions

$4+5+6+0+(-1)+(-2) = \boxed{12}$

OR

$\textbf{(B)}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 9

Problem 9

Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of odd integers among the 6 integers?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

Out of the first two integers, it's possible for both to be odd: for example, $11 + 15 = 26.$ But the next two integers, when added, increase the sum by $15,$ which is odd, so one of them must be odd and the other must be even: for example, $3 + 12 = 15.$ Finally, the next two integers increase the sum by $16,$ which is even, so we can have both be odd: for example, $9 + 7 = 16.$ Therefore, $\boxed{\textbf{(A)}\ 1}$ is the minimum number of integers that must be even.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 10

Problem 10

How many ordered pairs of positive integers (M,N) satisfy the equation $\frac{M}{6}=\frac{6}{N}$

$\textbf{(A)}\ 6 \qquad\textbf{(B)}\ 7 \qquad\textbf{(C)}\ 8 \qquad\textbf{(D)}\ 9 \qquad\textbf{(E)}\ 10$

Solution

Solution

$\frac{M}{6}=\frac{6}{N}$

is a ratio; therefore, you can cross-multiply.

$MN=36$

Now you find all the factors of 36:

$1\times 36=36$

$2\times 18=36$

$3\times 12=36$

$4\times 9=36$

$6\times 6=36$.

Now you can reverse the order of the factors for all of the ones listed above, because they are ordered pairs except for 6*6 since it is the same back if you reverse the order.

$4*2+1=9$

$\boxed{\textbf{(D)}\ 9}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 11

Problem 11

A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

$\textbf{(A)}\ 729 \qquad \textbf{(B)}\ 972 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 2187 \qquad \textbf{(E)}\ 2304$

Solution

Desserts must be chosen for 7 days: Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday.

There are 3 choices for dessert on Saturday: pie, ice cream, or pudding, as there must be cake on Friday and the same dessert may not be served two days in a row. Likewise, there are 3 choices for dessert on Thursday. Once dessert for Thursday is selected, there are 3 choices for dessert on Wednesday, once Wednesday's dessert is selected there are 3 choices for dessert on Tuesday, etc. Thus, there are 3 choices for dessert for each of 6 days, so the total number of possible dessert menus is 3^6, or $\boxed{\textbf{(A)}\ 729}$.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 12

Problem

Point B is due east of point A. Point C is due north of point B. The distance between points A and C is $10\sqrt{2}$, and $\angle BAC = 45^\circ$. Point D is 20 meters due north of point C. The distance AD is between which two integers?

$\textbf{(A)}\ 30\ \text{and}\ 31 \qquad\textbf{(B)}\ 31\ \text{and}\ 32 \qquad\textbf{(C)}\ 32\ \text{and}\ 33 \qquad\textbf{(D)}\ 33\ \text{and}\ 34 \qquad\textbf{(E)}\ 34\ \text{and}\ 35$

Solution

If point B is due east of point A and point C is due north of point B, $\angle CBA$ is a right angle. And if $\angle BAC = 45^\circ$, $\triangle CBA$ is a 45-45-90 triangle. Thus, the lengths of sides CB, BA, and AC are in the ratio $1:1:\sqrt{2}$, and CB is $10\sqrt{2} \div \sqrt{2} = 10$.

$\triangle DBA$ is clearly a right triangle with C on the side DB. DC is 20, so $DB = DC + CB = 20 + 10 = 30$.

By the Pythagorean Theorem, $DA = \sqrt{DB^2 + BA^2} = \sqrt{30^2 + 10^2} = \sqrt{1000}$.

$31^2 = 961$, and $32^2 = 1024$. Thus, $\sqrt{1000}$ must be between 31 and 32. The answer is $\boxed{B}$.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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Category: Introductory Geometry Problems

2012 AMC 10B Problems/Problem 13

Contents

Problem

It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

$\textbf{(A)}\ 36\qquad\textbf{(B)}\ 40\qquad\textbf{(C)}\ 42\qquad\textbf{(D)}\ 48\qquad\textbf{(E)}\ 52$

Solution 1

Let s be the speed of the escalator and c be the speed of Clea. Using $d = vt$, the first statement can be translated to the equation $d = 60c$. The second statement can be translated to $d = 24(c+s)$. Since the same distance is being covered in each scenario, we can set the two equations equal and solve for s. We find that $s = \dfrac{3c}{2}$. The problem asks for the time it takes her to ride down the escalator when she just stands on it. Since $t = \dfrac{d}{s}$ and $d = 60c$, we have $t = \dfrac{60c}{\dfrac{3c}{2}} = 40$ seconds. Answer choice $\boxed{B}$ is correct.

Solution 2

Let s be the speed of the escalator and c be the speed of Clea. Then Without loss of generality, assume that the length of the escalator be 1. Then $c = \dfrac{1}{60}$ and $c + s = \dfrac{1}{24}$, so $s = \dfrac{1}{24} - \dfrac{1}{60} = \dfrac{1}{40}$. Thus the time it takes for Clea to ride down the operating escalator when she just stands on it is $\dfrac{1}{\dfrac{1}{40}} = \boxed{\textbf{(B)}\ 40}$.

See Also

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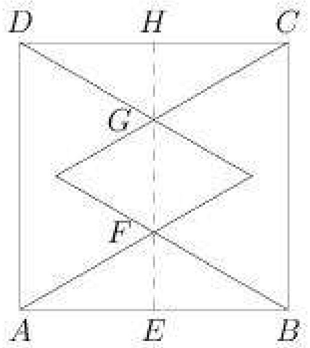
2012 AMC 10B Problems/Problem 14

Problem

Two equilateral triangles are contained in square whose side length is $2\sqrt{3}$. The bases of these triangles are the opposite side of the square, and their intersection is a rhombus. What is the area of the rhombus?

$\text{(A)}\ \frac{3}{2} \qquad \text{(B)}\ \sqrt{3} \qquad \text{(C)}\ 2\sqrt{2}-1 \qquad \text{(D)}\ 8\sqrt{3}-12 \qquad \text{(E)} \frac{4\sqrt{3}}{3}$

Solution



Observe that the rhombus is made up of two congruent equilateral triangles with side length equal to GF. Since AE has length $\sqrt{3}$ and triangle AEF is a 30-60-90 triangle, it follows that EF has length 1. By symmetry, HG also has length 1. Thus GF has length $2\sqrt{3}-2$. The formula for the area of an equilateral triangle of length s is $\frac{\sqrt{3}}{4}s^2$. It follows that the area of the rhombus is:

$$2 \times \frac{\sqrt{3}}{4}(2\sqrt{3}-2)^2 = 8\sqrt{3}-12$$

Thus, answer choice D is correct.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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Category: Introductory Geometry Problems

2012 AMC 10B Problems/Problem 15

Contents

Problem

In a round-robin tournament with 6 teams, each team plays one game against each other team, and each game results in one team winning and one team losing. At the end of the tournament, the teams are ranked by the number of games won. What is the maximum number of teams that could be tied for the most wins at the end on the tournament?

$\textbf{(A)}\ 2 \qquad\textbf{(B)}\ 3 \qquad\textbf{(C)}\ 4 \qquad\textbf{(D)}\ 5 \qquad\textbf{(E)}\ 6$

Solution

The total number of games in the tournament is $\frac{6\times5}{2}=15$. Here's a chart of 15 games:

```
|  1  2  3  4  5  6 |
|1 X  W  L  W  L  W |
|2 L  X  W  L  W  W |
|3 W  L  X  W  L  W |
|4 L  W  L  X  W  W |
|5 W  L  W  L  X  W |
|6 L  L  L  L  L  X |
```

The "X's" are for when it is where a team is set against itself, which cannot happen. The chart says that Team 6 has lost all of its matches, which means that each of the other teams won against it. Then, alternating Wins and Losses were tried. It shows that it is possible for 5 teams to tie for the same amount of wins, which in this case is 3 wins. Thus, the answer is $\boxed{\textbf{(D)}\ 5}$

Solution 2

Look above. Since there are 15 wins, and 15 losses in each of these games, you look and see that 15/3 = 5, so 5 teams can each have 3 wins, and since all the wins have been accounted for, you know that the last team must have lost all of their games.

Solution 2

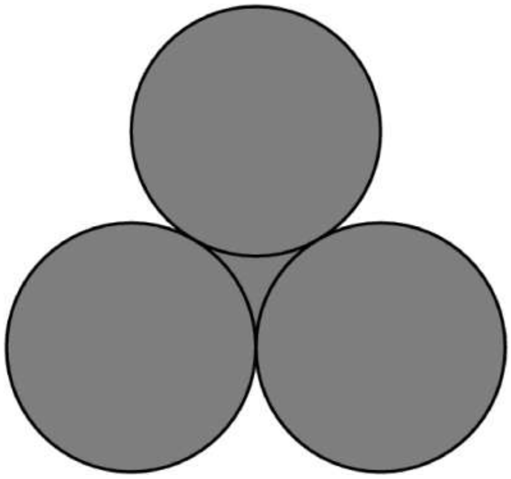
Look above. Since there are 15 wins, and 15 losses in each of these games, you look and see that 15/3 = 5, so 5 teams can each have 3 wins, and since all the wins have been accounted for, you know that the last team must have lost all of their games.

See Also

2012 AMC 10B Problems/Problem 16

Problem

Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?



$\textbf{(A)}\ 10\pi+4\sqrt{3}\qquad\textbf{(B)}\ 13\pi-\sqrt{3}\qquad\textbf{(C)}\ 12\pi+\sqrt{3}\qquad\textbf{(D)}\ 10\pi+9\qquad\textbf{(E)}\ 13\pi$

Solution

To determine the area of the figure, you can connect the centers of the circles to form an equilateral triangle with a side of length 4. We must find the area of this triangle to include the figure formed in between the circles. Since the equilateral triangle has two 30-60-90 triangles inside, we can find the height and the base of each 30-60-90 triangle from the ratios: $1:\sqrt{3}:2.$ The height is $2\sqrt{3}$ and the base is 2. Multiplying the height and base together with $\frac{1}{2}$, we get $2\sqrt{3}$. Since there are two 30-60-90 triangles in the equilateral triangle, we multiply the area of the 30-60-90 triangle by 2:

$$2\cdot2\sqrt{3}=4\sqrt{3}.$$

To find the area of the remaining sectors, which are $\frac{5}{6}$ of the original circles once we remove the triangle, we know that the sectors have a central angle of 300° since the equilateral triangle already covered that area. Since there are 3 $\frac{1}{6}$ pieces gone from the equilateral triangle, we have, in total, $\frac{1}{2}$ of a circle (with radius 2) gone. Each circle has an area of $\pi r^2 = 4pi$, so three circles gives a total area of 12π. Subtracting the half circle, we have:

$$12\pi-\frac{4\pi}{2}=12\pi-2\pi=10\pi.$$

Summing the areas from the equilateral triangle and the remaining circle sections gives us: $\boxed{\textbf{(A)} 10\pi+4\sqrt{3}}$.

See Also

2012 AMC 10B Problems/Problem 17

Problem

Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

$$\textbf{(A)}\ \frac{1}{8}\qquad\textbf{(B)}\ \frac{1}{4}\qquad\textbf{(C)}\ \frac{\sqrt{10}}{10}\qquad\textbf{(D)}\ \frac{\sqrt{5}}{6}\qquad\textbf{(E)}\ \frac{\sqrt{5}}{5}$$

Solution

Let's find the volume of the smaller cone first. We know that the circumference of the paper disk is 24π, so the circumference of the smaller cone would be $\frac{120}{360} \times 24\pi = 8\pi$. This means that the radius of the smaller cone is 4. Since the radius of the paper disk is 12, the slant height if the smaller cone would be 12. By the Pythagorean Theorem, the height of the cone is $\sqrt{12^2-4^2}=8\sqrt{2}$. Thus, the volume of the smaller cone is $\frac{1}{3} \times 4^2\pi \times 8\sqrt{2} = \frac{128\sqrt{2}}{3}\pi$.

Now, we need to find the volume of the larger cone. Using the same reason as above, we get that the radius is 8 and the slant height is 12. By the Pythagorean Theorem again, the height is $\sqrt{12^2-8^2}=4\sqrt{5}$. Thus, the volume of the larger cone is $\frac{1}{3} \times 8^2\pi \times 4\sqrt{5} = \frac{256\sqrt{5}}{3}\pi$.

The question asked for the ratio of the volume of the smaller cone to the larger cone. We need to find $\frac{\frac{128\sqrt{2}}{3}\pi}{\frac{256\sqrt{5}}{3}\pi}=\frac{\sqrt{10}}{10}$ after simplifying, or $\boxed{C}$.

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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Category: Introductory Geometry Problems

2012 AMC 10B Problems/Problem 18

Problem

Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive. For a person who does not have the disease, however, there is a 2% false positive rate--in other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease. Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p?

$\textbf{(A)}\ \frac{1}{98}\qquad\textbf{(B)}\ \frac{1}{9}\qquad\textbf{(C)}\ \frac{1}{11}\qquad\textbf{(D)}\ \frac{49}{99}\qquad\textbf{(E)}\ \frac{98}{99}$

Solution

This question can be solved by considering all the possibilities:

1 out of 500 people will have the disease and will be tested positive for the disease.

Out of the remaining 499 people, 2%, or 9.98 people, will be tested positive for the disease incorrectly.

Therefore, p can be found by taking $\frac{1}{1+9.98}$, which is closest to $\frac{1}{11}$, or $\boxed{\textbf{(C)}}$

See Also

2012 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2012))	
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2012 AMC 10B Problems/Problem 19

Problem

In rectangle $ABCD$, $AB=6$, $AD=30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of $BFDG$?

$\textbf{(A)}\ \frac{133}{2}\qquad\textbf{(B)}\ 67\qquad\textbf{(C)}\ \frac{135}{2}\qquad\textbf{(D)}\ 68\qquad\textbf{(E)}\ \frac{137}{2}$

Solution

The easiest way to find the area would be to find the area of $ABCD$ and subtract the areas of ABG and $CDF.$ You can easily get the area of ABG because you know $AB=6$ and $AG=15,$ so ABG's area is $15\cdot 6/2=45.$ However, for triangle $CDF,$ you don't know $CF.$ However, you can note that triangle BEF is similar to triangle CDF through AA. You see that $BE/DC=1/3.$ So, You can do $BF+3BF=30$ for $BF=15/2,$ and $CF=3BF=3(15/2)=45/2.$ Now, you can find the area of $CDF,$ which is $135/2.$ Now, you do $[ABCD]-[ABG]-[CDF]=180-45-135/2=135-135/2=\boxed{135/2}$, which is answer choice (C).

See Also

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Category: Introductory Geometry Problems

2012 AMC 12B Problems/Problem 14

Contents

Problem

Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Solution

Solution 1

The last number that Bernado says has to be between 950 and 999. Note that $1\rightarrow 2\rightarrow 52\rightarrow 104\rightarrow 154\rightarrow 308\rightarrow 358\rightarrow 716\rightarrow 766$ contains 4 doubling actions. Thus, we have $x \rightarrow 2x \rightarrow 2x+50 \rightarrow 4x+100 \rightarrow 4x+150 \rightarrow 8x+300 \rightarrow 8x+350 \rightarrow 16x+700$.

Thus, $950<16x+700<1000$. Then, $16x>250 \implies x \geq 16$. If $x=16$, we have $16x+700=956$. Working backwards from 956,

$$956 \rightarrow 478 \rightarrow 428 \rightarrow 214 \rightarrow 164 \rightarrow 82 \rightarrow 32 \rightarrow 16.$$

So the starting number is 16, and our answer is $1+6=\boxed{7}$, which is A.

Solution 2

Work backwards. The last number Bernardo produces must be in the range $[950,999]$. That means that before this, Silvia must produce a number in the range $[475,499]$. Before this, Bernardo must produce a number in the range $[425,449]$. Before this, Silvia must produce a number in the range $[213,224]$. Before this, Bernardo must produce a number in the range $[163,174]$. Before this, Silvia must produce a number in the range $[82,87]$. Before this, Bernardo must produce a number in the range $[32,37]$. Before this, Silvia must produce a number in the range $[16,18]$. Silvia could not have added 50 to any number before this to obtain a number in the range $[16,18]$, hence the minimum N is 16 with the sum of digits being $\boxed{\textbf{(A)}\ 7}$.

See Also

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2012 AMC 10B Problems/Problem 21

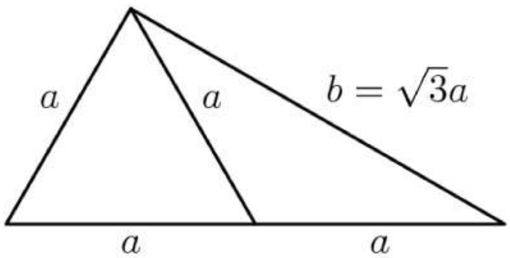
Problem

Four distinct points are arranged on a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

$\textbf{(A)}\ \sqrt{3}\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \sqrt{5}\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ \pi$

Solution

When you see that there are lengths a and 2a, one could think of 30-60-90 triangles. Since all of the other's lengths are a, you could think that $b=\sqrt{3}a$. Drawing the points out, it is possible to have a diagram where $b=\sqrt{3}a$. It turns out that $a, 2a,$ and b could be the lengths of a 30-60-90 triangle, and the other 3 a's can be the lengths of an equilateral triangle formed from connecting the dots. So, $b=\sqrt{3}a$, so $b:a=\sqrt{3}=(A)$



See Also

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Category: Introductory Geometry Problems

2012 AMC 10B Problems/Problem 22

Contents

Problem

Let $(a_1, a_2, \ldots a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 512 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 181,440 \qquad \textbf{(E)}\ 362,880$

Solution 1

If we have 1 as the first number, then the only possible list is $(1,2,3,4,5,6,7,8,9,10)$.

If we have 2 as the first number, then we have 9 ways to choose where the one goes, and the numbers ascend from the first number, 2, with the exception of the 1. For example, $(2,3,1,4,5,6,7,8,9,10)$, or $(2,3,4,1,5,6,7,8,9,10)$. There are $\binom{9}{1}$ ways to do so.

If we use 3 as the first number, we need to choose 2 spaces to be 2 and 1, respectively. There are $\binom{9}{2}$ ways to do this.

In the same way, the total number of lists is: $\binom{9}{0} + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \binom{9}{4} \ldots\ldots \binom{9}{9}$

By the binomial theorem, this is 2^9 = 512, or $\boxed{\textbf{(B)}}$

Solution 2

Arrange the spaces and put arrows pointing either up or down between them. Then for each arrangement of arrows there is one and only one list that corresponds to up. For example, all arrows pointing up is $(1,2,3,4,5\ldots10)$. There are 9 arrows, so the answer is 2^9 = 512 $\boxed{\textbf{(B)}}$

NOTE: Solution cited from: http://www.artofproblemsolving.com/Videos/external.php?video_id=269.

See Also

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2012 AMC 10B Problems/Problem 23

Problem

A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object?

$\textbf{(A)}\ \frac{\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{2\sqrt{2}}{3} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{2\sqrt{3}}{3} \qquad \textbf{(E)}\ \sqrt{2}$

Solution

This tetrahedron has the 4 vertices in these positions: on a corner (lets call this A) of the cube, and the other three corners (B, C, and D) adjacent to this corner. We can find the height of the remaining portion of the cube by finding the height of the tetrahedron. We can find the height of this tetrahedron in perspective to the equilateral triangle base (we call this height x) by finding the area of the tetrahedron in two ways. $\frac{1 \times 1}{2}$ is the area of the isosceles base of the tetrahedron. Multiply by the height, 1, and divide by 3, we have the volume of the tetrahedron as $\frac{1}{6}$. We set this area equal to one-third the product of our desired height and the area of the equilateral triangle base. First, find the area of the equilateral triangle: $[BCD] = \frac{\sqrt{2}^2 \times \sqrt{3}}{4} = \frac{\sqrt{3}}{2}$. So we have: $\frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot x = \frac{1}{6}$, and so $x = \frac{\sqrt{3}}{3}$.

Since we know what the height is, we can find the height of the remaining structure. The height of the structure if the tetrahedron was still on would simply be the space diagonal of the cube, $\sqrt{3}$, so we just subtract $\frac{\sqrt{3}}{3}$ from $\sqrt{3}$ to get $\frac{2\sqrt{3}}{3}$, or $\boxed{\textbf{(D)}}$.

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Category: Introductory Geometry Problems

2012 AMC 12B Problems/Problem 16

The following problem is from both the 2012 AMC 12B #16 and 2012 AMC 10B #24, so both problems redirect to this page.

Contents

Problem

Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

$\textbf{(A)}\ 108\qquad\textbf{(B)}\ 132\qquad\textbf{(C)}\ 671\qquad\textbf{(D)}\ 846\qquad\textbf{(E)}\ 1105$

Solutions

Solution 1

Let the ordered triple (a,b,c) denote that a songs are liked by Amy and Beth, b songs by Beth and Jo, and c songs by Jo and Amy. We claim that the only possible triples are $(1,1,1), (2,1,1), (1,2,1)(1,1,2)$.

To show this, observe these are all valid conditions. Second, note that none of a,b,c can be bigger than 3. Suppose otherwise, that $a=3$. Without loss of generality, say that Amy and Beth like songs 1, 2, and 3. Then because there is at least one song liked by each pair of girls, we require either b or c to be at least 1. In fact, we require either b or c to equal 1, otherwise there will be a song liked by all three. Suppose $b=1$. Then we must have $c=0$ since no song is liked by all three girls, a contradiction.

Case 1:How many ways are there for (a,b,c) to equal $(1,1,1)$? There are 4 choices for which song is liked by Amy and Beth, 3 choices for which song is liked by Beth and Jo, and 2 choices for which song is liked by Jo and Amy. The fourth song can be liked by only one of the girls, or none of the girls, for a total of 4 choices. So $(a,b,c)=(1,1,1)$ in $4\cdot3\cdot2\cdot4=96$ ways.

Case 2:To find the number of ways for $(a,b,c) = (2,1,1)$, observe there are $\binom{4}{2}=6$ choices of songs for the first pair of girls. There remain 2 choices of songs for the next pair (who only like one song). The last song is given to the last pair of girls. But observe that we let any three pairs of the girls like two songs, so we multiply by 3. In this case there are $6\cdot2\cdot3=36$ ways for the girls to like the songs.

That gives a total of $96+36=\boxed{132}$ ways for the girls to like the song, so the answer is $\textbf{(B)}$.

Solution 2

We begin by noticing that there are four ways to assign a song liked by both Amy and Beth, three ways to assign a song liked by both Amy and Jo (because Jo may not like the song liked by both Amy and Beth), and two ways to assign a song liked by both Beth and Jo (because both Beth and Jo may not like the song liked by the previous pairs. Additionally, there are $2\cdot2\cdot2=8$ ways to assign song preferences for the fourth song. Multiplying, we obtain an answer of $4\cdot3\cdot2\cdot8=192$. However, in doing so, we have committed an egregious error. We have in fact over counted the cases in which the fourth song is liked by two girls but not the third.

We proceed again by ignoring the cases in which the fourth song is liked by two girls but not the third. There are $4\cdot3\cdot2\cdot5=120$ of these cases. However, in doing so, we have committed yet another egregious error. The cases in which the fourth song is liked by two girls but not the third have not been accounted for!

In performing our past two calculations, we have, however, established that the answer has a lower bound of 120 and an upper bound of 192. As $\textbf{(B)}$ is the only answer within these bounds, we conclude that the answer must be $\textbf{(B)}$.

Solution 3: A Different Way of Looking at Solution 1

Let AB, BJ, and AJ denote a song that is liked by Amy and Beth (but not Jo), Beth and Jo (but not Amy), and Amy and Jo (but not Beth), respectively. Similarly, let $A, B, J,$ and N denote a song that is liked by only Amy, only Beth, only Jo, and none of them, respectively. Since we know that there is at least 1 AB, BJ, and AJ, they must be 3 songs out of the 4 that Amy, Beth, and Jo listened to. The fourth song can be of any type N, A, B, J, AB, BJ, and AJ (there is no ABJ because no song is liked by all three, as stated in the problem.) Therefore, we must find the number of ways to rearrange AB, BJ, AJ, and a song from the set $\{N, A, B, J, AB, BJ, AJ\}$.

Case 1: Fourth song = N, A, B, J

Note that in Case 1, all four of the choices for the fourth song are different from the first three songs.

Number of ways to rearrange = $(4!)$ rearrangements for each choice $* 4$ choices = 96.

Case 2: Fourth song = AB, BJ, AJ

Note that in Case 2, all three of the choices for the fourth song repeat somewhere in the first three songs.

Number of ways to rearrange = $(4!/2!)$ rearrangements for each choice $* 3$ choices = 36.

$96 + 36 = \boxed{\textbf{(B)}\ 132}$.

See Also

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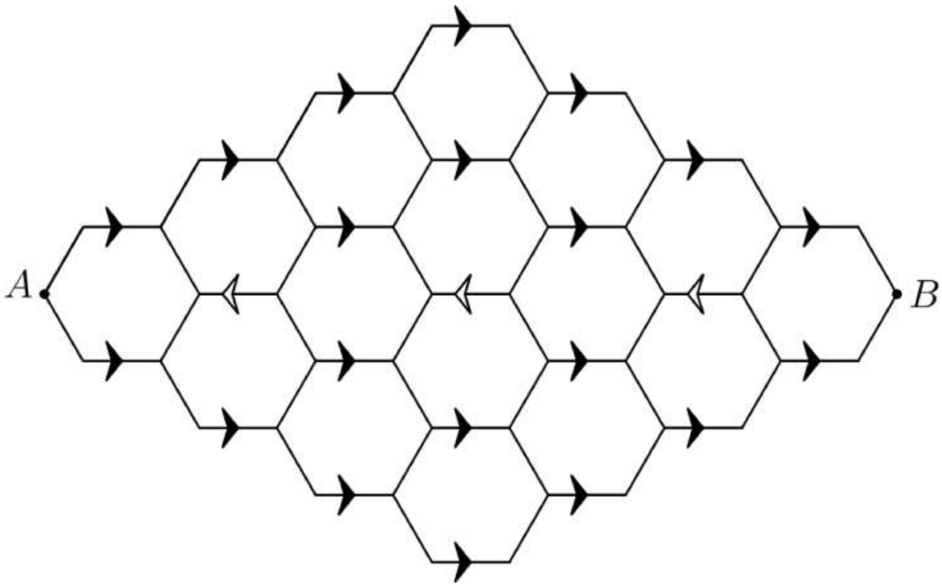
Category: Introductory Combinatorics Problems

2012 AMC 10B Problems/Problem 25

The following problem is from both the 2012 AMC 12B #22 and 2012 AMC 10B #25, so both problems redirect to this page.

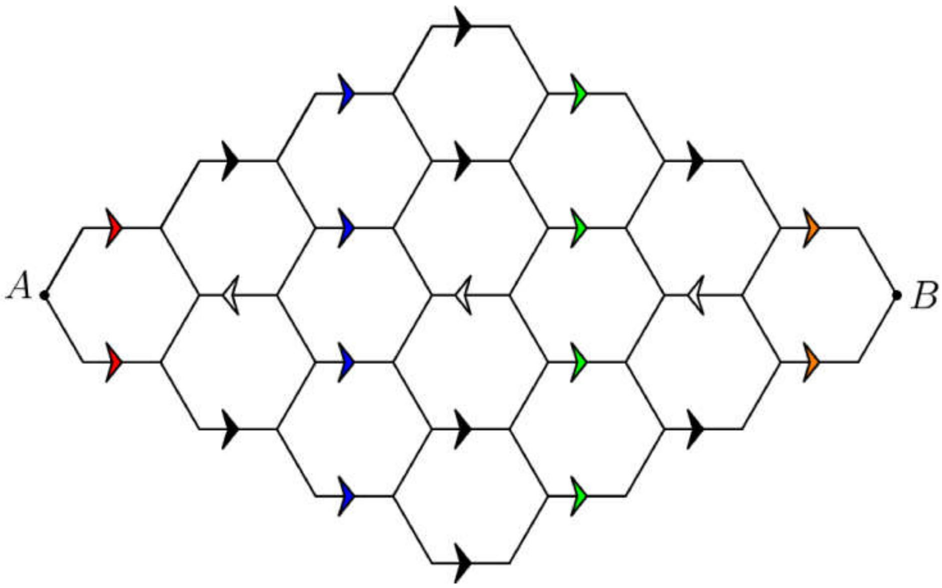
Problem 25

A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



$\textbf{(A)}\ 2112 \qquad \textbf{(B)}\ 2304 \qquad \textbf{(C)}\ 2368 \qquad \textbf{(D)}\ 2384 \qquad \textbf{(E)}\ 2400$

Solution 1

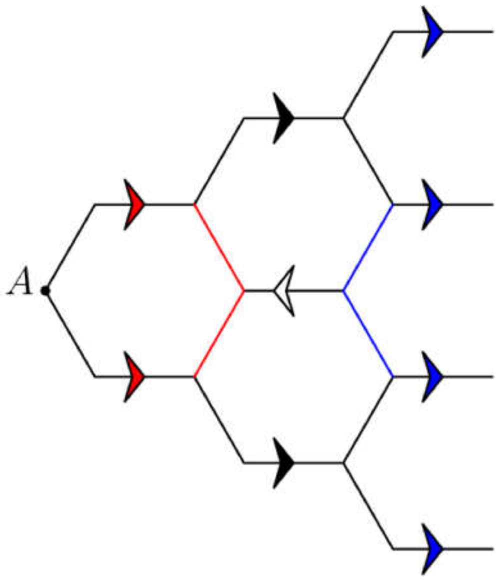


There is 1 way to get to any of the red arrows. From the first (top) red arrow, there are 2 ways to get to each of the first and the second (top 2) blue arrows; from the second (bottom) red arrow, there are 3 ways to get to each of the first and the second blue arrows. So there are in total 5 ways to get to each of the blue arrows.

From each of the first and second blue arrows, there are respectively 4 ways to get to each of the first and the second green arrows; from each of the third and the fourth blue arrows, there are respectively 8 ways to get to each of the first and the second green arrows. Therefore there are in total $5 \cdot (4+4+8+8) = 120$ ways to get to each of the green arrows.

Finally, from each of the first and second green arrows, there are respectively 2 ways to get to the first orange arrow; from each of the third and the fourth green arrows, there are 3 ways to get to the first orange arrow. Therefore there are $120 \cdot (2+2+3+3) = 1200$ ways to get to each of the orange arrows, hence 2400 ways to get to the point B. $\boxed{\textbf{(E)}\ 2400}$

Solution 2 (using the answer choices)



For every blue arrow, there are $2 \cdot 2 = 4$ ways to reach it without using the reverse arrow since the bug can choose any of 2 red arrows to pass through and 2 black arrows to pass through. If the bug passes through the white arrow, the red arrow that the bug travels through must be the closest to the first black arrow. Otherwise, the bug will have to travel through both red segments, which is impossible because now there is no path to take after the bug emerges from the reverse arrow. Similarly, with the blue segments, the second black arrow taken must be the one that is closest to the blue arrow that was taken. Also, it is trivial that the two black arrows taken must be different. Therefore, if the reverse arrow is taken, the blue arrow taken determines the entire path and there is 1 path for every arrow. Since the bug cannot return once it takes a blue arrow, the answer must be divisible by 5. $\boxed{\textbf{(E)}\ 2400}$ is the only answer that is.

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