2013 AMC 10A

Problem 1

A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?

出租车的起步价是 1.5 美元，每开 1 英里还需额外支付 0.25 美元，那么 5 英里的里程需要支付多少美元？

$\textbf{(A)}\ 2.25 \qquad \textbf{(B)}\ 2.50 \qquad \textbf{(C)}\ 2.75 \qquad \textbf{(D)}\ 3.00 \qquad \textbf{(E)}\ 3.75$

Problem 2

Alice is making a batch of cookies and needs $2\frac{1}{2}$ cups of sugar. Unfortunately, her measuring cup holds only $\frac{1}{4}$ cup of sugar. How many times must she fill that cup to get the correct amount of sugar?

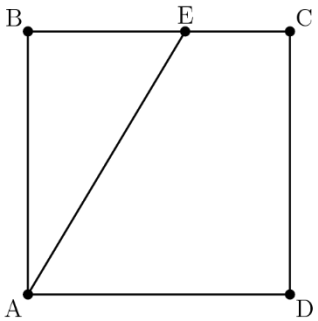
Alice 正在做一批曲奇饼，需要$2\frac{1}{2}$杯糖。不幸的是，她的量杯只能一次装满$\frac{1}{4}$杯的糖。那么她需要装满多少杯量杯才能得到所需要总量的糖？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 20$

Problem 3

Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

正方形 *ABCD* 的边长为 10。点 *E* 在$\overline{BC}$上，$\triangle ABE$的面积为 40。问 *BE* 的长是多少?



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 4

A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

一支垒球队打了 10 场比赛，得分为 1，2，3，4，5，6，7，8，9 和 10 分，其中恰好 5 场比赛比对手少 1 分，在其他 5 场比赛中，得分是对手分数的 2 倍，问他们的对手总共得分是多少？

$\textbf{(A)}\ 35 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 55$

Problem 5

Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid $\$105$, Dorothy paid $\$125$, and Sammy paid $\$175$. In order to share costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

Tom，Dorothy 和 Sammy 一起去度假，并且同意均摊费用。旅途中，Tom 支付了 105 美元，Dorothy 支付了 125 美元，Sammy 支付了 175 美元。为了平摊费用，Tom 给 Sammy 支付了 t 美元，Dorothy 给 Sammy 支付了 d 美元。问$t-d$是多少？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$

Problem 6

Joey and his five brothers are ages 3, 5, 7, 9, 11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

Joey 和他的 5 个兄弟的年龄是 3，5，7，9，11 和 13 岁，一天下午他的两个年龄之和为 16 的兄弟去看电影，另外 2 个年龄小于 10 岁的兄弟去打棒球，Jocy 和 5 岁的兄弟待在家里，问 Joey 多少岁？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Problem 7

A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

一个学生需要从一张课程单上选择 4 门课程组成一个课程项目，且这张课程单上有：英语，代数，几何，历史，艺术，和拉丁语这些课程可供选择。这个课程项目必须包含英语和至少一门数学课程，那么这个课程项目的课程有多少种选择方法？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 16$

Problem 8

What is the value of $\dfrac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}$?

下式的值是多少

$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ \frac{5}{3} \qquad \textbf{(D)}\ 2013 \qquad \textbf{(E)}\ 2^{4024}$

Problem 9

In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

在最近的篮球比赛中，Shenile 只尝试三分球和二分球，三分球的成功率是 20%，二分球的成功率是 30%。已知 Shenille 总共尝试了 30 次投篮。问她总共得了多少分?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

Problem 10

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

一束花包含粉色玫瑰，红色玫瑰，粉色康乃馨和红色康乃馨。粉色花的三分之一是玫瑰，红色花的四分之三是康乃馨，所有花的十分之六是粉色的。问所有花的百分之多少是康乃馨？

(A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

Problem 11

A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

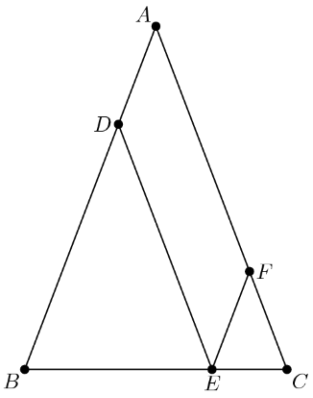
一个学生会需要从它的所有成员里选择一个 2 人的欢迎委员会和一个 3 人的计划委员会。恰好有 10 种方法可以为欢迎委员会选择 2 人。每个学生都可以服务欢迎委员会或者计划委员会。为计划委员会选择 3 人总共有多少种不同的方法?

(A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 25

Problem 12

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?

在$\triangle ABC$中， AB=AC=28，BC=20。点 D，E，F 分别在边$\overline{AB}$，$\overline{BC}$和$\overline{AC}$上，满足$\overline{DE}$和$\overline{EF}$分别平行于$\overline{AC}$和$\overline{AB}$，问平行四边形 $ADEF$ 的周长是多少？



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

Problem 13

How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

有多少个这样的三位数，满足：不能被 5 整除，各个位上数字之和小于 20，且第一个数字和第三个数字相等？

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 66 \qquad \textbf{(D)}\ 68 \qquad \textbf{(E)}\ 70$

Problem 14

A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

从一个边长为 3 的正方体的每个角落切掉一个边长为 1 的小正方体，所剩立体图形有多少条边？

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 72 \qquad \textbf{(D)}\ 84 \qquad \textbf{(E)}\ 108$

Problem 15

Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

一个三角形的两条边长度分别是 10 和 15。第三条边上的高的长度是另外两条已知边上高的长度的平均值。问第三条边有多长？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 18$

Problem 16

A triangle with vertices $(6, 5)$, $(8, -3)$, and $(9, 1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

一个顶点为（6，5），（8，−3）和（9，1）的三角形关于直线 x=8 对称后，得到第二个三角形。那么这 2 个三角形的并集所形成的图形的面积是多少？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ \frac{28}{3} \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ \frac{31}{3} \qquad \textbf{(E)}\ \frac{32}{3}$

Problem 17

Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

Daphne 定期被她的三个最好的朋友访问:Alice，Beatrix 和 Claire。Alice 每三天访问一次，Beatrix 每四天访问一次，Claire 每五天访问一次。昨天三个朋友都访问了 Daphne。问接下来的 365 天内，有多少天在那天恰好有 2 个朋友来访问她？

(A) 48 **(B)** 54 **(C)** 60 **(D)** 66 **(E)** 72

Problem 18

Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

已知四个点$A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$。四边形 *ABCD* 被一条通过 *A* 点的直线分成面积相等的两块区域。这条直线和$\overline{CD}$交于点$\left(\frac{p}{q}, \frac{r}{s}\right)$，这里的两个分数已经是最简分数。问$p + q + r + s$是多少？

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

Problem 19

In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

2013 的 10 进制以 3 结尾，若把 2013 写成 9 进制，则是$(2676)_9$，它以 6 结尾。若 2013 的 *b* 进制以 3 结尾，这样的正整数 *b* 有多少个？

(A) 6 **(B)** 9 **(C)** 13 **(D)** 16 **(E)** 18

Problem 20

A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

一个单位正方形（边长为 1 的正方形）绕着它的中心旋转 45° 。那么这个正方形的内部区域所扫过的区域的面积是多少?

$\textbf{(A)}\ 1-\frac{\sqrt{2}}{2}+\frac{\pi}{4}\qquad \textbf{(B)}\ \frac{1}{2}+\frac{\pi}{4}\qquad \textbf{(C)}\ 2-\sqrt{2}+\frac{\pi}{4}$

$\textbf{(D)}\ \frac{\sqrt{2}}{2}+\frac{\pi}{4}\qquad \textbf{(E)}\ 1+\frac{\sqrt{2}}{4}+\frac{\pi}{8}$

Problem 21

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The kth pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12th pirate receive?

12 个海盗同意以以下方案分一箱金币：第 k 个海盗拿走箱子里剩下金币的$\frac{k}{12}$。开始箱子里金币的总数量是这种分配方案能够使得每个海盗都能分得整数枚金币的最小正整数。问第 12 个海盗得到多少枚金币?

$\textbf{(A)}\ 720\qquad \textbf{(B)}\ 1296\qquad \textbf{(C)}\ 1728\qquad \textbf{(D)}\ 1925\qquad \textbf{(E)}\ 3850$

Problem 22

Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

6 个半径为 1 的球的球心位于一个边长为 2 的正六边形的 6 个顶点上，这 6 个球和一个球心在六边形中心的大球相内切。第 8 个球和这 6 个小球都外切，且和这个大球相内切。问这第 8 个球的半径是多少?

$\textbf{(A)}\ \sqrt{2}\qquad \textbf{(B)}\ \frac{3}{2}\qquad \textbf{(C)}\ \frac{5}{3}\qquad \textbf{(D)}\ \sqrt{3}\qquad \textbf{(E)}\ 2$

Problem 23

In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

$\triangle ABC$中， AB=86 ，AC=97。一个以 A 为圆心，AB 为半径的圆和线段$\overline{BC}$交于点 B 和 X. 并且线段$\overline{BX}$和$\overline{CX}$的长度都是整数，问 BC 是多长？

(A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

Problem 24

Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

市中高中和北方高中准备进行一场双陆棋比赛，每个学校有 3 名选手。比赛规则规定：每个学校的每个选手都要和另一个学校的每个选手打 2 场比赛。一共分 6 个回合，每个回合里有 3 场比赛同时进行。问有多少种安排比赛的方法?

(A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

Problem 25

All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

一个正八边形的 20 条对角线都连接了起来。那么在八边形的内部（不包括八边形的边上）有多少个不同的点，使得八边形的 2 条或者多条对角线相交于此点？

(A) 49 **(B)** 65 **(C)** 70 **(D)** 96 **(E)** 128

2013 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	E	C	B	D	C	C	B	E	A	C	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	D	E	B	B	C	C	D	B	D	E	A	