2013 AMC 10A Problems/Problem 1

Problem

A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?

$\textbf{(A)}\ 2.25 \qquad \textbf{(B)}\ 2.50 \qquad \textbf{(C)}\ 2.75 \qquad \textbf{(D)}\ 3.00 \qquad \textbf{(E)}\ 3.75$

Solution

There are five miles which need to be traveled. The cost of these five miles is $(0.25 \cdot 5) = 1.25$. Adding this to 1.50, we get $\boxed{\textbf{(C)}\ 2.75}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 2

Problem

Alice is making a batch of cookies and needs $2\frac{1}{2}$ cups of sugar. Unfortunately, her measuring cup holds only $\frac{1}{4}$ cup of sugar. How many times must she fill that cup to get the correct amount of sugar?

$\textbf{(A)}\ 8 \qquad\textbf{(B)}\ 10 \qquad\textbf{(C)}\ 12 \qquad\textbf{(D)}\ 16 \qquad\textbf{(E)}\ 20$

Solution

To get how many cups we need, we realize that we simply need to divide the number of cups needed by the number of cups collected in her measuring cup each time. Thus, we need to evaluate the fraction $\frac{2\frac{1}{2}}{\frac{1}{4}}$. Simplifying, this is equal to $\frac{5}{2}(4) = \boxed{\textbf{(B)}\ 10}$

See Also

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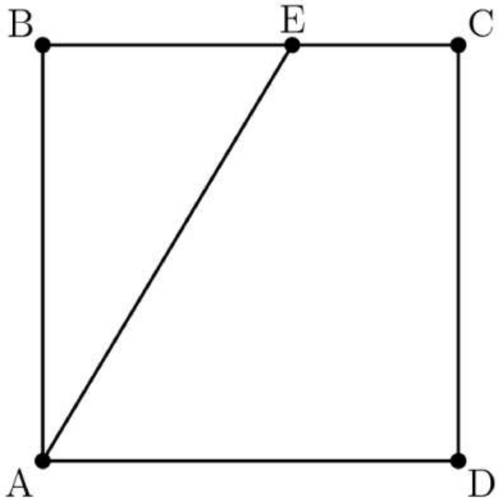
2013 AMC 10A Problems/Problem 3

Problem

Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Solution

We know that the area of $\triangle ABE$ is equal to $\dfrac{AB(BE)}{2}$. Plugging in $AB = 10$, we get $80 = 10BE$. Dividing, we find that $BE = \boxed{\textbf{(E)}\ 8}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 4

Problem

A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

$\textbf{(A)}\ 35 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 55$

Solution

We know that, for the games where they scored an odd number of runs, they cannot have scored twice as many runs as their opponents, as odd numbers are not divisible by 2. Thus, from this, we know that the five games where they lost by one run were when they scored 1, 3, 5, 7, and 9 runs, and the others are where they scored twice as many runs. We can make the following chart:

Them	Opponent
1	2
2	1
3	4
4	2
5	6
6	3
7	8
8	4
9	10
10	5

The sum of their opponent's scores is $2+1+4+2+6+3+8+4+10+5=\boxed{\textbf{(C)}\ 45}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 5

Contents

Problem

Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid $105, Dorothy paid $125, and Sammy paid $175. In order to share costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$

Solution 1

The total amount paid is $105+125+175=405$. To get how much each should have paid, we do $405/3=135$.

Thus, we know that Tom needs to give Sammy 30 dollars, and Dorothy 10 dollars. This means that $t-d=30-10=\boxed{\textbf{(B)}\ 20}$.

Solution 2

The difference in the money that Tom paid and Dorothy paid is 20. In order for them both to have paid the same amount, Tom must pay 20 more than Dorothy. The answer is $\boxed{(B) 20}$.

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 6

Problem

Joey and his five brothers are ages 3, 5, 7, 9, 11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Solution

Because the 5-year-old stayed home, we know that the 11-year-old did not go to the movies, as the 5-year-old did not and $11+5=16$. Also, the 11-year-old could not have gone to play baseball, as he is older than 10. Thus, the 11-year-old must have stayed home, so Joey is $\boxed{\textbf{(D)}\ 11}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 7

Contents

Problem

A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 8\qquad\textbf{(C)}\ 9\qquad\textbf{(D)}\ 12\qquad\textbf{(E)}\ 16$

Solution 1

Let us split this up into two cases.

Case 1: The student chooses both algebra and geometry.

This means that 3 courses have already been chosen. We have 3 more options for the last course, so there are 3 possibilities here.

Case 2: The student chooses one or the other.

Here, we simply count how many ways we can do one, multiply by 2, and then add to the previous.

WLOG assume the mathematics course is algebra. This means that we can choose 2 of History, Art, and Latin, which is simply $\dbinom{3}{2}=3$. If it is geometry, we have another 3 options, so we have a total of 6 options if only one mathematics course is chosen.

Thus, overall, we can choose a program in $6+3=\boxed{\textbf{(C)}\ 9}$ ways

Solution 2

We can use complementary counting. Since there must be an English class, we will add that to our list of classes for 3 remaining spots for the classes. We are also told that there needs to be at least one math class. This calls for complementary counting. The total number of ways of choosing 3 classes out of the 5 is $\dbinom{5}{3}$. The total number of ways of choosing only non-mathematical classes is $\dbinom{3}{3}$. Therefore the amount of ways you can pick classes with at least one math class is $\dbinom{5}{3}-\dbinom{3}{3}=10-1=\boxed{\textbf{(C)}\ 9}$ ways.

See Also

2013 AMC 10A Problems/Problem 8

Problem

What is the value of $\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}$?

$\textbf{(A)}\ -1\qquad\textbf{(B)}\ 1\qquad\textbf{(C)}\ \frac{5}{3}\qquad\textbf{(D)}\ 2013\qquad\textbf{(E)}\ 2^{4024}$

Solution

Factoring out, we get: $\frac{2^{2012}(2^2+1)}{2^{2012}(2^2-1)}$.

Cancelling out the 2^{2012} from the numerator and denominator, we see that it simplifies to $\boxed{\textbf{(C)}\ \frac{5}{3}}$.

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 9

Problem

In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

$\textbf{(A)}\ 12 \qquad\textbf{(B)}\ 18 \qquad\textbf{(C)}\ 24 \qquad\textbf{(D)}\ 30 \qquad\textbf{(E)}\ 36$

Solution

Let the number of attempted three-point shots made be x and the number of attempted two-point shots be y. We know that $x+y=30$, and we need to evaluate $(0.2\cdot3)x + (0.3\cdot2)y$, as we know that the three-point shots are worth 3 points and that she made 20% of them and that the two-point shots are worth 2 and that she made 30% of them.

Simplifying, we see that this is equal to $0.6x + 0.6y = 0.6(x+y)$. Plugging in $x+y=30$, we get $0.6(30) = \boxed{\textbf{(B)}\ 18}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 10

Problem

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

$\textbf{(A)}\ 15 \qquad\textbf{(B)}\ 30 \qquad\textbf{(C)}\ 40 \qquad\textbf{(D)}\ 60 \qquad\textbf{(E)}\ 70$

Solution

Let the total amount of flowers be x. Thus, the number of pink flowers is $0.6x$, and the number of red flowers is $0.4x$. The number of pink carnations is $\frac{2}{3}(0.6x) = 0.4x$ and the number of red carnations is $\frac{3}{4}(0.4x) = 0.3x$. Summing these, the total number of carnations is $0.4x + 0.3x = 0.7x$. Dividing, we see that $\frac{0.7x}{x} = 0.7 = \boxed{\textbf{(E)}\ 70\%}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 11

Problem

A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 25$

Solution

Let the number of students on the council be x. We know that there are $\binom{x}{2}$ ways to choose a two person team. This gives that $x(x-1) = 20$, which has a positive integer solution of 5.

If there are 5 people on the welcoming committee, then there are $\binom{5}{3} = \boxed{\textbf{(A)}\ 10}$ ways to choose a three-person committee.

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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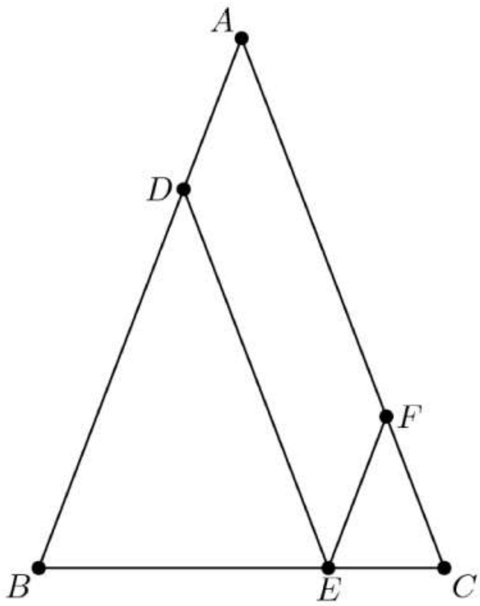
2013 AMC 10A Problems/Problem 12

Problem

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



$\textbf{(A) }48\qquad\textbf{(B) }52\qquad\textbf{(C) }56\qquad\textbf{(D) }60\qquad\textbf{(E) }72$

Solution

Note that because $\overline{DE}$ and $\overline{EF}$ are parallel to the sides of $\triangle ABC$, the internal triangles $\triangle BDE$ and $\triangle EFC$ are similar to $\triangle ABC$, and are therefore also isosceles triangles.

It follows that $BD = DE$. Thus, $AD + DE = AD + DB = AB = 28$.

Since opposite sides of parallelograms are equal, the perimeter is $2 * (AD + DE) = \boxed{\textbf{(C) }56}$.

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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2013 AMC 10A Problems/Problem 13

Problem

How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

$\textbf{(A)}\ 52 \qquad\textbf{(B)}\ 60 \qquad\textbf{(C)}\ 66 \qquad\textbf{(D)}\ 68 \qquad\textbf{(E)}\ 70$

Solution

These three digit numbers are of the form xyx. We see that $x \neq 0$ and $x \neq 5$, as $x = 0$ does not yield a three-digit integer and $x = 5$ yields a number divisible by 5.

The second condition is that the sum $2x + y < 20$. When x is 1, 2, 3, or 4, y can be any digit from 0 to 9, as $2x < 10$. This yields $10(4) = 40$ numbers.

When $x = 6$, we see that $12 + y < 20$ so $y < 8$. This yields 8 more numbers.

When $x = 7$, $14 + y < 20$ so $y < 6$. This yields 6 more numbers.

When $x = 8$, $16 + y < 20$ so $y < 4$. This yields 4 more numbers.

When $x = 9$, $18 + y < 20$ so $y < 2$. This yields 2 more numbers.

Summing, we get $40 + 8 + 6 + 4 + 2 = \boxed{\textbf{(B)}\ 60}$

See Also

<table>
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2013 AMC 10A Problems/Problem 14

Problem

A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

$\textbf{(A) }36\qquad\textbf{(B) }60\qquad\textbf{(C) }72\qquad\textbf{(D) }84\qquad\textbf{(E) }108$

Solution

We can use Euler's polyhedron formula that says that $F+V=E+2$. We know that there are originally 6 faces on the cube, and each corner cube creates 3 more. $6+8(3)=30$. In addition, each cube creates 7 new vertices while taking away the original 8, yielding $8(7)=56$ vertices. Thus $E+2=56+30$, so $E=\boxed{\textbf{(D) }84}$

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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Category: Introductory Geometry Problems

2013 AMC 10A Problems/Problem 15

Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

$\textbf{(A)}\ 6 \qquad\textbf{(B)}\ 8 \qquad\textbf{(C)}\ 9 \qquad\textbf{(D)}\ 12 \qquad\textbf{(E)}\ 18$

Solution 1 (Using the Answer Choices)

The shortest side length has the longest altitude perpendicular to it. The average of the two altitudes given will be between the lengths of the two altitudes, therefore the length of the side perpendicular to that altitude will be between 10 and 15. The only answer choice that meets this requirement is $\boxed{\textbf{(D)}\ 12}$.

Solution 2

Let the height to the side of length 15 be h_1, the height to the side of length 10 be h_2, the area be A, and the height to the unknown side be h_3.

Because the area of a triangle is $\frac{bh}{2}$, we get that $15(h_1) = 2A$ and $10(h_2) = 2A$, so, setting them equal, $h_2 = \frac{3h_1}{2}$. From the problem, we know that $2h_3 = h_1 + h_2$. Substituting, we get that

$$h_3 = 1.25h_1.$$

Thus, the side length is going to be $\frac{2A}{1.25h_1} = \frac{15}{\frac{5}{4}} = \boxed{\textbf{(D)}\ 12}$.

See Also

2013 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2013))	
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Category: Introductory Geometry Problems

2013 AMC 10A Problems/Problem 16

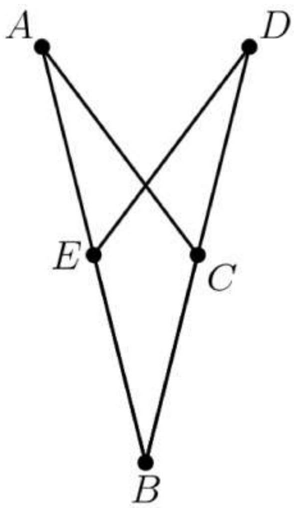
Problem

A triangle with vertices $(6,5)$, $(8,-3)$, and $(9,1)$ is reflected about the line $x=8$ to create a second triangle. What is the area of the union of the two triangles?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ \frac{28}{3} \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ \frac{31}{3} \qquad \textbf{(E)}\ \frac{32}{3}$

Solution

Let A be at $(6,5)$, B be at $(8,-3)$, and C be at $(9,1)$. Reflecting over the line $x=8$, we see that $A'=D=(10,5)$, $B'=B$ (as the x-coordinate of B is 8), and $C'=E=(7,1)$. Line AB can be represented as $y=-4x+29$, so we see that E is on line AB.



We see that if we connect A to D, we get a line of length 4 (between $(6,5)$ and $(10,5)$). The area of $\triangle ABD$ is equal to $\frac{bh}{2}=\frac{4(8)}{2}=16$.

Now, let the point of intersection between AC and DE be F. If we can just find the area of $\triangle ADF$ and subtract it from 16, we are done.

We realize that because the diagram is symmetric over $x=8$, the intersection of lines AC and DE should intersect at an x-coordinate of 8. We know that the slope of DE is $\frac{5-1}{10-7}=\frac{4}{3}$. Thus, we can represent the line going through E and D as $y-1=\frac{4}{3}(x-7)$. Plugging in $x=8$, we find that the y-coordinate of F is $\frac{7}{3}$. Thus, the height of $\triangle ADF$ is $5-\frac{7}{3}=\frac{8}{3}$. Using the formula for the area of a triangle, the area of $\triangle ADF$ is $\frac{16}{3}$.

To get our final answer, we must subtract this from 16. $[ABD]-[ADF]=16-\frac{16}{3}=\boxed{\textbf{(E)}\ \frac{32}{3}}$

See Also

2013 AMC 10A Problems/Problem 17

Problem

Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

$\textbf{(A)}\ 48\qquad\textbf{(B)}\ 54\qquad\textbf{(C)}\ 60\qquad\textbf{(D)}\ 66\qquad\textbf{(E)}\ 72$

Solution

The 365-day time period can be split up into 6 60-day time periods, because after 60 days, all three of them visit again (Least common multiple of 3, 4, and 5). You can find how many times each pair of visitors can meet by finding the LCM of their visiting days and dividing that number by 60. Remember to subtract 1, because you do not wish to count the 60th day, when all three visit.

A and B visit $\frac{60}{3\cdot4}-1=4$ times. B and C visit $\frac{60}{4\cdot5}-1=2$ times. C and A visit $\frac{60}{3\cdot5}-1=3$ times.

This is a total of 9 visits per 60 day period. Therefore, the total number of 2-person visits is $9\cdot6=\boxed{\textbf{(B)}\ 54}$.

See Also

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2013 AMC 10A Problems/Problem 18

Problem

Let points $A=(0,0)$, $B=(1,2)$, $C=(3,3)$, and $D=(4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $(\frac{p}{q},\frac{r}{s})$, where these fractions are in lowest terms. What is $p+q+r+s$?

$\textbf{(A)}\ 54 \qquad \textbf{(B)}\ 58 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 75$

Solution

First, we shall find the area of quadrilateral $ABCD$. This can be done in any of three ways:

Pick's Theorem: $[ABCD]=I+\frac{B}{2}-1=5+\frac{7}{2}-1=\frac{15}{2}$.

Splitting: Drop perpendiculars from B and C to the x-axis to divide the quadrilateral into triangles and trapezoids, and so the area is $1+5+\frac{3}{2}=\frac{15}{2}$.

Shoelace Theorem: The area is half of $|1\cdot 3-2\cdot 3-3\cdot 4|=15$, or $\frac{15}{2}$.

$[ABCD]=\frac{15}{2}$. Therefore, each equal piece that the line separates $ABCD$ into must have an area of $\frac{15}{4}$.

Call the point where the line through A intersects $\overline{CD}$ E. We know that $[ADE]=\frac{15}{4}=\frac{bh}{2}$. Furthermore, we know that $b=4$, as $AD=4$. Thus, solving for h, we find that $2h=\frac{15}{4}$, so $h=\frac{15}{8}$. This gives that the y coordinate of E is $\frac{15}{8}$.

Line CD can be expressed as $y=-3x+12$, so the x coordinate of E satisfies $\frac{15}{8}=-3x+12$. Solving for x, we find that $x=\frac{27}{8}$.

From this, we know that $E=(\frac{27}{8},\frac{15}{8})$. $27+15+8+8=\boxed{\textbf{(B)}\ 58}$

See Also

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2013 AMC 10A Problems/Problem 19

Problem

In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 18$

Solution

We want the integers b such that $2013 \equiv 3 \pmod{b} \Rightarrow b$ is a factor of 2010. Since $2010 = 2 \cdot 3 \cdot 5 \cdot 67$, it has $(1+1)(1+1)(1+1)(1+1) = 16$ factors. Since b cannot equal $1, 2,$ or 3, as these cannot have the digit 3 in their base representations, our answer is $16 - 3 = \boxed{\textbf{(C)}\ 13}$

See Also

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Category: Introductory Number Theory Problems

2013 AMC 10A Problems/Problem 20

Contents

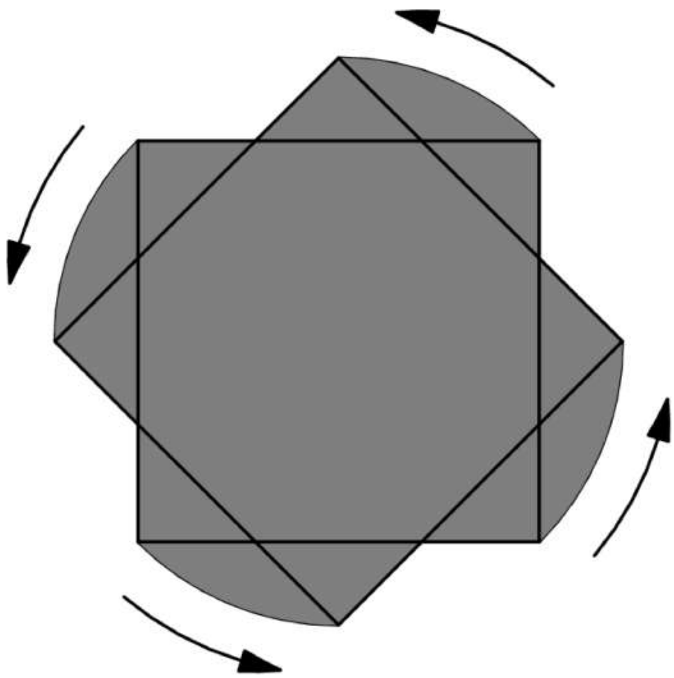
Problem

A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

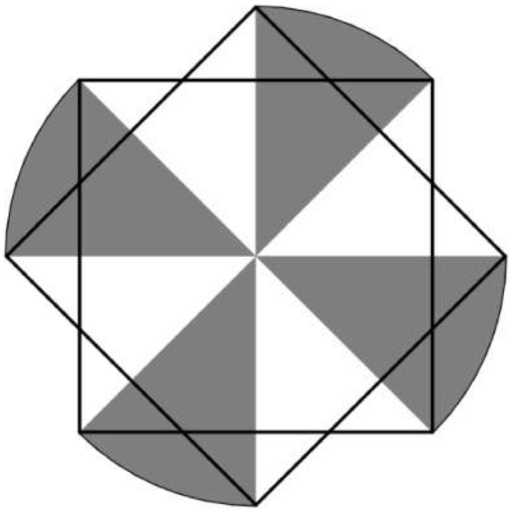
$$\textbf{(A)}\ 1-\frac{\sqrt2}{2}+\frac{\pi}{4}\qquad\textbf{(B)}\ \frac{1}{2}+\frac{\pi}{4}\qquad\textbf{(C)}\ 2-\sqrt2+\frac{\pi}{4}\qquad\textbf{(D)}\ \frac{\sqrt2}{2}+\frac{\pi}{4}\qquad\textbf{(E)}\ 1+\frac{\sqrt2}{4}+\frac{\pi}{8}$$

Solution 1

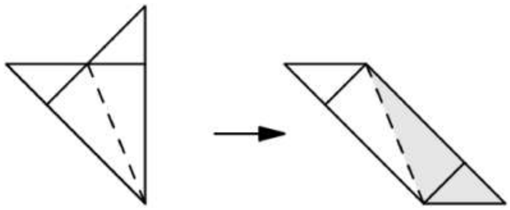
First, we need to see what this looks like. Below is a diagram.



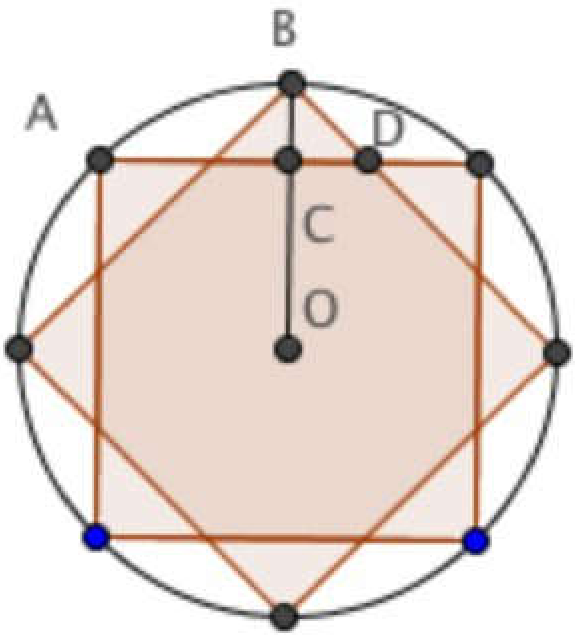
For this square with side length 1, the distance from center to vertex is $r=\frac{1}{\sqrt2}$, hence the area is composed of a semicircle of radius r, plus 4 times a parallelogram with height $\frac{1}{2}$ and base $\frac{\sqrt2}{2(1+\sqrt2)}$. That is to say, the total area is $\frac{1}{2}\pi(1/\sqrt2)^2+4\frac{\sqrt2}{4(1+\sqrt2)}=\boxed{\textbf{(C)}\ 2-\sqrt2+\frac{\pi}{4}}$.



(To turn each dart-shaped piece into a parallelogram, cut along the dashed line and flip over one half.)



Solution 2



Let O be the center of the square and C be the intersection of OB and $\overline{AD}$. The desired area consists of the unit square, plus 4 regions congruent to the region bounded by arc AB, $\overline{AC}$, and $\overline{BC}$, plus 4 triangular regions congruent to right triangle BCD. The area of the region bounded by arc AB, $\overline{AC}$, and $\overline{BC}$ is $\frac{\text{Area of Circle} - \text{Area of Square}}{28}$. Since the circle has radius $\frac{1}{\sqrt{2}}$, the area of the region is $\frac{\frac{\pi}{2}-1}{8}$, so 4 times the area of that region is $\frac{\pi}{4}-\frac{1}{2}$. Now we find the area of $\triangle BCD$. $BC = BO - OC = \frac{\sqrt{2}}{2} - \frac{1}{2}$

. Since $\triangle BCD$ is a $45-45-90$ right triangle, the area of $\triangle BCD$ is $\frac{BC^2}{2}=\frac{\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)^2}{2}$, so 4 times the area of $\triangle BCD$ is $\frac{3}{2}-\sqrt{2}$. Finally, the area of the whole region is $1+\left(\frac{3}{2}-\sqrt{2}\right)+\left(\frac{\pi}{4}-\frac{1}{2}\right)=\frac{\pi}{4}+2-\sqrt{2}$.

See Also

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Category: Introductory Geometry Problems

2013 AMC 10A Problems/Problem 21

Problem

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

$\textbf{(A)}\ 720\qquad\textbf{(B)}\ 1296\qquad\textbf{(C)}\ 1728\qquad\textbf{(D)}\ 1925\qquad\textbf{(E)}\ 3850$

Solution

Let x be the number of coins. After the k^{th} pirate takes his share, $\frac{12-k}{12}$ of the original amount is left. Thus, we know that

$x\cdot\frac{11}{12}\cdot\frac{10}{12}\cdot\frac{9}{12}\cdot\frac{8}{12}\cdot\frac{7}{12}\cdot\frac{6}{12}\cdot\frac{5}{12}\cdot\frac{4}{12}\cdot\frac{3}{12}\cdot\frac{2}{12}\cdot\frac{1}{12}$ must be an integer. Simplifying, we get

$x\cdot\frac{11}{12}\cdot\frac{5}{6}\cdot\frac{1}{2}\cdot\frac{7}{12}\cdot\frac{1}{2}\cdot\frac{5}{12}\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{6}\cdot\frac{1}{12}$. Now, the minimal x is the denominator of this fraction multiplied out, obviously. We mentioned before that this product must be an integer. Specifically, it is an integer and it is the amount that the 12^{th} pirate receives, as he receives $\frac{12}{12}=1=$ all of what is remaining.

Thus, we know the denominator is cancelled out, so the number of gold coins received is going to be the product of the numerators, $11\cdot 5\cdot 7\cdot 5=\boxed{\textbf{(D)}\ 1925}$.

See Also

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2013 AMC 10A Problems/Problem 22

Contents

Problem

Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

$\textbf{(A)}\ \sqrt{2}\qquad\textbf{(B)}\ \frac{3}{2}\qquad\textbf{(C)}\ \frac{5}{3}\qquad\textbf{(D)}\ \sqrt{3}\qquad\textbf{(E)}\ 2$

Solution 1

Set up an isosceles triangle between the center of the 8th sphere and two opposite ends of the hexagon. Then set up another triangle between the point of tangency of the 7th and 8th spheres, and the points of tangency between the 7th sphere and 2 of the original spheres on opposite sides of the hexagon. Express each side length of the triangles in terms of r (the radius of sphere 8) and h (the height of the first triangle). You can then use Pythagorean Theorem to set up two equations for the two triangles, and find the values of h and r.

$(1+r)^2=2^2+h^2$

$(3\sqrt{2})^2=3^2+(h+r)^2$

$r=\boxed{\textbf{(B)}\ \frac{3}{2}}$

Solution 2

We have a regular hexagon with side length 2 and six spheres on each vertex with radius 1 that are internally tangent, therefore, drawing radii to the tangent points would create this regular hexagon.

Imagine a 2D overhead view. There is a larger sphere which the 6 spheres are internally tangent to, with center in the center of the hexagon. To find the radius of the larger sphere we must first, either by prior knowledge or by deducing from the angle sum that the hexagon can be split into 6 equilateral triangles from its vertices, that the radius is two more than the small radius, or 3.

Now imagine the figure in three dimensions. The 8th sphere must be sitting atop of the 6 spheres, which is the only possibility for it to be tangent to all the 6 small spheres externally and the larger sphere internally. The ring of 6 small spheres is symmetrical and the 8th sphere will be resting atop it with its center aligned with the diameter of the large sphere.

We can now create a right triangle with one leg being the line from a vertex of the hexagon to the center of the hexagon and one leg being the line from the center of the 7th sphere to the center of the 8th sphere. Let the radius of our 8th sphere be r. As previously mentioned, the distance from the center of the hexagon to one of its vertices is 2. The distance between the centers will be $3-r$. The hypotenuse will be $r+1$.

We now have a right triangle. Applying the Pythagorean Theorem, $2^2+(3-r)^2=(1+r)^2$. Expanding and solving for r, we find $r=\frac{12}{8}=\boxed{\textbf{(B)} \frac{3}{2}}$.

See Also

2013 AMC 10A Problems/Problem 23

Contents

Problem

In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

$\textbf{(A)}\ 11\qquad\textbf{(B)}\ 28\qquad\textbf{(C)}\ 33\qquad\textbf{(D)}\ 61\qquad\textbf{(E)}\ 72$

Solution 1 (Power of a Point)

Let $BX = q$, $CX = p$, and AC meet the circle at Y and Z, with Y on AC. Then $AZ = AY = 86$. Using the Power of a Point (Secant-Secant Power Theorem), we get that $p(p+q) = 11(183) = 11 * 3 * 61$. We know that $p+q>p$, so p is either 3, 11, or 33. We also know that $p>11$ by the triangle inequality on $\triangle ACX$. p is 33. Thus, we get that $BC = p+q = \boxed{\textbf{(D) }61}$.

Solution 2 (Stewart's Theorem)

Let x represent CX, and let y represent BX. Since the circle goes through B and X, $AB = AX = 86$. Then by Stewart's Theorem,

$$xy(x+y) + 86^2 (x+y) = 97^2 y + 86^2 x.$$

$$x^2 y + xy^2 + 86^2 x + 86^2 y = 97^2 y + 86^2 x$$

$$x^2 + xy + 86^2 = 97^2$$

(Since y cannot be equal to 0, dividing both sides of the equation by y is allowed.)

$$x(x+y) = (97+86)(97-86)$$

$$x(x+y) = 2013$$

The prime factors of 2013 are 3, 11, and 61. Obviously, $x < x+y$. In addition, by the Triangle Inequality, $BC < AB + AC$, so $x+y < 183$. Therefore, x must equal 33, and $x+y$ must equal $\boxed{\textbf{(D) }61}$.

See Also

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2013 AMC 10A Problems/Problem 24

Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

$\textbf{(A)}\ 540 \qquad\textbf{(B)}\ 600 \qquad\textbf{(C)}\ 720 \qquad\textbf{(D)}\ 810 \qquad\textbf{(E)}\ 900$

Solution 1

Let us label the players of the first team A, B, and C, and those of the second team, X, Y, and Z.

1. One way of scheduling all six distinct rounds could be:

Round 1---->AX BY CZ
Round 2---->AX BZ CY
Round 3---->AY BX CZ
Round 4---->AY BZ CX
Round 5---->AZ BX CY
Round 6---->AZ BY CX

The above mentioned schedule ensures that each player of one team plays twice with each player from another team. Now you can generate a completely new schedule by permuting those 6 rounds and that can be done in $6!$=720 ways.

2. One can also make the schedule in such a way that two rounds are repeated.

(a)

Round 1---->AX BZ CY
Round 2---->AX BZ CY
Round 3---->AY BX CZ
Round 4---->AY BX CZ
Round 5---->AZ BY CX
Round 6---->AZ BY CX

(b)

Round 1---->AX BY CZ
Round 2---->AX BY CZ
Round 3---->AY BZ CX
Round 4---->AY BZ CX
Round 5---->AZ BX CY
Round 6---->AZ BX CY

As mentioned earlier any permutation of (a) and (b) will also give us a new schedule. For both (a) and (b) the number of permutations are $\frac{6!}{2!2!2!}$ = 90

So the total number of schedules is $720+90+90$ = $\boxed{\textbf{(E)} 900}$.

Solution 2

Label the players of the first team A, B, and C, and those of the second team, X, Y, and Z. We can start by assigning an opponent to person A for all 6 games. Since A has to play each of X, Y, and Z twice, there are $\frac{6!}{2!2!2!}=90$ ways to do this. We can assume that the opponents for A in the 6 rounds are X, X, Y, Y, Z, Z and multiply by 90 afterwards.

Notice that for every valid assignment of the opponents of A and B, there is only 1 valid assignment of opponents for C. More specifically, the opponents for C are the leftover opponents after the opponents for A and B are chosen in each round. Therefore, all we have to do is assign the opponents for B. This is the same as finding the number of permutations of X, Y, and Z that do not have a X in the first two spots, an Y in the next two spots, and a Z in the final two spots.

We can use casework to find this by using the fact that after we put down the X's and Y's first there is 1 way to put down the Z's (the two remaining spots).

If X's are put in the middle 2 spots, then there is 1 way to assign spots to Y, namely the last 2 spots. (If one of the last two spots are left empty, there will have to be a Z there, which which not valid).

If X's are put in the last 2 spots, then there is 1 way to assign spots to Y.

Finally, if one X was put in on of the middle two spots and one X was put in one of the last two spots, there are $2 * 2$ ways to assign spots to X and $2 * 1$ ways to assign spots to Y (one of the first two spots and the remaining spot in the last 2).

There are $1 + 1 + 2 \cdot 2 \cdot 2 \cdot 1 = 10$ ways to assign opponents to B and $90 \cdot 10 = 900$ ways to order the games. $\boxed{\textbf{(E)} 900}$

See Also

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Category: Introductory Combinatorics Problems

2013 AMC 10A Problems/Problem 25

Contents

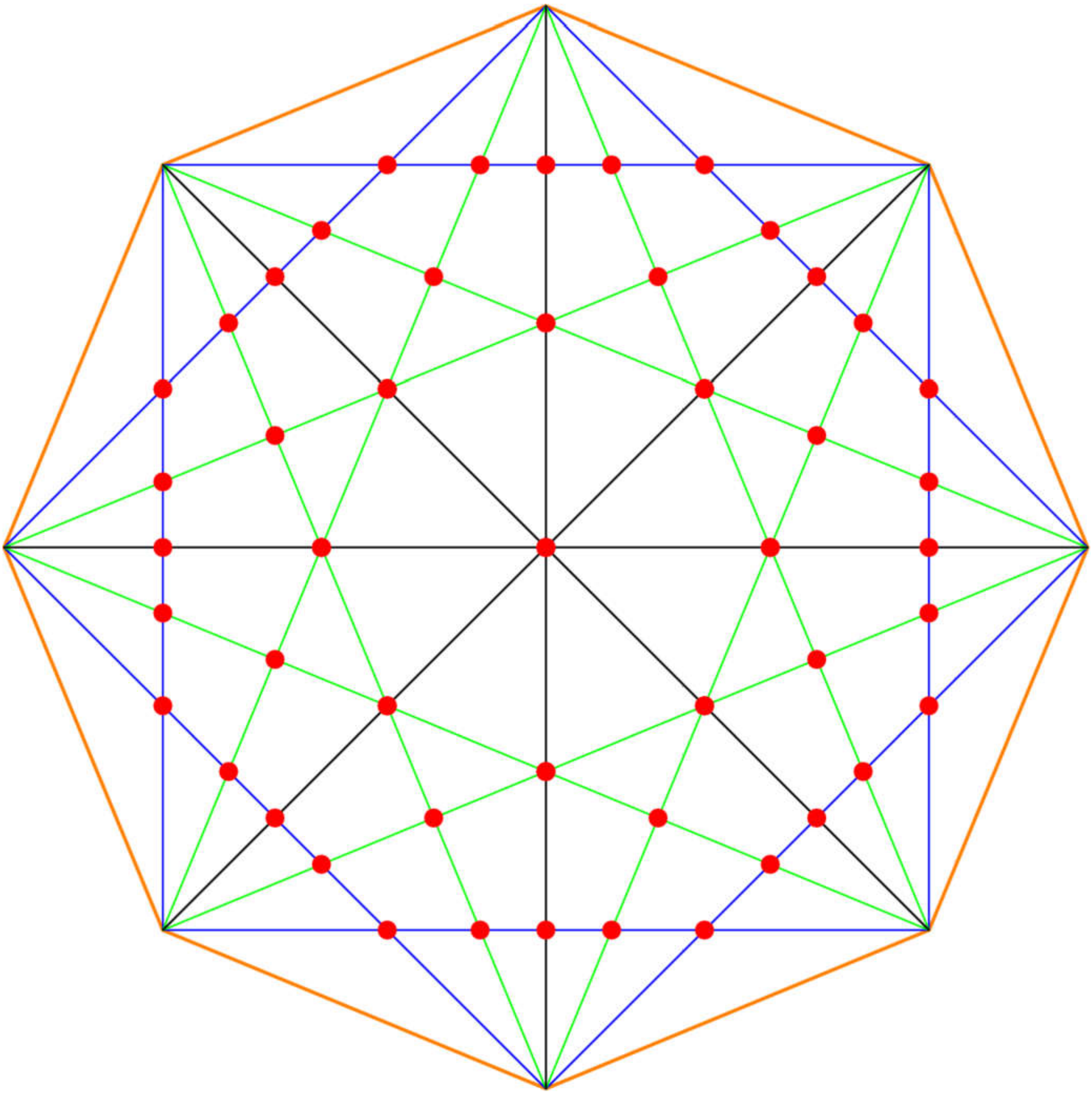
Problem

All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

$\textbf{(A)}\ 49 \qquad \textbf{(B)}\ 65 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 96 \qquad \textbf{(E)}\ 128$

Solution 1 (Drawing: The best way)

If you draw a good diagram like the one below, it is easy to see that there are $\boxed{\textbf{(A)}\ 49}$ points.



Solution 2 (Elimination)

Let the number of intersections be x. We know that $x \leq \binom{8}{4} = 70$, as every 4 points forms a quadrilateral with intersecting diagonals. However, four diagonals intersect in the center, so we need to subtract $\binom{4}{2} - 1 = 5$ from this count. $70 - 5 = 65$. Note that diagonals like $\overline{AD}$, $\overline{CG}$, and $\overline{BE}$ all intersect at the same point. There are 8 of this type with three diagonals intersecting at the same point, so we need to subtract 2 of the $\binom{3}{2}$ (one is kept as the actual intersection). In the end, we obtain $65 - 16 = \boxed{\textbf{(A)}\ 49}$.

See Also