2014 AMC 10B

Problem 1

Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

Leah 有 13 枚硬币，它们都是 1 分或者 5 分硬币，假设她比现在再多一枚 5 分硬币，那么她拥有的 5 分和 1 分硬币的数目就一样多了，问她现在所拥有的所有硬币总共价值多少？

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 35 \qquad \textbf{(C)}\ 37 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 41$

Problem 2

What is $\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$?

$\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$的值是多少？

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 64$

Problem 3

Peter drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles how long was Peter's trip?

Peter 在石子路上行驶了他总路程的前三分之一，接下来的 20 英里走的是石板路，最后剩下的总路程的五分之一走的是泥路。问 Peter 的总路程是多少英里？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ \dfrac{400}{11} \qquad \textbf{(C)}\ \dfrac{75}{2} \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ \dfrac{300}{7}$

Problem 4

Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

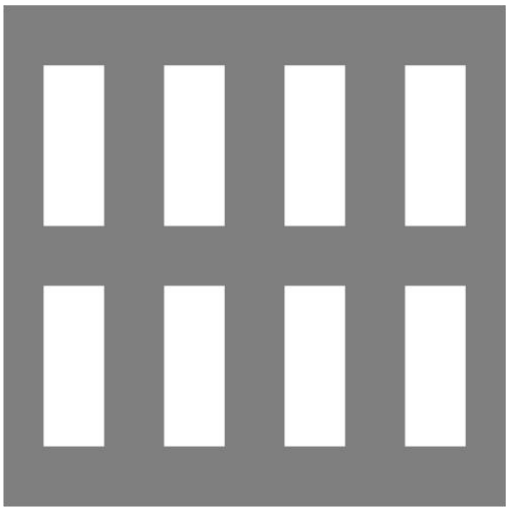
Susie 花钱买了 4 个玛芬蛋糕和 3 根香蕉。Calvin 花了 Susie 两倍的钱买了 2 个玛芬蛋糕和 16 根香蕉，问一个玛芬的价格是一根香蕉价格的几倍？

$\textbf{(A)}\ \frac{3}{2} \qquad \textbf{(B)}\ \frac{5}{3} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{13}{4}$

Problem 5

Camden constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5:2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?

Camden 用 8 块大小相同的玻璃做了一个正方形的窗户，如图所示，每块玻璃的高和宽之比为 5:2，并且玻璃周围以及玻璃之间的窗框的宽度是 2 英寸。正方形窗户的边长是多少英寸?



$\textbf{(A)}\ 26 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 34$

Problem 6

Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

Orvin 带着足够多的钱去商店买 30 只气球，当他到达商店，他发现商店正在对气球做促销降价活动：以原价买一只气球，那么第二只气球价格降价$\frac{1}{3}$，最多可买到多少只气球？

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 34 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 38 \qquad \textbf{(E)}\ 39$

Problem 7

Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

假设 *A*>*B*>0 且 *A* 比 *B* 大$x\%$。*x* 是多少？

$\textbf{(A)}\ 100\left(\frac{A-B}{B}\right) \qquad \textbf{(B)}\ 100\left(\frac{A+B}{B}\right) \qquad \textbf{(C)}\ 100\left(\frac{A+B}{A}\right) \qquad \textbf{(D)}\ 100\left(\frac{A-B}{A}\right) \qquad \textbf{(E)}\ 100\left(\frac{A}{B}\right)$

Problem 8

A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

一辆卡车每 t 秒可以行驶$\frac{b}{6}$英尺。一码等于 3 英尺。问 3 分钟内这辆卡车可以行驶多少码?

$\textbf{(A)}\ \frac{b}{1080t} \qquad \textbf{(B)}\ \frac{30t}{b} \qquad \textbf{(C)}\ \frac{30b}{t} \qquad \textbf{(D)}\ \frac{10t}{b} \qquad \textbf{(E)}\ \frac{10b}{t}$

Problem 9

For real numbers w and z, $\dfrac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}=2014$. What is $\dfrac{w+z}{w-z}$?

对于实数 w 和 z，满足 $\dfrac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}=2014$。$\dfrac{w+z}{w-z}$的值是多少？

$\textbf{(A)}\ -2014 \qquad \textbf{(B)}\ \dfrac{-1}{2014} \qquad \textbf{(C)}\ \dfrac{1}{2014} \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2014$

Problem 10

In the addition shown below $A, B, C,$ and D are distinct digits. How many different values are possible for D?

在如下图所示的加法中，A，B，C，D 都是不同的数字。D 有多少种不同的取值？

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 11

For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) two successive 15% discounts

(2) three successive 10% discounts

(3) a 25% discount followed by a 5% discount

What is the smallest possible positive integer value of n?

对于一个顾客来说，一次性降价 $n\%$ 比下面任何降价方案都有利：

（1）连续 2 次降价 15%

（2）连续 3 次降价 10%

（3）第一次降价 25%，第二次降价 5%

问 n 的最小可能的正整数值是多少？

$\textbf{(A)}\ 27 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 29 \qquad \textbf{(D)}\ 31 \qquad \textbf{(E)}\ 33$

Problem 12

The largest divisor of $2,014,000,000$ is itself. What is its fifth largest divisor?

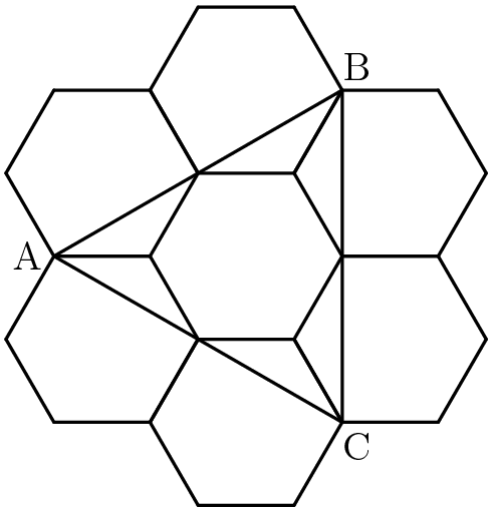
2，014，000，000 的最大因子是它自己，问它的第五大因子是多少？

(A) $125,875,000$ **(B)** $201,400,000$ **(C)** $251,750,000$ **(D)** $402,800,000$ **(E)** $503,500,000$

Problem 13

Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?

6 个正六边形把一个边长为 1 的正三角形包围了起来，如图所示。问$\triangle ABC$的面积是多少?



(A) $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

Problem 14

Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2$?

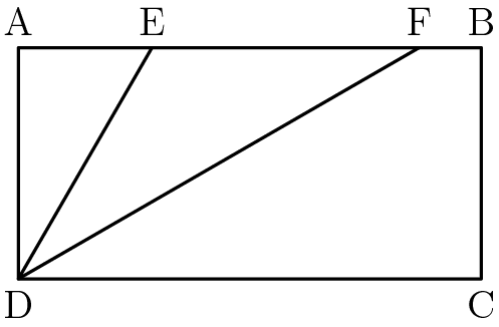
Danica 开着她的新车在路上行驶了整数个小时，平均速度为 55 英里每小时。一开始，里程表的读数是 *abc* 英里，这里 *abc* 是一个三位整数且 $a \geqslant 1$， $a+b+c \leqslant 7$ 行程结束后，里程表读数为 *cba* 英里。问$a^2+b^2+c^2$是多少？

(A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

Problem 15

In rectangle $ABCD$, $DC = 2 \cdot CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?

在长方形 *ABCD* 中，*DC*=2 • *CB*，点 *E* 和 *F* 在线段$\overline{AB}$上满足$\overline{ED}$和$\overline{FD}$三等分$\angle ADC$，如图所示.那么$\triangle DEF$的面积和矩形 *ABCD* 的面积的比值是多少？



(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{6}}{8}$ **(C)** $\frac{3\sqrt{3}}{16}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

Problem 16

Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

掷了 4 个标准的六面骰子，问 4 个骰子中至少有 3 个显示的数字相同的概率是多少？

(A) $\frac{1}{36}$ **(B)** $\frac{7}{72}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{6}$

Problem 17

What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

一个数是$10^{1002} - 4^{501}$的因子，且同时又是 2 的幂，问这个数最大是多少？

(A) 2^{1002} **(B)** 2^{1003} **(C)** 2^{1004} **(D)** 2^{1005} **(E)** 2^{1006}

Problem 18

A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

11 个正整数形成的一组数，其平均值是 10，中位数是 9，唯一的众数是 8。这组数里最大可能的整数是多少?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 31 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 35$

Problem 19

Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

两个同心圆半径分别是 1 和 2。从外圆上随机均匀的选择 2 个点，这 2 个点的连线和内圆相交的概率是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{2-\sqrt{2}}{2} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 20

For how many integers x is the number $x^4 - 51x^2 + 50$ negative?

使得$x^4 - 51x^2 + 50$为负数的整数 x 有多少个？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 16$

Problem 21

Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles at A and B are acute. What is the length of the shorter diagonal of $ABCD$?

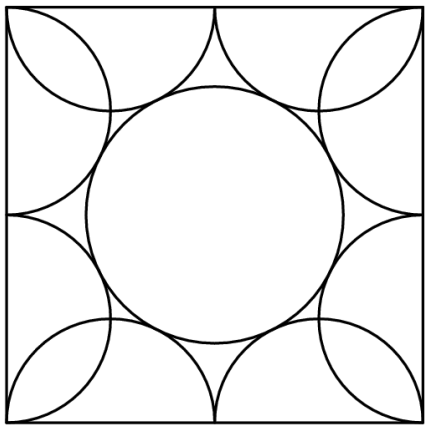
梯形 *ABCD* 两条平行的边$\overline{AB}$和$\overline{CD}$的长度分别是 33 和 21。另外两条边的长度是 10 和 14。位于顶点 *A* 和 *B* 处的角是锐角。则 *ABCD* 较短的那条对角线的长度是多少?

$\textbf{(A)}\ 10\sqrt{6} \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 8\sqrt{10} \qquad \textbf{(D)}\ 18\sqrt{2} \qquad \textbf{(E)}\ 26$

Problem 22

Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?

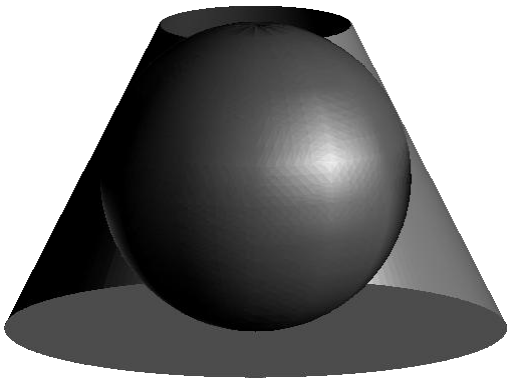
8 个半圆如图所示在边长为 2 的正方形的内部沿着正方形的边分布。求与这些半圆都相切的圆的半径是多少?



(A) $\frac{1+\sqrt{2}}{4}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{3}+1}{4}$ (D) $\frac{2\sqrt{3}}{5}$ (E) $\frac{\sqrt{5}}{3}$

Problem 23

A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



一个球与一个被切掉顶部的正圆锥相内切，如图所示，这个被切掉顶部的正圆锥的体积是球体积的 2 倍。那么这个被切掉顶部的正圆锥的下底面的半径和上底面的半径之比是多少?

(A) $\frac{3}{2}$ (B) $\frac{1+\sqrt{5}}{2}$ (C) $\sqrt{3}$ (D) 2 (E) $\frac{3+\sqrt{5}}{2}$

Problem 24

The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

现在要将数字 1，2，3，4，5 排列到一个圆上，有这样一个要求：对于 1 到 15 的每一个 *n* 来说，我们都能找到在圆上连续出现的 1 个或多个数，它们的和为 *n*。若对于某一种排列这个要求无法满足，那么就称这种排列是坏的，如果一种排列可以通过旋转或者对称得到另外一种排列，那么这两种排列被认为是一样的。问一共有多少种不同的坏的排列？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 25

In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

在一个池塘里，一共有 11 个百合花叶片排成一行，编号为 0 到 10。一只青蛙坐在叶片 1 上，当青蛙在叶片 *N* 上时，0<*N*<10，它就会以$\frac{N}{10}$的概率跳到叶片$N+1$上。每一跳相互之间都是独立的，和前一跳无关，若青蛙跳到叶片 0 上，就会被一条在那里静静等待的蛇吃掉。若青蛙跳到叶片 10 上，它就会离开池塘，永不回来，问青蛙能够逃走而不被蛇吃掉的概率是多少？

$\textbf{(A)}\ \frac{32}{79} \qquad \textbf{(B)}\ \frac{161}{384} \qquad \textbf{(C)}\ \frac{63}{146} \qquad \textbf{(D)}\ \frac{7}{16} \qquad \textbf{(E)}\ \frac{1}{2}$

2014 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	E	B	A	C	A	E	A	C	C	C	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	A	B	D	E	D	C	B	B	E	B	C	