2014 AMC 10A Problems/Problem 1

Problem

What is $10 \cdot \left(\tfrac{1}{2} + \tfrac{1}{5} + \tfrac{1}{10}\right)^{-1}?$

$\textbf{(A)}\ 3 \qquad\textbf{(B)}\ 8 \qquad\textbf{(C)}\ \frac{25}{2} \qquad\textbf{(D)}\ \frac{170}{3} \qquad\textbf{(E)}\ 170$

Solution

We have

$$10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$$

Making the denominators equal gives

$$\implies 10 \cdot \left(\frac{5}{10} + \frac{2}{10} + \frac{1}{10}\right)^{-1}$$

$$\implies 10 \cdot \left(\frac{5+2+1}{10}\right)^{-1}$$

$$\implies 10 \cdot \left(\frac{8}{10}\right)^{-1}$$

$$\implies 10 \cdot \left(\frac{4}{5}\right)^{-1}$$

$$\implies 10 \cdot \frac{5}{4}$$

$$\implies \frac{50}{4}$$

Finally, simplifying gives

$$\implies \boxed{\textbf{(C)}\ \frac{25}{2}}$$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 2

Problem

Roy's cat eats $\frac{1}{3}$ of a can of cat food every morning and $\frac{1}{4}$ of a can of cat food every evening. Before feeding his cat on Monday morning, Roy opened a box containing 6 cans of cat food. On what day of the week did the cat finish eating all the cat food in the box?

$\textbf{(A)}\ \text{Tuesday} \qquad \textbf{(B)}\ \text{Wednesday} \qquad \textbf{(C)}\ \text{Thursday} \qquad \textbf{(D)}\ \text{Friday} \qquad \textbf{(E)}\ \text{Saturday}$

Solution

Each day, the cat eats $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ of a can of cat food. Therefore, the cat food will last for $\frac{6}{\frac{7}{12}}=\frac{72}{7}$ days, which is greater than 10 days but less than 11 days.

Because the number of days is greater than 10 and less than 11, the cat will finish eating in on the 11th day, which is equal to 10 days after Monday, or $\boxed{\textbf{(C)}\ \text{Thursday}}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Prealgebra Problems

2014 AMC 10A Problems/Problem 3

Problem

Bridget bakes 48 loaves of bread for her bakery. She sells half of them in the morning for $\$2.50$ each. In the afternoon she sells two thirds of what she has left, and because they are not fresh, she charges only half price. In the late afternoon she sells the remaining loaves at a dollar each. Each loaf costs $\$0.75$ for her to make. In dollars, what is her profit for the day?

$\textbf{(A)}\ 24\qquad\textbf{(B)}\ 36\qquad\textbf{(C)}\ 44\qquad\textbf{(D)}\ 48\qquad\textbf{(E)}\ 52$

Solution

She first sells one-half of her 48 loaves, or $\frac{48}{2}=24$ loaves. Each loaf sells for $\$2.50$, so her total earnings in the morning is equal to

$$24\cdot\$2.50=\$60$$

This leaves 24 loaves left, and Bridget will sell $\frac{2}{3}\times 24=16$ of them for a price of $\$\frac{2.50}{2}=\$1.25$. Thus, her total earnings for the afternoon is

$$16\cdot\$1.25=\$20$$

Finally, Bridget will sell the remaining $24-16=8$ loaves for a dollar each. This is a total of $\$1\cdot 8=\$8$

The total amount of money she makes is equal to $60+20+8=\$88$.

However, since Bridget spends $\$0.75$ making each loaf of bread, the total cost to make the bread is equal to $\$0.75\cdot 48=\$36$.

Her total profit is the amount of money she spent subtracted from the amount of money she made, which is

$$88-36=52\implies\boxed{\textbf{(E)}\ 52}$$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Prealgebra Problems

2014 AMC 10A Problems/Problem 4

The following problem is from both the 2014 AMC 12A #3 and 2014 AMC 10A #4, so both problems redirect to this page.

Contents

Problem

Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 5\qquad\textbf{(E)}\ 6$

Solution 1

Attack this problem with very simple casework. The only possible locations for the yellow house (Y) is the 3rd house and the last house.

Case 1: Y is the 3rd house.

The only possible arrangement is $B-O-Y-R$

Case 2: Y is the last house.

There are two possible ways:

$B-O-R-Y$ and

so our answer is $\boxed{\textbf{(B)} 3}$

Solution 2

There are 24 possible arrangements of the houses. The number of ways with the blue house next to the yellow house is $3!\cdot 2! = 12$, as we can consider the arrangements of O, (RB), and Y. Thus there are $24-12$ arrangements with the blue and yellow houses non-adjacent.

Exactly half of these have the orange house before the red house by symmetry, and exactly half of those have the blue house before the yellow house (also by symmetry), so our answer is $12\cdot\frac{1}{2}\cdot\frac{1}{2}=3$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 5

The following problem is from both the 2014 AMC 12A #5 and 2014 AMC 10A #5, so both problems redirect to this page.

Problem

On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

Without loss of generality, let there be 20 students(the least whole number possible) who took the test. We have 2 students score 70 points, 7 students score 80 points, 6 students score 90 points and 5 students score 100 points.

The median can be obtained by eliminating members from each group. The median is 90 points.

The mean is equal to the total number of points divided by the number of people, which gives 87

Thus, the difference between the median and the mean is equal to $90-87=\boxed{\textbf{(C)}\ 3}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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All AMC 12 Problems and Solutions	

Category: Introductory Algebra Problems

2014 AMC 10A Problems/Problem 6

The following problem is from both the 2014 AMC 12A #4 and 2014 AMC 10A #6, so both problems redirect to this page.

Contents

Problem

Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

$\textbf{(A)}\ \frac{bde}{ac}\qquad\textbf{(B)}\ \frac{ac}{bde}\qquad\textbf{(C)}\ \frac{abde}{c}\qquad\textbf{(D)}\ \frac{bcde}{a}\qquad\textbf{(E)}\ \frac{abc}{de}$

Solution 1

We need to multiply b by $\frac{d}{a}$ for the new cows and $\frac{e}{c}$ for the new time, so the answer is $b\cdot \frac{d}{a}\cdot \frac{e}{c}=\frac{bde}{ac}$, or $\boxed{\textbf{(A)}}$.

Solution 2

We plug in $a=2$, $b=3$, $c=4$, $d=5$, and $e=6$. Hence the question becomes "2 cows give 3 gallons of milk in 4 days. How many gallons of milk do 5 cows give in 6 days?"

If 2 cows give 3 gallons of milk in 4 days, then 2 cows give $\frac{3}{4}$ gallons of milk in 1 day, so 1 cow gives $\frac{3}{4\cdot 2}$ gallons in 1 day. This means that 5 cows give $\frac{5\cdot 3}{4\cdot 2}$ gallons of milk in 1 day. Finally, we see that 5 cows give $\frac{5\cdot 3\cdot 6}{4\cdot 2}$ gallons of milk in 6 days. Substituting our values for the variables, this becomes $\frac{dbe}{ac}$, which is $\boxed{\textbf{(A)}\ \frac{bde}{ac}}$.

Solution 3

We see that the the number of cows is inversely proportional to the number of days and directly proportional to the gallons of milk. So our constant is $\frac{ac}{b}$.

Let g be the answer to the question. We have $\frac{de}{g}=\frac{ac}{b}\implies gac=bde\implies g=\frac{bde}{ac}\implies\boxed{\textbf{(A)}\ \frac{bde}{ac}}$

Solution 4

The problem specifics "rate," so it would be wise to first find the rate at which cows produce milk. We can find the rate of work/production by finding the gallons produced by a single cow in a single day. To do this, we divide the amount produced by the number of cows and number of days

$$\implies \text{rate} = \frac{b}{ac}$$

Now that we have the gallons produced by a single cow in a single day, we simply multiply that rate by the number of cows and the number of days

$$\implies \frac{bde}{ac} \implies \boxed{\textbf{(A)}}$$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Algebra Problems

2014 AMC 10A Problems/Problem 7

Problem

Nonzero real numbers x, y, a, and b satisfy $x < a$ and $y < b$. How many of the following inequalities must be true?

$\textbf{(I)}\ x+y < a+b$

$\textbf{(II)}\ x-y < a-b$

$\textbf{(III)}\ xy < ab$

$\textbf{(IV)}\ \frac{x}{y} < \frac{a}{b}$

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 1\qquad\textbf{(C)}\ 2\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ 4$

Solution

First, we note that $\textbf{(I)}$ must be true by adding our two original inequalities.

$$x < a, y < b$$

$$\implies x+y < a+b$$

Though one may be inclined to think that $\textbf{(II)}$ must also be true, it is not, for we cannot subtract inequalities.

In order to prove that the other inequalities are false, we only need to provide one counterexample. Let's try substituting

$$x=-3, y=-2, a=1, b=1$$

$\textbf{(III)}$ states that $xy < ab \implies (-3)(-2) < 1\cdot 1 \implies 6 < 1$. This is also false, thus $\textbf{(III)}$ is false.

$\textbf{(IV)}$ states that $\frac{x}{y} < \frac{a}{b} \implies \frac{-3}{-2} < \frac{1}{1} \implies 1.5 < 1$. This is false, so $\textbf{(IV)}$ is false.

One of our four inequalities is true, hence, our answer is $\boxed{\textbf{(B)}\ 1}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 8

Problem

Which of the following number is a perfect square?

$\textbf{(A)}\ \dfrac{14!15!}{2}\qquad\textbf{(B)}\ \dfrac{15!16!}{2}\qquad\textbf{(C)}\ \dfrac{16!17!}{2}\qquad\textbf{(D)}\ \dfrac{17!18!}{2}\qquad\textbf{(E)}\ \dfrac{18!19!}{2}$

Solution

Note that for all positive n, we have

$$\dfrac{n!(n+1)!}{2}$$

$$\implies \dfrac{(n!)^2\cdot(n+1)}{2}$$

$$\implies (n!)^2\cdot\dfrac{n+1}{2}$$

We must find a value of n such that $(n!)^2\cdot\dfrac{n+1}{2}$ is a perfect square. Since $(n!)^2$ is a perfect square, we must also have $\frac{n+1}{2}$ be a perfect square.

In order for $\frac{n+1}{2}$ to be a perfect square, $n+1$ must be twice a perfect square. From the answer choices, $n+1=18$ works, thus, $n=17$ and our desired answer is $\boxed{\textbf{(D)}\ \frac{17!18!}{2}}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Number Theory Problems

2014 AMC 10A Problems/Problem 9

Problem

The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution 1

We find that the area of the triangle is $6\times\sqrt{3}=6\sqrt{3}$. By the Pythagorean Theorem, we have that the length of the hypotenuse is $\sqrt{(2\sqrt{3})^2+6^2}=4\sqrt{3}$. Dropping an altitude from the right angle to the hypotenuse, we can calculate the area in another way.

Let h be the third height of the triangle. We have $4\sqrt{3}h=2\times 6\sqrt{3}=12\sqrt{3}\implies h=\boxed{\textbf{(C)}\ 3}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 10

The following problem is from both the 2014 AMC 12A #9 and 2014 AMC 10A #10, so both problems redirect to this page.

Contents

Problem

Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

$\textbf{(A)}\ a+3\qquad\textbf{(B)}\ a+4\qquad\textbf{(C)}\ a+5\qquad\textbf{(D)}\ a+6\qquad\textbf{(E)}\ a+7$

Solution 1

Let $a=1$. Our list is $\{1,2,3,4,5\}$ with an average of $15\div 5=3$. Our next set starting with 3 is $\{3,4,5,6,7\}$. Our average is $25\div 5=5$.

Therefore, we notice that $5=1+4$ which means that the answer is $\boxed{\textbf{(B)}\ a+4}$.

Solution 2

We are given that

$$b=\frac{a+a+1+a+2+a+3+a+4}{5}$$

$$\implies b=a+2$$

We are asked to find the average of the 5 consecutive integers starting from b in terms of a. By substitution, this is

$$\frac{a+2+a+3+a+4+a+5+a+6}{5}=a+4$$

Thus, the answer is $\boxed{\textbf{(B)}\ a+4}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 11

The following problem is from both the 2014 AMC 12A #8 and 2014 AMC 10A #11, so both problems redirect to this page.

Problem

A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least $\$50$

Coupon 2: $\$20$ off the listed price if the listed price is at least $\$100$

Coupon 3: 18% off the amount by which the listed price exceeds $\$100$

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

$\textbf{(A)}\ \$179.95 \qquad \textbf{(B)}\ \$199.95 \qquad \textbf{(C)}\ \$219.95 \qquad \textbf{(D)}\ \$239.95 \qquad \textbf{(E)}\ \$259.95$

Solution 1

Let the listed price be x. Since all the answer choices are above $\$100$, we can assume $x > 100$. Thus the discounts after the coupons are used will be as follows:

Coupon 1: $x \times 10\% = .1x$

Coupon 2: 20

Coupon 3: $18\% \times (x - 100) = .18x - 18$

For coupon 1 to give a greater price reduction than the other coupons, we must have $.1x > 20 \implies x > 200$ and $.1x > .18x - 18 \implies .08x < 18 \implies x < 225$.

From the first inequality, the listed price must be greater than $\$200$, so answer choices $\textbf{(A)}$ and $\textbf{(B)}$ are eliminated.

From the second inequality, the listed price must be less than $\$225$, so answer choices $\textbf{(D)}$ and $\textbf{(E)}$ are eliminated.

The only answer choice that remains is $\boxed{\textbf{(C)}\ \$219.95}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2014))	
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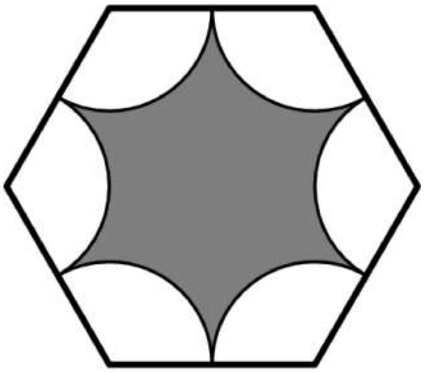
All AMC 12 Problems and Solutions	

2014 AMC 10A Problems/Problem 12

Problem

A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown What is the area of the shaded region?



$\textbf{(A)}\ 27\sqrt{3}-9\pi \qquad \textbf{(B)}\ 27\sqrt{3}-6\pi \qquad \textbf{(C)}\ 54\sqrt{3}-18\pi \qquad \textbf{(D)}\ 54\sqrt{3}-12\pi$
$\textbf{(E)}\ 108\sqrt{3}-9\pi$

Solution

The area of the hexagon is equal to $\frac{3(6)^2\sqrt{3}}{2} = 54\sqrt{3}$ by the formula for the area of a hexagon.

We note that each interior angle of the regular hexagon is 120° which means that each sector is $\frac{1}{3}$ of the circle it belongs to. The area of each sector is $\frac{9\pi}{3} = 3\pi$. The area of all six is $6 \times 3\pi = 18\pi$.

The shaded area is equal to the area of the hexagon minus the sum of the area of all the sectors, which is equal to $\boxed{\textbf{(C)}\ 54\sqrt{3}-18\pi}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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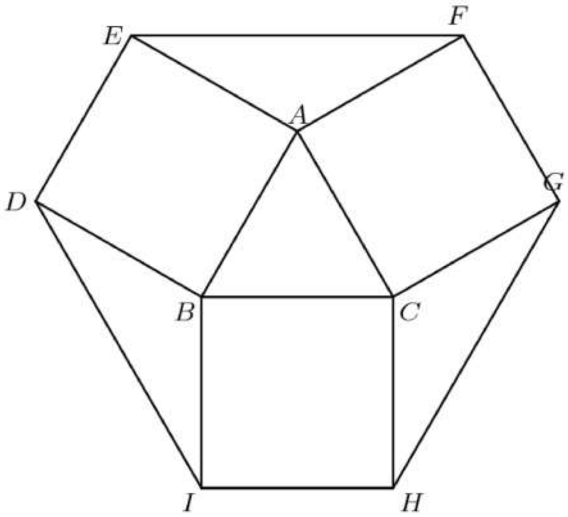
Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 13

Contents

Problem

Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



$\textbf{(A)}\ \dfrac{12+3\sqrt3}4\qquad\textbf{(B)}\ \dfrac92\qquad\textbf{(C)}\ 3+\sqrt3\qquad\textbf{(D)}\ \dfrac{6+3\sqrt3}2\qquad\textbf{(E)}\ 6$

Solution 1

The area of the equilateral triangle is $\frac{\sqrt{3}}{4}$. The area of the three squares is $3\times 1=3$.

Since $\angle C = 360$, $\angle GCH = 360 - 90 - 90 - 60 = 120$.

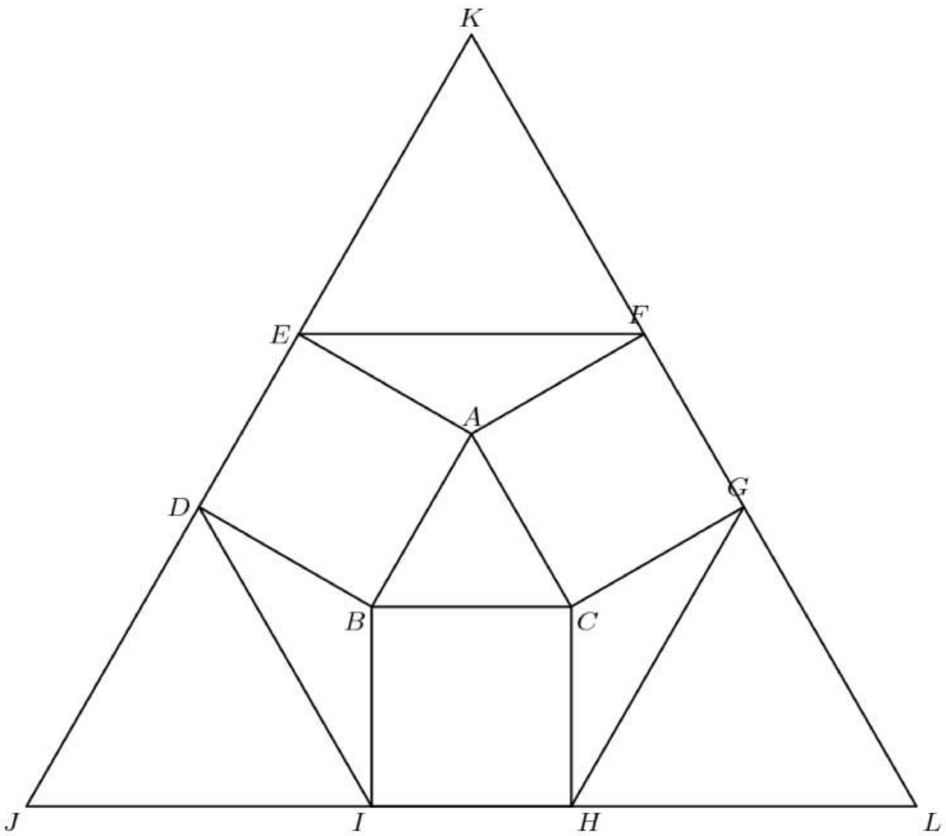
Dropping an altitude from C to GH allows to create a $30-60-90$ triangle since $\triangle GCH$ is isosceles.

This means that the height of $\triangle GCH$ is $\frac{1}{2}$ and half the length of GH is $\frac{\sqrt{3}}{2}$. Therefore, the area of each isosceles triangle is $\frac{1}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}$. Multiplying by 3 yields $\frac{3\sqrt{3}}{4}$ for all three isosceles triangles.

Therefore, the total area is $3+\frac{\sqrt{3}}{4}+\frac{3\sqrt{3}}{4}=3+\frac{4\sqrt{3}}{4}=3+\sqrt{3}\implies\boxed{\textbf{(C)}\ 3+\sqrt3}$.

Solution 2

As seen in the previous solution, segment GH is $\sqrt{3}$. Think of the picture as one large equilateral triangle, $\triangle JKL$ with the sides of $2\sqrt{3}+1$, by extending EF, GH, and DI to points J, K, and L, respectively. This makes the area of $\triangle JKL$ $\frac{\sqrt{3}}{4}(2\sqrt{3}+1)^2=\frac{12+13\sqrt{3}}{4}$.



Triangles $\triangle DIJ$, $\triangle EFK$, and $\triangle GHL$ have sides of $\sqrt{3}$, so their total area is $3(\frac{\sqrt{3}}{4}(\sqrt{3})^2) = \frac{9\sqrt{3}}{4}$.

Now, you subtract their total area from the area of $\triangle JKL$:

$$\frac{12+13\sqrt{3}}{4} - \frac{9\sqrt{3}}{4} = \frac{12+13\sqrt{3}-9\sqrt{3}}{4} = \frac{12+4\sqrt{3}}{4} = 3+\sqrt{3} \implies \boxed{\textbf{(C)}\ 3+\sqrt{3}}$$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 14

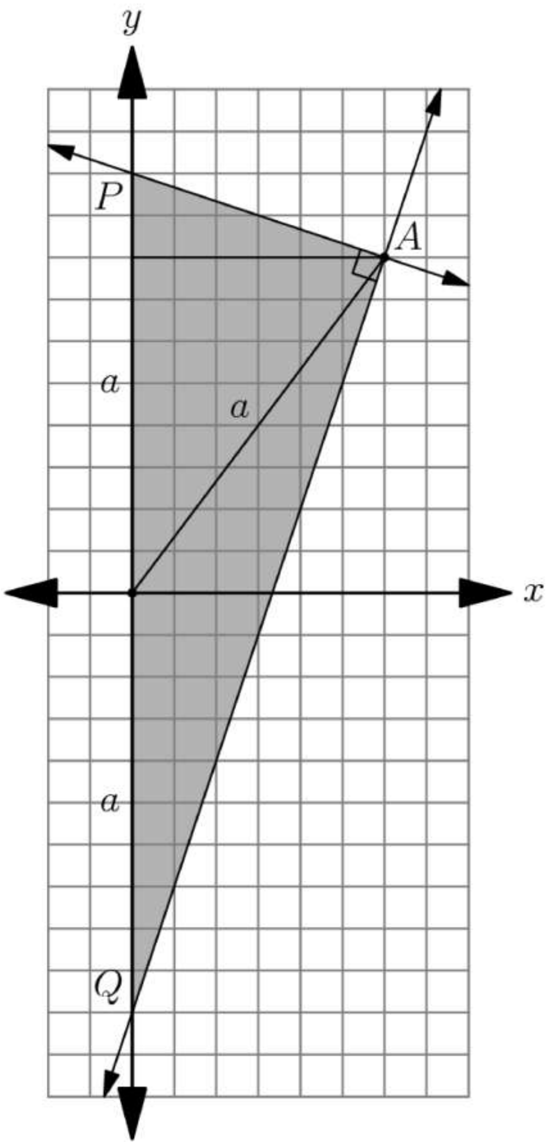
Contents

Problem

The y-intercepts, P and Q, of two perpendicular lines intersecting at the point $A(6,8)$ have a sum of zero. What is the area of $\triangle APQ$?

$\textbf{(A)}\ 45\qquad\textbf{(B)}\ 48\qquad\textbf{(C)}\ 54\qquad\textbf{(D)}\ 60\qquad\textbf{(E)}\ 72$

Solution 1



Note that if the y-intercepts have a sum of 0, the distance from the origin to each of the intercepts must be the same. Call this distance a. Since the $\angle PAQ = 90^\circ$, the length of the median to the midpoint of the hypotenuse is equal to half the length of the hypotenuse. Since the median's length is $\sqrt{6^2+8^2}=10$, this means

$a = 10$, and the length of the hypotenuse is $2a = 20$. Since the x-coordinate of A is the same as the altitude to the hypotenuse, $[APQ] = \frac{20 \cdot 6}{2} = \boxed{\textbf{(D) } 60}$.

Solution 2

We can let the two lines be

$$y = mx + b$$

$$y = -\frac{1}{m}x - b$$

This is because the lines are perpendicular, hence the m and $-\frac{1}{m}$, and the sum of the y-intercepts is equal to 0, hence the $b, -b$.

Since both lines contain the point $(6, 8)$, we can plug this into the two equations to obtain

$$8 = 6m + b$$

and

$$8 = -6\frac{1}{m} - b$$

Adding the two equations gives

$$16 = 6m + \frac{-6}{m}$$

Multiplying by m gives

$$16m = 6m^2 - 6$$

$$\implies 6m^2 - 16m - 6 = 0$$

$$\implies 3m^2 - 8m - 3 = 0$$

Factoring gives

$$(3m + 1)(m - 3) = 0$$

We can just let $m = 3$, since the two values of m do not affect our solution - one is the slope of one line and the other is the slope of the other line.

Plugging $m = 3$ into one of our original equations, we obtain

$$8 = 6(3) + b$$

$$\implies b = 8 - 6(3) = -10$$

Since $\triangle APQ$ has hypotenuse $2|b| = 20$ and the altitude to the hypotenuse is equal to the the x-coordinate of point A, or 6, the area of $\triangle APQ$ is equal to

$$\frac{20 \cdot 6}{2} = \boxed{\textbf{(D) } 60}$$

See Also

2014 AMC 10A Problems/Problem 15

The following problem is from both the 2014 AMC 12A #11 and 2014 AMC 10A #15, so both problems redirect to this page.

Contents

Problem

David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

$\textbf{(A) }140\qquad\textbf{(B) }175\qquad\textbf{(C) }210\qquad\textbf{(D) }245\qquad\textbf{(E) }280$

Solution 1 (Algebra)

Note that he drives at 50 miles per hour after the first hour and continues doing so until he arrives.

Let d be the distance still needed to travel after the first 1 hour. We have that $\dfrac{d}{50}+1.5=\dfrac{d}{35}$, where the 1.5 comes from 1 hour late decreased to 0.5 hours early.

Simplifying gives $7d+525=10d$, or $d=175$.

Now, we must add an extra 35 miles traveled in the first hour, giving a total of $\boxed{\textbf{(C) }210}$ miles.

Solution 2 (Answer Choices)

Instead of spending time thinking about how one can set up an equation to solve the problem, one can simply start checking the answer choices. Quickly checking, we know that neither choice $\textbf{(A)}$ or choice $\textbf{(B)}$ work, but $\textbf{(C)}$ does. We can verify as follows. After 1 hour at 35 mph, David has 175 miles left. This then takes him 3.5 hours at 50 mph. But $210/35=6\text{ hours}$. Since $1+3.5=4.5\text{ hours}$ is 1.5 hours less than 6, our answer is $\boxed{\textbf{(C) }210}$.

See Also

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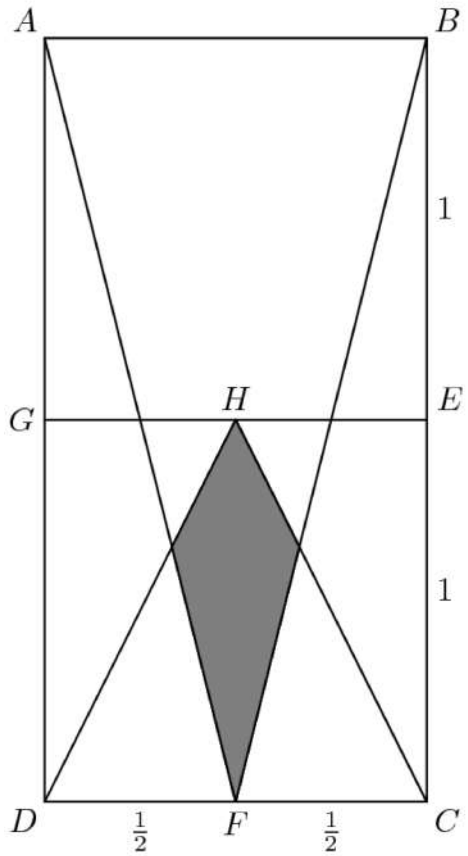
2014 AMC 10A Problems/Problem 16

Contents

Problem

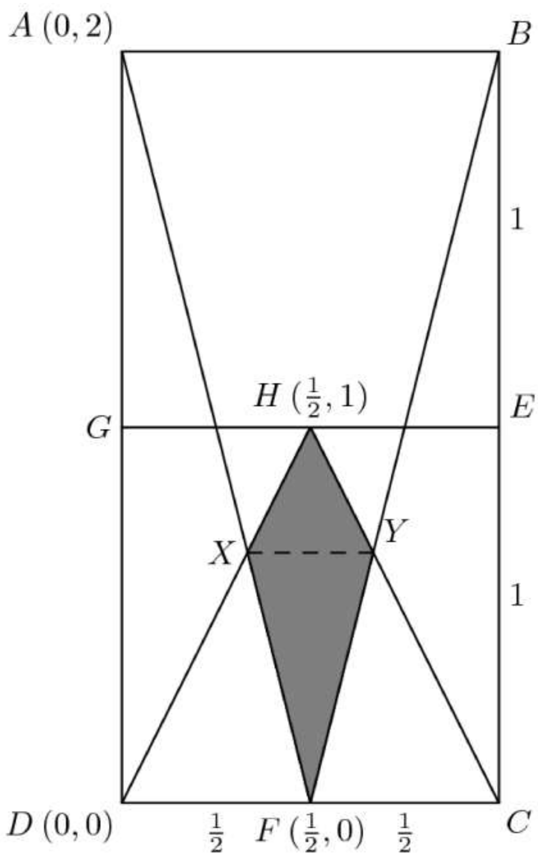
In rectangle $ABCD$, $AB=1$, $BC=2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?



$\textbf{(A) }\dfrac{1}{12}\qquad\textbf{(B) }\dfrac{\sqrt{3}}{18}\qquad\textbf{(C) }\dfrac{\sqrt{2}}{12}\qquad\textbf{(D) }\dfrac{\sqrt{3}}{12}\qquad\textbf{(E) }\dfrac{1}{6}$

Solution 1

Denote $D=(0,0)$. Then $A=(0,2), F=\left(\dfrac{1}{2},0\right), H=\left(\dfrac{1}{2},1\right)$. Let the intersection of AF and DH be X, and the intersection of BF and CH be Y. Then we want to find the coordinates of X so we can find XY. From our points, the slope of AF is $\left(\dfrac{-2}{\frac{1}{2}}\right)=-4$, and its y-intercept is just 2. Thus the equation for AF is $y=-4x+2$. We can also quickly find that the equation of DH is $y=2x$. Setting the equations equal, we have $2x = -4x + 2 \implies x = \frac{1}{3}$. Because of symmetry, we can see that the distance from Y to BC is also $\frac{1}{3}$, so $XY = 1 - 2 \cdot \frac{1}{3} = \frac{1}{3}$. Now the area of the kite is simply the product of the two diagonals over 2. Since the length $HF = 1$, our answer is $\frac{\frac{1}{3} \cdot 1}{2} = \boxed{\textbf{(E)}\ \frac{1}{6}}$.



Solution 2

Let the area of the shaded region be x. Let the other two vertices of the kite be I and J with I closer to AD than J. Note that $[ABCD] = [ABF] + [DCH] - x + [ADI] + [BCJ]$. The area of ABF is 1 and the area of DCH is $\frac{1}{2}$. We will solve for the areas of ADI and BCJ in terms of x by noting that the area of each triangle is the length of the perpendicular from I to AD and J to BC respectively. Because the area of x = $\frac{1}{2} * IJ$ based on the area of a kite formula, $\frac{ab}{2}$ for diagonals of length a and b, $IJ = 2x$. So each perpendicular is length $\frac{1-2x}{2}$. So taking our numbers and plugging them into $[ABCD] = [ABF] + [DCH] - x + [ADI] + [BCJ]$ gives us $2 = \frac{5}{2} - 3x$ Solving this equation for x gives us $x = \boxed{\textbf{(E)}\ \frac{1}{6}}$

Solution 3

From the diagram in Solution 1, let e be the height of XHY and f be the height of XFY. It is clear that their sum is 1 as they are parallel to GD. Let k be the ratio of the sides of the similar triangles XFY and AFB, which are similar because XY is parallel to AB and the triangles share angle F. Then $k = f/2$, as 2 is the height of AFB. Since XHY and DHC are similar for the same reasons as XFY and AFB, the height of XHY will be equal to the base, like in DHC, making $XY = e$. However, XY is also the base of XFY, so $k = e/AB$ where $AB = 1$ so $k = e$. Subbing into $k = f/2$ gives a system of linear equations, $e + f = 1$ and $e = f/2$. Solving yields $e = XY = 1/3$ and $f = 2/3$, and since the area of the kite is simply the product of the two diagonals over 2 and $HF = 1$, our answer is $\frac{\frac{1}{3} \cdot 1}{2} = \boxed{\textbf{(E)}\ \frac{1}{6}}$.

Solution 4

Let the unmarked vertices of the shaded area be labeled I and J, with I being closer to GD than J. Noting that kite $HJFI$ can be split into triangles HJI and JIF. Because HJI and JIF are similar to HDC and ABF, we know that the distance from line segment JI to H is half the distance from JI to F. Because kite $HJFI$ is orthodiagonal, we multiply $(1 * (1/3))/2 = \boxed{\textbf{(E)}\ \frac{1}{6}}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 17

Contents

Problem

Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

$\textbf{(A)}\ \dfrac16\qquad\textbf{(B)}\ \dfrac{13}{72}\qquad\textbf{(C)}\ \dfrac7{36}\qquad\textbf{(D)}\ \dfrac5{24}\qquad\textbf{(E)}\ \dfrac29$

Solution 1 (Clean Counting)

First, we note that there are $1, 2, 3, 4,$ and 5 ways to get sums of $2, 3, 4, 5, 6$ respectively--this is not too hard to see. With any specific sum, there is exactly one way to attain it on the other die. This means that the probability that two specific dice have the same sum as the other is

$$\frac{1}{6}\left(\frac{1+2+3+4+5}{36}\right)=\frac{5}{72}.$$

Since there are $\binom{3}{1}$ ways to choose which die will be the one with the sum of the other two, our answer is $3\cdot\frac{5}{72}=\boxed{\textbf{(D)}\ \frac{5}{24}}$.

Solution 2 (Casework)

Since there are 6 possible values for the number on each dice, there are $6^3=216$ total possible rolls.

The possible results of the 3 dice such that the sum of the values of two of the die is equal to the value of the third die are, without considering the order of the die,

$$(1,1,2),(1,2,3),(1,3,4),(1,4,5),(1,5,6),(2,2,4),(2,3,5),(2,4,6),(3,3,6)$$

There are $\frac{3!}{2}=3$ ways to order the first, sixth, and ninth results, and there are $3!=6$ ways to order the other results.

Therefore, there are a total of $3\times 3+6\times 6=45$ ways to roll the dice such that 2 of the dice sum to the other die, so our answer is

$$\frac{45}{216}=\boxed{\textbf{(D)}\ \frac{5}{24}}$$

See Also

2014 AMC 10A Problems/Problem 18

Problem

A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?

$\textbf{(A)}\ 16 \qquad\textbf{(B)}\ 17 \qquad\textbf{(C)}\ 25 \qquad\textbf{(D)}\ 26 \qquad\textbf{(E)}\ 27$

Solution

Let the points be $A=(x_1,0)$, $B=(x_2,1)$, $C=(x_3,5)$, and $D=(x_4,4)$

Note that the difference in y value of B and C is 4. By rotational symmetry of the square, the difference in x value of A and B is also 4. Note that the difference in y value of A and B is 1. We now know that AB, the side length of the square, is equal to $\sqrt{1^2+4^2}=\sqrt{17}$, so the area is $\boxed{\textbf{(B)}\ 17}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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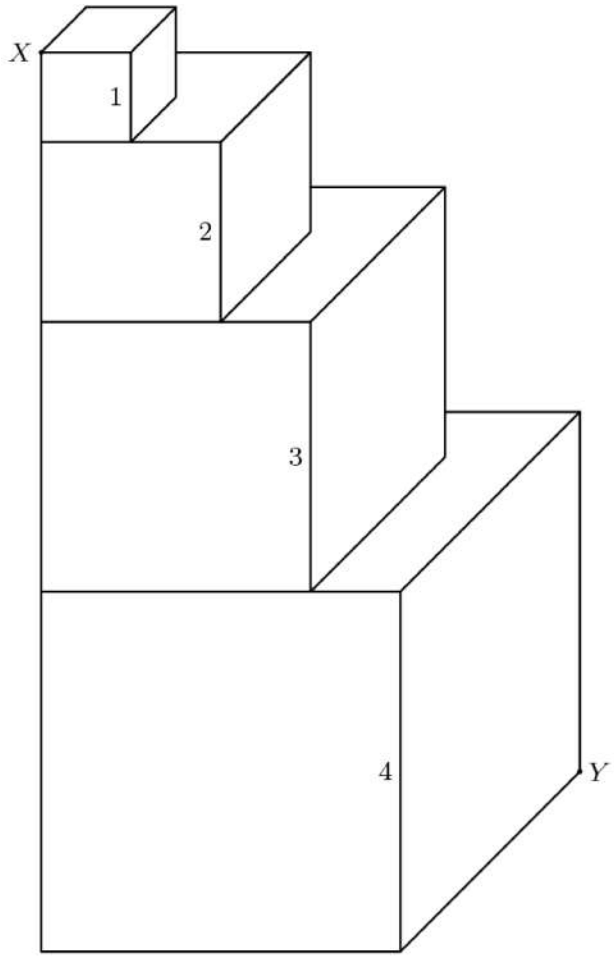
Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 19

Problem

Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of $\overline{XY}$ contained in the cube with edge length 3?

$\textbf{(A)}\ \frac{3\sqrt{33}}{5} \qquad \textbf{(B)}\ 2\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{33}}{3} \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 3\sqrt{2}$



Solution

By Pythagorean Theorem in three dimensions, the distance XY is $\sqrt{4^2+4^2+10^2}=2\sqrt{33}$.

Let the length of the segment XY that is inside the cube with side length 3 be x. By similar triangles, $\frac{x}{3}=\frac{2\sqrt{33}}{10}$, giving $x=\boxed{\textbf{(A)}\ \frac{3\sqrt{33}}{5}}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 20

The following problem is from both the 2014 AMC 12A #16 and 2014 AMC 10A #20, so both problems redirect to this page.

Problem

The product $(8)(888\dots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

$\textbf{(A)}\ 901 \qquad\textbf{(B)}\ 911 \qquad\textbf{(C)}\ 919 \qquad\textbf{(D)}\ 991 \qquad\textbf{(E)}\ 999$

Solution

We can list the first few numbers in the form $8*(8....8)$

$8*8=64$

$8*88=704$

$8*888=7104$

$8*8888=71104$

$8*88888=711104$

By now it's clear that the numbers will be in the form 7, $k-2$ 1s, and 04. We want to make the numbers sum to 1000, so $7+4+(k-2)=1000$. Solving, we get $k=991$, meaning the answer is $\boxed{\text{(D)}}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Number Theory Problems

2014 AMC 10A Problems/Problem 21

Problem

Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

$\textbf{(A)}\ -20 \qquad \textbf{(B)}\ -18 \qquad \textbf{(C)}\ -15 \qquad \textbf{(D)}\ -12 \qquad \textbf{(E)}\ -8$

Solution

Note that when $y = 0$, the x values of the equations should be equal by the problem statement. We have that

$$0 = ax + 5 \implies x = -\frac{5}{a}$$

$$0 = 3x + b \implies x = -\frac{b}{3}$$

Which means that

$$-\frac{5}{a} = -\frac{b}{3} \implies ab = 15$$

The only possible pairs (a, b) then are $(a, b) = (1, 15), (3, 5), (5, 3), (15, 1)$. These pairs give respective x-values of $-5, -\frac{5}{3}, -1, -\frac{1}{3}$ which have a sum of $\boxed{\textbf{(E)}\ -8}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Algebra Problems

2014 AMC 10A Problems/Problem 22

Contents

Problem

In rectangle $ABCD$, $\overline{AB}=20$ and $\overline{BC}=10$. Let E be a point on $\overline{CD}$ such that $\angle CBE=15^\circ$. What is $\overline{AE}$?

$\textbf{(A)}\ \frac{20\sqrt{3}}{3}\qquad\textbf{(B)}\ 10\sqrt{3}\qquad\textbf{(C)}\ 18\qquad\textbf{(D)}\ 11\sqrt{3}\qquad\textbf{(E)}\ 20$

Solution (Trigonometry)

Note that $\tan 15^\circ=\frac{EC}{10}\Rightarrow EC=20-10\sqrt{3}$. (If you do not know the tangent half-angle formula, it is $\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}$). Therefore, we have $DE=10\sqrt{3}$. Since ADE is a $30-60-90$ triangle, $AE=2\cdot AD=2\cdot 10=\boxed{\textbf{(E)}\ 20}$

Solution 2 (non-trig)

Let F be a point on line $\overline{CD}$ such that points C and F are distinct and that $\angle EBF=15^\circ$. By the angle bisector theorem, $\frac{\overline{BC}}{\overline{BF}}=\frac{\overline{CE}}{\overline{EF}}$. Since $\triangle BFC$ is a $30-60-90$ right triangle, $\overline{CF}=\frac{10\sqrt{3}}{3}$ and $\overline{BF}=\frac{20\sqrt{3}}{3}$. Additionally,

$$\overline{CE}+\overline{EF}=\overline{CF}=\frac{10\sqrt{3}}{3}$$

Now, substituting in the obtained values, we get $\frac{10}{\frac{20\sqrt{3}}{3}}=\frac{\overline{CE}}{\overline{EF}}\Rightarrow\frac{2\sqrt{3}}{3}\overline{CE}=\overline{EF}$ and $\overline{CE}+\overline{EF}=\frac{10\sqrt{3}}{3}$. Substituting the first equation into the second yields $\frac{2\sqrt{3}}{3}\overline{CE}+\overline{CE}=\frac{10\sqrt{3}}{3}\Rightarrow\overline{CE}=20-10\sqrt{3}$, so $\overline{DE}=10\sqrt{3}$. Because $\triangle ADE$ is a $30-60-90$ triangle, $\overline{AE}=\boxed{\textbf{(E)}\ 20}$.

Solution 3 (Trigonometry)

By Law of Sines

$$\frac{BC}{\sin 75^\circ}=\frac{EC}{\sin 15^\circ}\rightarrow\frac{10}{\frac{\sqrt{2}+\sqrt{6}}{4}}=\frac{EC}{\frac{\sqrt{6}-\sqrt{2}}{4}}\rightarrow\frac{10}{EC}=\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\rightarrow EC=\frac{10}{2+\sqrt{3}}=20{-}10\sqrt{3}.$$

Thus, $DE=20-(20-10\sqrt{3})=10\sqrt{3}$.

We see that $\triangle ADE$ is a $30-60-90$ triangle, leaving $\overline{AE} = \boxed{\textbf{(E)}\ 20}$.

See Also

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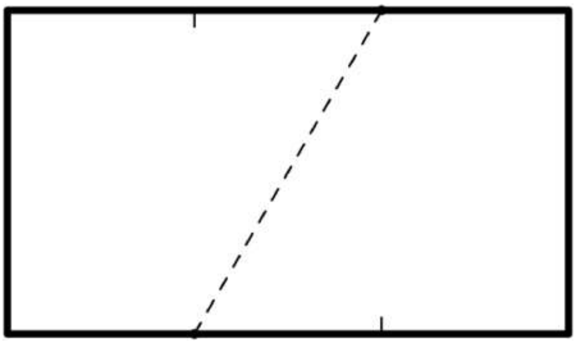
Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 23

Contents

Problem

A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B:A$?



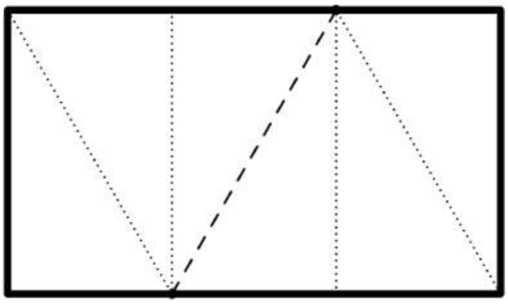
$\textbf{(A)}\ 1:2 \qquad\textbf{(B)}\ 3:5 \qquad\textbf{(C)}\ 2:3 \qquad\textbf{(D)}\ 3:4 \qquad\textbf{(E)}\ 4:5$

Solution 1

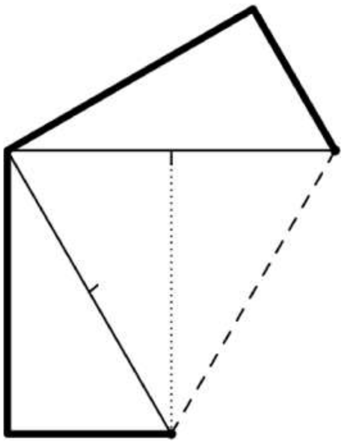
I have no clue how to draw pictures on here, so I'll give instructions. Find the midpoint of the dotted line. Draw a line perpendicular to it. From the point this line intersects the top of the paper, draw lines to each endpoint of the dotted line. These two lines plus the dotted line form a triangle which is the double-layered portion of the folded paper. WLOG, assume the width of the paper is 1 and the length is $\sqrt{3}$. The triangle we want to find has side lengths $\frac{2\sqrt{3}}{3}$, $\sqrt{\left(\frac{\sqrt{3}}{3}\right)^2+1}=\frac{2\sqrt{3}}{3}$, and $\sqrt{\left(\frac{\sqrt{3}}{3}\right)^2+1}=\frac{2\sqrt{3}}{3}$. It is an equilateral triangle with height $\frac{\sqrt{3}}{3}\cdot\sqrt{3}=1$, and area $\frac{\frac{2\sqrt{3}}{3}\cdot 1}{2}=\frac{\sqrt{3}}{3}$. The area of the paper is $1\cdot\sqrt{3}=\sqrt{3}$, and the folded paper has area $\sqrt{3}-\frac{\sqrt{3}}{3}=\frac{2\sqrt{3}}{3}$. The ratio of the area of the folded paper to that of the original paper is thus $\boxed{\textbf{(C)}\ 2:3}$

Solution 2

Our original paper can be divided like this:



After the fold across the dotted line, our paper becomes:



Since our original sheet of paper has six congruent $30-60-90$ triangles and and our new one has four, the ratio of the area $B:A$ is equal to $4:6 \implies \boxed{\textbf{(C)}\ 2:3}$

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Geometry Problems

2014 AMC 10A Problems/Problem 24

Contents

Problem

A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the nth iteration, listing $n+3$ and skipping n. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the $500{,}000$th the number in the sequence?

$\textbf{(A)}\ 996{,}506\qquad\textbf{(B)}\ 996{,}507\qquad\textbf{(C)}\ 996{,}508\qquad\textbf{(D)}\ 996{,}509\qquad\textbf{(E)}\ 996{,}510$

Solution 1

If we list the rows by iterations, then we get

$1, 2, 3, 4$

$13, 14, 15, 16, 17, 18$ etc.

so that the $500,000$th number is the 506th number on the 997th row. ($4+5+6+7......+999=499,494$) The last number of the 996th row (when including the numbers skipped) is $499,494+(1+2+3+4.....+996)=996,000$, (we add the $1-996$ because of the numbers we skip) so our answer is $996,000+506=\boxed{\textbf{(A)}996,506}$

Solution 2

Let's start with natural numbers, with no skips in between.

$1, 2, 3, 4, 5, ..., 500000$

All we need to do is count how many numbers are skipped, n, and "push" (add on to) 500000 however many numbers are skipped.

Clearly, $\frac{999(1000)}{2}<500,000<\frac{1000(1001)}{2}$. This means that the number of skipped number "blocks" in the sequence is $999-3=996$ because we started counting from 4.

Therefore $n=\frac{996(997)}{2}=496,506$, and the answer is $496,506+500000=\boxed{\textbf{(A)}996,506}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10A Problems/Problem 25

The following problem is from both the 2014 AMC 12A #22 and 2014 AMC 10A #25, so both problems redirect to this page.

Problem

The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m,n) are there such that $1 \leq m \leq 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$

$\textbf{(A)}\ 278 \qquad \textbf{(B)}\ 279 \qquad \textbf{(C)}\ 280 \qquad \textbf{(D)}\ 281 \qquad \textbf{(E)}\ 282$

Solution

Between any two consecutive powers of 5 there are either 2 or 3 powers of 2 (because $2^2 < 5^1 < 2^3$). Consider the intervals $(5^0, 5^1), (5^1, 5^2), \dots (5^{866}, 5^{867})$. We want the number of intervals with 3 powers of 2.

From the given that $2^{2013} < 5^{867} < 2^{2014}$, we know that these 867 intervals together have 2013 powers of 2. Let x of them have 2 powers of 2 and y of them have 3 powers of 2. Thus we have the system

$$x + y = 867$$

$$2x + 3y = 2013$$

from which we get $y = 279$, so the answer is $\boxed{\textbf{(B)}}$.

See Also

2014 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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