2014 AMC 10B Problems/Problem 1

Problem

Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

$\textbf{(A)}\ 33\qquad\textbf{(B)}\ 35\qquad\textbf{(C)}\ 37\qquad\textbf{(D)}\ 39\qquad\textbf{(E)}\ 41$

Solution

If Leah has 1 more nickel, she has 14 total coins. Because she has the same number of nickels and pennies, she has 7 nickels and 7 pennies. This is after the nickel has been added, so we must subtract 1 nickel to get 6 nickels and 7 pennies. Therefore, Leah has $6\cdot5+7=\boxed{37 (\textbf{C})}$ cents.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 2

Problem

What is $\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 64$

Solution

We can synchronously multiply 2^3 to the polynomials both above and below the fraction bar. Thus,

$$\frac{2^3+2^3}{2^{-3}+2^{-3}}=\frac{2^6+2^6}{1+1}=2^6.$$

Hence, the fraction equals to $\boxed{64 (\textbf{E})}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 3

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Problem

Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles how long was Randy's trip?

$\textbf{(A)}\ 30\qquad\textbf{(B)}\ \frac{400}{11}\qquad\textbf{(C)}\ \frac{75}{2}\qquad\textbf{(D)}\ 40\qquad\textbf{(E)}\ \frac{300}{7}$

Solution 1

Let the total distance be x. We have $\frac{x}{3}+20+\frac{x}{5}=x$, or $\frac{8x}{15}+20=x$. Subtracting $\frac{8x}{15}$ from both sides gives us $20=\frac{7x}{15}$. Multiplying by $\frac{15}{7}$ gives us $x=\boxed{\text{E}}$

Solution 2

The first third of his distance added to the last one-fifth of his distance equals $\frac{8}{15}$. Therefore, $\frac{7}{15}$ of his distance is 20. Let x be his total distance, and solve for x. Therefore, x is equal to $\frac{300}{7}$, or E.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 4

Problem

Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

$\textbf{(A)}\ \frac{3}{2}\qquad\textbf{(B)}\ \frac{5}{3}\qquad\textbf{(C)}\ \frac{7}{4}\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ \frac{13}{4}$

Solution

Let m be the cost of a muffin and b be the cost of a banana. From the given information,

$$2m+16b=2(4m+3b)=8m+6b \Rightarrow 10b=6m \Rightarrow m=\frac{10}{6}b=\text{(B)}\ \boxed{\frac{5}{3}b}$$

.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 5

Problem

Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5 : 2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?

$\textbf{(A)}\ 26\qquad\textbf{(B)}\ 28\qquad\textbf{(C)}\ 30\qquad\textbf{(D)}\ 32\qquad\textbf{(E)}\ 34$

Solution

We note that the total length must be the same as the total height because the window is a square. Calling the width of each small rectangle $2x$, and the height $5x$, we can see that the length is composed of 4 widths and 5 bars of length 2. This is equal to two heights of the small rectangles as well as 3 bars of 2. Thus, $4(2x)+5(2)=2(5x)+3(2)$. We quickly find that $x=2$. The total side length is $4(4)+5(2)=2(10)+3(2)=26$, or $\boxed{\textbf{(A)}}$.

See Also

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Category: Introductory Geometry Problems

2014 AMC 10B Problems/Problem 6

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Problem 6

Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 34 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 38 \qquad \textbf{(E)}\ 39$

Solution

Since he pays $\frac{2}{3}$ the price for every second balloon, the price for two balloons is $\frac{5}{3}$. Thus, if he had enough money to buy 30 balloons before, he now has enough to buy $30 \cdot \frac{2}{\frac{5}{3}} = 30 \cdot \frac{6}{5} = \boxed{\text{(C)}\ 36}$.

Solution 2

Suppose each balloon costs 3 dollars. Therefore, Orvin brought 90 dollars. Since every second balloon costs $\frac{2}{3} \cdot 3 = 2$ dollars, Orvin gets 2 balloons for 5 dollars. Therefore, Orvin gets $\frac{2 \cdot 90}{5} = \boxed{\text{(C)}\ 36}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 7

Problem

Suppose $A > B > 0$ and A is x% greater than B. What is x?

$\textbf{(A)}\ 100\left(\frac{A-B}{B}\right) \qquad \textbf{(B)}\ 100\left(\frac{A+B}{B}\right) \qquad \textbf{(C)}\ 100\left(\frac{A+B}{A}\right) \qquad \textbf{(D)}\ 100\left(\frac{A-B}{A}\right) \qquad \textbf{(E)}\ 100\left(\frac{A}{B}\right)$

Solution

We have that A is $x\%$ greater than B, so $A = \frac{100+x}{100}(B)$. We solve for x. We get

$$\frac{A}{B} = \frac{100+x}{100}$$

$$100\frac{A}{B} = 100+x$$

$$100\left(\frac{A}{B}-1\right) = x$$

$$\boxed{100\left(\frac{A-B}{B}\right)(\textbf{A})} = x.$$

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 8

Problem

A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

$\textbf{(A)}\ \frac{b}{1080t} \qquad \textbf{(B)}\ \frac{30t}{b} \qquad \textbf{(C)}\ \frac{30b}{t} \qquad \textbf{(D)}\ \frac{10t}{b} \qquad \textbf{(E)}\ \frac{10b}{t}$

Solution

Converting feet to yards and minutes to second, we see that the truck travels $\frac{b}{18}$ yards every t seconds for 180 seconds. We see that he does $\frac{180}{t}$ cycles of $\frac{b}{18}$ yards. Multiplying, we get $\frac{180b}{18t}$, or $\frac{10b}{t}$, or $\boxed{\textbf{(E)}}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 9

Problem

For real numbers w and z,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}=2014.$$

What is $\frac{w+z}{w-z}$?

$\textbf{(A) }-2014\qquad\textbf{(B) }\frac{-1}{2014}\qquad\textbf{(C) }\frac{1}{2014}\qquad\textbf{(D) }1\qquad\textbf{(E) }2014$

Solution

Multiply the numerator and denominator of the LHS by wz to get $\frac{z+w}{z-w}=2014$. Then since $z+w=w+z$ and $w-z=-(z-w)$, $\frac{w+z}{w-z}=-\frac{z+w}{z-w}=-2014$, or choice $\boxed{A}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 10

Problem

In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution

Note from the addition of the last digits that $A+B=D$ or $D+10$. From the addition of the frontmost digits, $A+B$ cannot have a carry, since the answer is still a five-digit number. Therefore $A+B=D$.

Using the second or fourth column, this then implies that $C=0$, so that $B+C=B$ and $C+D=D$. Note that all of the remaining equalities are now satisfied: $A+B=D, B+C=B$, and $B+A=D$. Thus, if we have any A, B, D such that $A+B=D$ then the addition will be satisfied. Since the digits must be distinct, the smallest possible value of D is $1+2=3$, and the largest possible value is 9. Any of these values can be obtained by taking $A=1, B=D-1$. Thus we have that $3 \leq D \leq 9$, so the number of possible values is $\boxed{\textbf{(C)}\ 7}$

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 11

Problem

For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) two successive 15% discounts

(2) three successive 10% discounts

(3) a 25% discount followed by a 5% discount

What is the smallest possible positive integer value of n?

$\textbf{(A)}\ \ 27\qquad\textbf{(B)}\ 28\qquad\textbf{(C)}\ 29\qquad\textbf{(D)}\ 31\qquad\textbf{(E)}\ 33$

Solution

Let the original price be x. Then, for option 1, the discounted price is $(1-.15)(1-.15)x=.7225x$. For option 2, the discounted price is $(1-.1)(1-.1)(1-.1)x=.729x$. Finally, for option 3, the discounted price is $(1-.25)(1-.05)=.7125x$. Therefore, n must be greater than $\max(x-.7225x,x-.729x,x-.7125x)$. It follows n must be greater than $.2875$. We multiply this by 100 to get the percent value, and then round up because n is the smallest integer that provides a greater discount than 28.75, leaving us with the answer of $\boxed{\textbf{(C)}\ 29}$

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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2014 AMC 10B Problems/Problem 12

Problem

The largest divisor of $2,014,000,000$ is itself. What is its fifth-largest divisor?

$\textbf{(A) }125,875,000\qquad\textbf{(B) }201,400,000\qquad\textbf{(C) }251,750,000\qquad\textbf{(D) }402,800,000\qquad\textbf{(E) }503,500,000$

Solution

Note that $2,014,000,000$ is divisible by $2,\ 4,\ 5,\ 8$. Then, the fifth largest factor would come from divisibility by 8, or $251,750,000$, or $\boxed{\textbf{(C)}}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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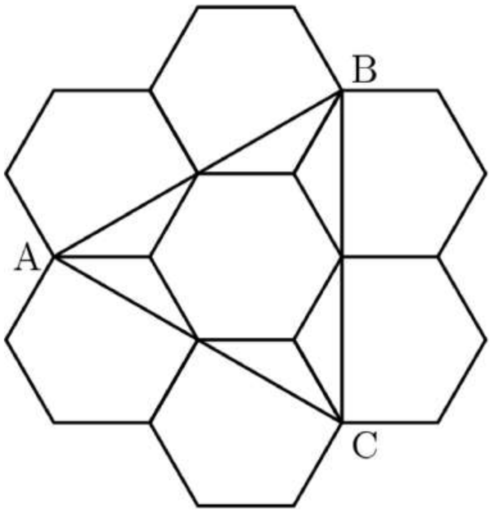
2014 AMC 10B Problems/Problem 13

Problem

Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



$\textbf{(A) }2\sqrt3\qquad\textbf{(B) }3\sqrt3\qquad\textbf{(C) }1+3\sqrt2\qquad\textbf{(D) }2+2\sqrt3\qquad\textbf{(E) }3+2\sqrt3$

Solution

We note that the 6 triangular sections in $\triangle ABC$ can be put together to form a hexagon congruent to each of the seven other hexagons. By the formula for the area of the hexagon, we get the area for each hexagon as $\frac{3\sqrt{3}}{2}$. The area of $\triangle ABC$, which is equivalent to two of these hexagons together, is $\boxed{\textbf{(B)} 3\sqrt{3}}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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Category: Introductory Geometry Problems

2014 AMC 10B Problems/Problem 14

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Problem

Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2$?

$\textbf{(A)}\ 26\qquad\textbf{(B)}\ 27\qquad\textbf{(C)}\ 36\qquad\textbf{(D)}\ 37\qquad\textbf{(E)}\ 41$

Solution

Let $h \in \mathbb{N}$ be the number of hours Danica drove. Note that abc can be expressed as $100 \cdot a + 10 \cdot b + c$. From the given information, we have $100a + 10b + c + 55h = 100c + 10b + a$. This can be simplified into $99a + 55h = 99c$ by subtraction, which can further be simplified into $9a + 5h = 9c$ by dividing both sides by 11. Thus we must have $h \equiv 0 \pmod{9}$. However, if $h \geq 15$, then $\min\{c\} \geq \dfrac{9+5(15)}{9} > 9$, which is impossible since c must be a digit. The only value of h that is divisible by 9 and less than or equal to 14 is $h = 9$.

From this information, $9a + 5(9) = 9c \Rightarrow a + 5 = c$. Combining this with the inequalities $a+b+c \leq 7$ and $a \geq 1$, we have $a + b + a + 5 \leq 7 \Rightarrow 2a + b \leq 2$, which implies $1 \leq a \leq 1$, so $a = 1$, $b = 0$, and $c = 6$. Thus $a^2+b^2+c^2 = 1 + 0 + 36 = \boxed{37\ \textbf{(D)}}$

Solution 2

Danica drives m miles, such that $m > 0$ and m is a multiple of 55. Therefore, m must have an units digit of either 0 or $5.$ If the units digit of m is $0,$ then $a = c$ which would imply that Danica did not drive at all. Thus, $c > a.$ Therefore, $|a - c| = 5,$ and because $a + b + c \le 7, c > a,$ we have $(a, c) = (1, 6).$ Finally, b then must be 0 due to $a + b + c \le 7,$ and $a^2 + b^2 + c^2 = 1^2 + 0^2 + 6^2 = \boxed{\textbf{(D)}\ 37}$

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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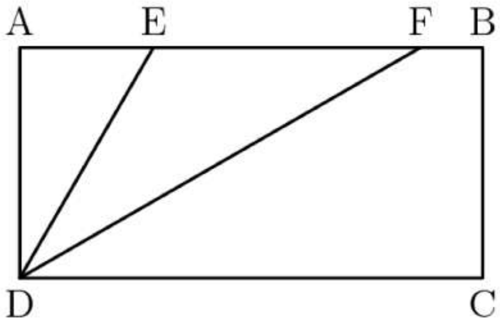
2014 AMC 10B Problems/Problem 15

Problem

In rectangle $ABCD$, $DC = 2CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



$\textbf{(A)}\ \frac{\sqrt{3}}{6}\qquad\textbf{(B)}\ \frac{\sqrt{6}}{8}\qquad\textbf{(C)}\ \frac{3\sqrt{3}}{16}\qquad\textbf{(D)}\ \frac{1}{3}\qquad\textbf{(E)}\ \frac{\sqrt{2}}{4}$

Solution

Let the length of AD be x, so that the length of AB is $2x$ and $[ABCD] = 2x^2$.

Because $ABCD$ is a rectangle, $\angle ADC = 90^\circ$, and so $\angle ADE = \angle EDF = \angle FDC = 30^\circ$. Thus $\triangle DAE$ is a $30-60-90$ right triangle; this implies that $\angle DEF = 180^\circ - 60^\circ = 120^\circ$, so $\angle EFD = 180^\circ - (120^\circ + 30^\circ) = 30^\circ$. Now drop the altitude from E of $\triangle DEF$, forming two $30-60-90$ triangles.

Because the length of AD is x, from the properties of a $30-60-90$ triangle the length of AE is $\frac{x\sqrt{3}}{3}$ and the length of DE is thus $\frac{2x\sqrt{3}}{3}$. Thus the altitude of $\triangle DEF$ is $\frac{x\sqrt{3}}{3}$, and its base is $2x$, so its area is $\frac{1}{2}(2x)\left(\frac{x\sqrt{3}}{3}\right) = \frac{x^2\sqrt{3}}{3}$.

To finish, $\frac{[\triangle DEF]}{[ABCD]} = \frac{\frac{x^2\sqrt{3}}{3}}{2x^2} = \boxed{\textbf{(A)}\ \frac{\sqrt{3}}{6}}$

See Also

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2014 AMC 10B Problems/Problem 16

Problem

Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

$\textbf {(A) } \frac{1}{36} \qquad \textbf {(B) } \frac{7}{72} \qquad \textbf {(C) } \frac{1}{9} \qquad \textbf {(D) } \frac{5}{36} \qquad \textbf {(E) } \frac{1}{6}$

Solution

We split this problem into 2 cases.

First, we calculate the probability that all four are the same. After the first dice, all the number must be equal to that roll, giving a probability of $1*\frac{1}{6}*\frac{1}{6}*\frac{1}{6}=\frac{1}{216}$.

Second, we calculate the probability that three are the same and one is different. After the first dice, the next two must be equal and the third different. There are 4 orders to roll the different dice, giving $4*1*\frac{1}{6}*\frac{1}{6}*\frac{5}{6}=\frac{5}{54}$.

Adding these up, we get $\frac{7}{72}$, or $\boxed{\textbf{(B)}}$.

See Also

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2014 AMC 10B Problems/Problem 17

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Problem 17

What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

$\textbf{(A) } 2^{1002} \qquad\textbf{(B) } 2^{1003} \qquad\textbf{(C) } 2^{1004} \qquad\textbf{(D) } 2^{1005} \qquad\textbf{(E) } 2^{1006}$

Solution 1

We begin by factoring the 2^{1002} out. This leaves us with $5^{1002} - 1$.

We factor the difference of squares, leaving us with $(5^{501} - 1)(5^{501} + 1)$. We note that all even powers of 5 more than two end in $\ldots 625$. Also, all odd powers of five more than 2 end in $\ldots 125$. Thus, $(5^{501} + 1)$ would end in $\ldots 126$ and thus would contribute one power of two to the answer, but not more.

We can continue to factor $(5^{501} - 1)$ as a difference of cubes, leaving us with $(5^{167} - 1)$ times an odd number. $(5^{167} - 1)$ ends in $\ldots 124$, contributing two powers of two to the final result.

Adding these extra 3 powers of two to the original 1002 factored out, we obtain the final answer of $\textbf{(D) } 2^{1005}$.

Solution 2

First, we can write the expression in a more primitive form which will allow us to start factoring.

$$10^{1002} - 4^{501} = 2^{1002} \cdot 5^{1002} - 2^{1002}$$

Now, we can factor out 2^{1002}. This leaves us with $5^{1002} - 1$. Call this number N. Thus, our final answer will be 2^{1002+k}, where k is the largest power of 2 that divides N. Now we can consider , since $k \leq 4$ by the answer choices.

Note that

$$\begin{aligned} 5^1 &\equiv 5 \pmod{16} \\ 5^2 &\equiv 9 \pmod{16} \\ 5^3 &\equiv 13 \pmod{16} \\ 5^4 &\equiv 1 \pmod{16} \\ 5^5 &\equiv 5 \pmod{16} \\ &\vdots \end{aligned}$$

The powers of 5 cycle in $\mod 16$ with a period of 4. Thus,

$$5^{1002} \equiv 5^2 \equiv 9 \pmod{16} \implies 5^{1002} - 1 \equiv 8 \pmod{16}$$

This means that N is divisible by $8 = 2^3$ but not $16 = 2^4$, so $k = 3$ and our answer is $2^{1002+3} = \boxed{\textbf{(D) } 2^{1005}}$.

See Also

2014 AMC 10B Problems/Problem 18

Problem

A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

$\textbf{(A)}\ 24\qquad\textbf{(B)}\ 30\qquad\textbf{(C)}\ 31\qquad\textbf{(D)}\ 33\qquad\textbf{(E)}\ 35$

Solution

We start off with the fact that the median is 9, so we must have $a, b, c, d, e, 9, f, g, h, i, j$, listed in ascending order. Note that the integers do not have to be distinct.

Since the mode is 8, we have to have at least 2 occurrences of 8 in the list. If there are 2 occurrences of 8 in the list, we will have $a, b, c, 8, 8, 9, f, g, h, i, j$. In this case, since 8 is the unique mode, the rest of the integers have to be distinct. So we minimize a, b, c, f, g, h, i in order to maximize j. If we let the list be $1, 2, 3, 8, 8, 9, 10, 11, 12, 13, j$, then $j = 11 \times 10 - (1 + 2 + 3 + 8 + 8 + 9 + 10 + 11 + 12 + 13) = 33$.

Next, consider the case where there are 3 occurrences of 8 in the list. Now, we can have two occurrences of another integer in the list. We try $1, 1, 8, 8, 8, 9, 9, 10, 10, 11, j$. Following the same process as above, we get $j = 11 \times 10 - (1 + 1 + 8 + 8 + 8 + 9 + 9 + 10 + 10 + 11) = 35$. As this is the highest choice in the list, we know this is our answer. Therefore, the answer is $\boxed{\textbf{(E)}\ 35}$

See Also

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2014 AMC 10B Problems/Problem 19

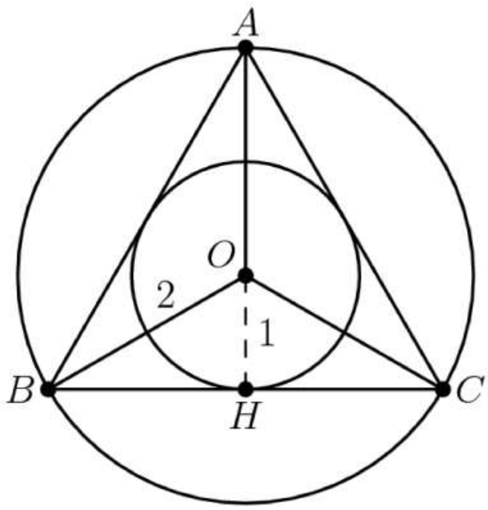
Problem

Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

$\textbf{(A)}\ \frac{1}{6}\qquad\textbf{(B)}\ \frac{1}{4}\qquad\textbf{(C)}\ \frac{2-\sqrt{2}}{2}\qquad\textbf{(D)}\ \frac{1}{3}\qquad\textbf{(E)}\ \frac{1}{2}$

Solution

Let the center of the two circles be O. Now pick an arbitrary point A on the boundary of the circle with radius 2. We want to find the range of possible places for the second point, A', such that AA' passes through the circle of radius 1. To do this, first draw the tangents from A to the circle of radius 1. Let the intersection points of the tangents (when extended) with circle of radius 2 be B and C. Let H be the foot of the altitude from O to $\overline{BC}$. Then we have the following diagram.



We want to find $\angle BOC$, as the range of desired points A' is the set of points on minor arc $\overset{\frown}{BC}$. This is because B and C are part of the tangents, which "set the boundaries" for A'. Since $OH = 1$ and $OB = 2$ as shown in the diagram, $\triangle OHB$ is a $30-60-90$ triangle with $\angle BOH = 60^\circ$. Thus, $\angle BOC = 120^\circ$, and the probability A' lies on the minor arc $\overset{\frown}{BC}$ is thus $\dfrac{120}{360} = \boxed{\textbf{(D)}\ \frac{1}{3}}$.

See Also

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2014 AMC 10B Problems/Problem 20

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Problem

For how many integers x is the number $x^4 - 51x^2 + 50$ negative?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 16$

Solution 1

First, note that $50+1=51$, which motivates us to factor the polynomial as $(x^2-50)(x^2-1)$. Since this expression is negative, one term must be negative and the other positive. Also, the first term is obviously smaller than the second, so $x^2-50<0<x^2-1$. Solving this inequality, we find $1<x^2<50$. There are exactly 12 integers x that satisfy this inequality, $\pm 2, 3, 4, 5, 6, 7$.

Thus our answer is $\boxed{\textbf{(C)}\ 12}$

Solution 2

Since the x^4-51x^2 part of x^4-51x^2+50 has to be less than -50 (because we want x^4-51x^2+50 to be negative), we have the inequality $x^4-51x^2<-50$ --> $x^2(x^2-51)<-50$. x^2 has to be positive, so (x^2-51) is negative. Then we have $x^2<51$. Try answers to find $\boxed{\textbf{(C)}\ 12}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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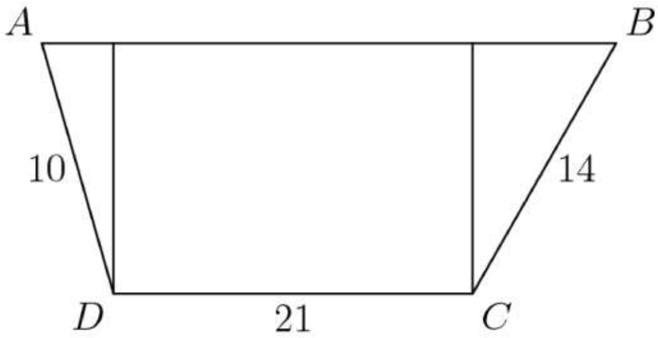
2014 AMC 10B Problems/Problem 21

Problem

Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of $ABCD$?

$\textbf{(A) }10\sqrt{6}\qquad\textbf{(B) }25\qquad\textbf{(C) }8\sqrt{10}\qquad\textbf{(D) }18\sqrt{2}\qquad\textbf{(E) }26$

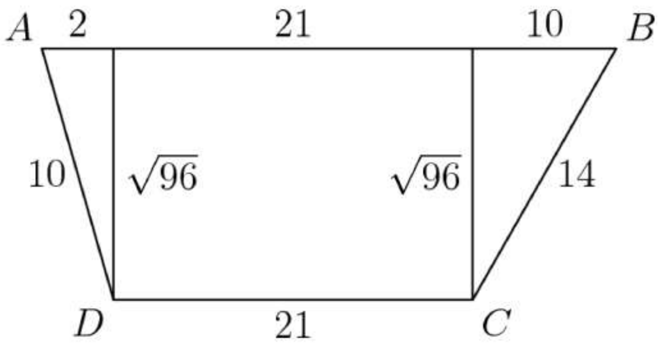
Solution



In the diagram, $\overline{DE} \perp \overline{AB}, \overline{FC} \perp \overline{AB}$. Denote $\overline{AE} = x$ and $\overline{DE} = h$. In right triangle AED, we have from the Pythagorean theorem: $x^2+h^2=100$. Note that since $EF = DC$, we have $BF = 33-DC-x = 12-x$. Using the Pythagorean theorem in right triangle BFC, we have $(12-x)^2 + h^2 = 196$.

We isolate the h^2 term in both equations, getting $h^2 = 100-x^2$ and $h^2 = 196 - (12-x)^2$.

Setting these equal, we have $100 - x^2 = 196 - 144 + 24x - x^2 \implies 24x = 48 \implies x = 2$. Now, we can determine that $h^2 = 100 - 4 \implies h = \sqrt{96}$.



The two diagonals are $\overline{AC}$ and $\overline{BD}$. Using the Pythagorean theorem again on $\triangle AFC$ and $\triangle BED$, we can find these lengths to be $\sqrt{96 + 529} = 25$ and $\sqrt{96 + 961} = \sqrt{1057}$. Since $\sqrt{96 + 529} < \sqrt{96 + 961}$, 25 is the shorter length, so the answer is $\boxed{\textbf{(B) }25}$.

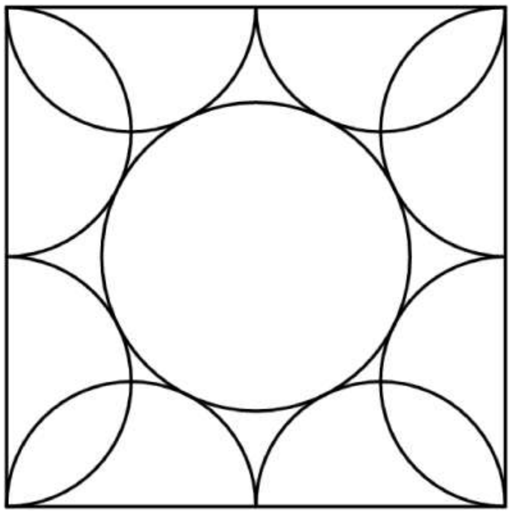
See Also

2014 AMC 10B Problems/Problem 22

Problem

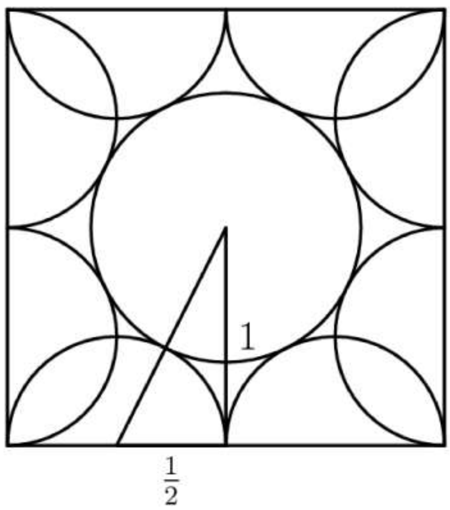
Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?

$\text{(A) } \frac{1+\sqrt2}4 \quad \text{(B) } \frac{\sqrt5-1}2 \quad \text{(C) } \frac{\sqrt3+1}4 \quad \text{(D) } \frac{2\sqrt3}5 \quad \text{(E) } \frac{\sqrt5}3$



Solution

We connect the centers of the circle and one of the semicircles, then draw the perpendicular from the center of the middle circle to that side, as shown.



We will start by creating an equation by the Pythagorean theorem:

$$\sqrt{1^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}.$$

Let's call r as the radius of the circle that we want to find. We see that the hypotenuse of the bold right triangle is $\frac{1}{2}+r$, and thus r is $\boxed{\textbf{(B)} \frac{\sqrt5-1}2}$

See Also

2014 AMC 10B Problems/Problem 23

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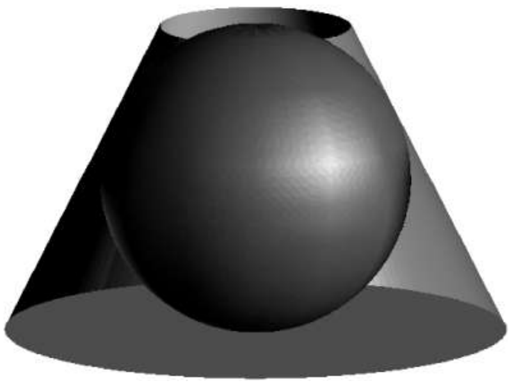
Problem

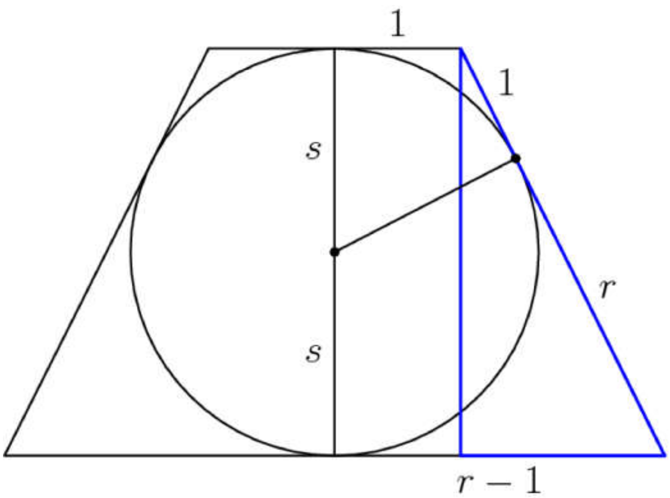
A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?

$\textbf{(A)}\ \dfrac32 \quad \textbf{(B)}\ \dfrac{1+\sqrt5}2 \quad \textbf{(C)}\ \sqrt3 \quad \textbf{(D)}\ 2 \quad \textbf{(E)}\ \dfrac{3+\sqrt5}2$



Solution 1

First, we draw the vertical cross-section passing through the middle of the frustum. Let the top base have a diameter of 2, and the bottom base have a diameter of 2r.



Then using the Pythagorean theorem we have: $(r+1)^2 = (2s)^2 + (r-1)^2$, which is equivalent to: $r^2+2r+1 = 4s^2+r^2-2r+1$. Subtracting r^2-2r+1 from both sides, $4r = 4s^2$

Solving for s, we end up with

$$s = \sqrt{r}.$$

Next, we can find the volume of the frustum and of the sphere. Since we know $V_{frustum} = 2V_{sphere}$, we can solve for s using $V_{frustum} = \frac{\pi h}{3}(R^2 + r^2 + Rr)$ we get:

$$V_{frustum} = \frac{\pi \cdot 2\sqrt{r}}{3}(r^2 + r + 1)$$

Using $V_{sphere} = \frac{4r^3\pi}{3}$, we get

$$V_{sphere} = \frac{4(\sqrt{r})^3\pi}{3}$$

so we have:

$$\frac{\pi \cdot 2\sqrt{r}}{3}(r^2 + r + 1) = 2 \cdot \frac{4(\sqrt{r})^3\pi}{3}.$$

Dividing by $\frac{2\pi\sqrt{r}}{3}$, we get

$$r^2 + r + 1 = 4r$$

which is equivalent to

$$r^2 - 3r + 1 = 0$$

$r = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$, so

$$r = \frac{3 + \sqrt{5}}{2} \longrightarrow \boxed{\textbf{(E)}}$$

Solution 2

Similar to above, draw a smaller cone top with the base of the smaller circle with radius r_1 and height h. The smaller right triangle is similar to the blue highlighted one in Solution 1. Then $\frac{r_1}{r_2 - r_1} = \frac{h}{2R}$ where R is the radius of the sphere. Then $h = \frac{2Rr_1}{r_2 - r_1}$.

From the Pythagorean theorem on the blue triangle in Solution 1, we get similarly that $R = \sqrt{r_2 r_1}$.

From the volume requirements, we get that $\frac{8}{3}\pi R^3 = \frac{\pi r_2^2(h + 2R) - \pi r_1^2 h}{3}$ which yields $8R^3 = r_2^2(h + 2R) - r_1^2 h$.

The small right triangle on top is similar to the big right triangle of the entire big cone. So $\frac{r_1}{r_2} = \frac{h}{h + 2R} \implies h + 2R = \frac{r_2 h}{r_1}$.

Substituting yields $8R^3 = \frac{h(r_2^3 - r_1^3)}{r_1}$.

Substituting $R = \sqrt{r_2 r_1}$ and $h = \frac{2Rr_1}{r_2 - r_1}$ yields $8(r_2 r_1)^{\frac{3}{2}} = 2(r_2 r_1)^{\frac{1}{2}}(r_2^2 + r_2 r_1 + r_1^2)$ which yields $r_2^2 - 3r_2 r_1 + r_1^2 = 0$.

Solving in r_2 yields $r_2 = \frac{3r_1 \pm r_1\sqrt{5}}{2}$ so $\frac{r_2}{r_1} = \frac{3 + \sqrt{5}}{2}$.

2014 AMC 10B Problems/Problem 24

The following problem is from both the 2014 AMC 12B #18 and 2014 AMC 10B #24, so both problems redirect to this page.

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Problem

The numbers $1, 2, 3, 4, 5$ are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

$\textbf{(A)}\ 1 \qquad\textbf{(B)}\ 2 \qquad\textbf{(C)}\ 3 \qquad\textbf{(D)}\ 4 \qquad\textbf{(E)}\ 5$

Solution 1

We see that there are $5!$ total ways to arrange the numbers. However, we can always rotate these numbers so that, for example, the number 1 is always at the top of the circle. Thus, there are only $4!$ ways under rotation, which is not difficult to list out. We systematically list out all 24 cases.

Now, we must examine if they satisfy the conditions. We can see that by choosing one number at a time, we can always obtain subsets with sums $1, 2, 3, 4,$ and 5. By choosing the full circle, we can obtain 15. By choosing everything except for $1, 2, 3, 4,$ and 5, we can obtain subsets with sums of $10, 11, 12, 13,$ and 14.

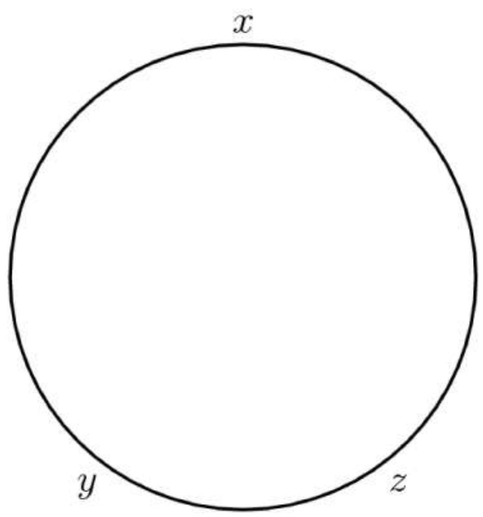
This means that we now only need to check for $6, 7, 8,$ and 9. However, once we have found a set summing to 6, we can choose everything else and obtain a set summing to 9, and similarly for 7 and 8. Thus, we only need to check each case for whether or not we can obtain 6 or 7.

We find that there are only 4 arrangements that satisfy these conditions. However, each of these is a reflection of another. We divide by 2 for these reflections to obtain a final answer of $\boxed{\textbf{(B)}\ 2}$.

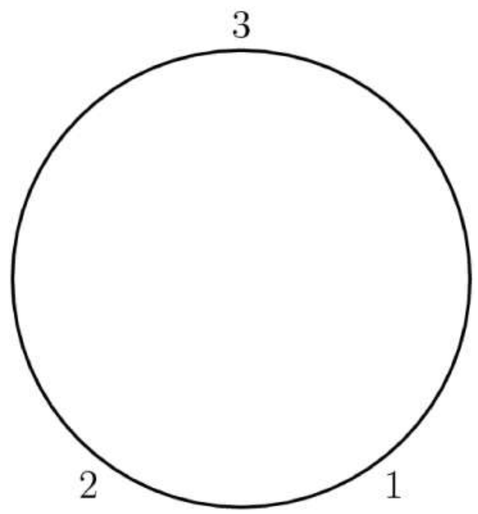
Solution 2

Like Solution 1, we note that the numbers from $1, 2, 3, 4,$ and 5 are always going to be able to be made. Also, by selecting all numbers but one of $1, 2, 3, 4, 5$, we can obtain the numbers from $10, 11, 12, 13, 14,$ and 15 will always be made from all the numbers. So we must check only 6 through 9. But if one circle can make a 6, we can select all of the other numbers and get a 9. Similarly, we can do the same for 7 and 8. So we must check only for 6 and 7.

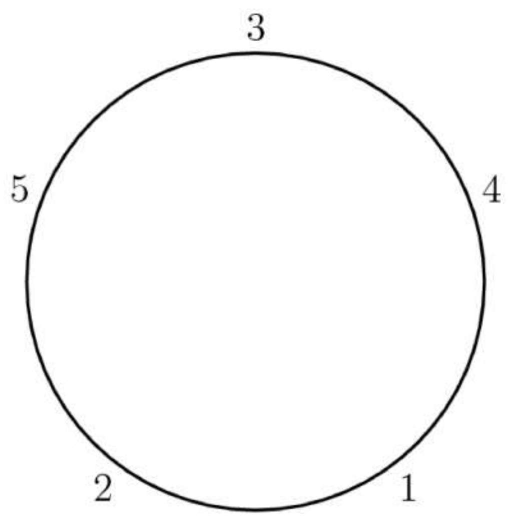
We can make 6 by having $4, 2$, or $3, 2, 1$, or $5, 1$. We can start with the group of three. To seperate $3, 2, 1$ from eachother, they must be grouped two together and one seperate, like this.



Now, we note that x is next to both blank spots, so we can't have a number from one of the pairs. So since we can't have 1, because it is part of the $5, 1$ pair, and we can't have 2 there, because it's part of the $4, 2$ pair, we must have 3 inserted into the x spot. We can insert 1 and 2 in y and z interchangably, since reflections are considered the same.

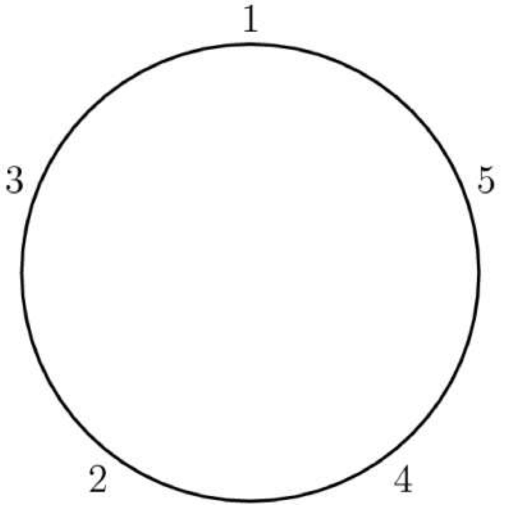


We have 4 and 5 left to insert. We can't place the 4 next to the 2 or the 5 next to the 1, so we must place 4 next to the 1 and 5 next to the 2.



This is the only solution to make 6 "bad."

Next we move on to 7, which can be made by $3, 4$, or $5, 2$, or $4, 2, 1$. We do this the same way as before. We start with the three group. Since we can't have 4 or 2 in the top slot, we must have one there, and 4 and 2 are next to eachother on the bottom. When we have 3 and 5 left to insert, we place them such that we don't have the two pairs adjacent.



This is the only solution to make 7 "bad."

We've covered all needed cases, and the two examples we found are distinct, therefore the answer is $\boxed{\textbf{(B) } 2}$.

See Also

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2014 AMC 10B Problems/Problem 25

Problem

In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0<N<10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

$\textbf{(A) }\frac{32}{79}\qquad \textbf{(B) }\frac{161}{384}\qquad \textbf{(C) }\frac{63}{146}\qquad \textbf{(D) }\frac{7}{16}\qquad \textbf{(E) }\frac{1}{2}$

Solution

Notice that the probabilities are symmetrical around the fifth lily pad. If the frog is on the fifth lily pad, there is a $\frac{1}{2}$ chance that it escapes and a $\frac{1}{2}$ that it gets eaten. Now, let P_k represent the probability that the frog escapes if it is currently on pad k. We get the following system of 5 equations:

$$P_1=\frac{9}{10}\cdot P_2$$

$$P_2=\frac{1}{5}\cdot P_1+\frac{4}{5}\cdot P_3$$

$$P_3=\frac{3}{10}\cdot P_2+\frac{7}{10}\cdot P_4$$

$$P_4=\frac{2}{5}\cdot P_3+\frac{3}{5}\cdot P_5$$

$$P_5=\frac{1}{2}$$

We want to find P_1, since the frog starts at pad 1. Solving the above system yields $P_1=\frac{63}{146}$, so the answer is $\boxed{(C)}$.

See Also

2014 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2014))	
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