2015 AMC 10B

Problem 1

What is the value of $2-(-2)^{-2}$?

$2-(-2)^{-2}$的值是多少？

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ \frac{1}{16} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ \frac{9}{4} \qquad \textbf{(E)}\ 6$

Problem 2

Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

Marie 连续做了 3 项耗时相同的任务，中间没有休息。她从下午 1:00 开始做第一个任务，在下午 2:40 完成第二个任务。她何时完成第三个任务?

$\textbf{(A)}$ 3:10 PM $\qquad \textbf{(B)}$ 3:30 PM $\qquad \textbf{(C)}$ 4:00 PM $\qquad \textbf{(D)}$ 4:10 PM $\qquad \textbf{(E)}$ 4:30 PM

Problem 3

Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

Isaac 把一个整数写了 2 遍，另一个整数写了 3 遍，这 5 个数之和为 100.其中一个数是 28，另一个数是多少？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 14 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Problem 4

Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of pizza they consumed?

4 个兄弟姐妹订购了一块大的比萨饼.Alex 吃了整块的$\frac{1}{5}$，Beth 吃了$\frac{1}{3}$，Cyril 吃了$\frac{1}{4}$，Dan 吃了剩下的。下面那个选项是将这 4 个兄弟姐妹按照他们所吃的比萨饼的量降序排列？

$\textbf{(A)}$ Alex, Beth, Cyril, Dan $\textbf{(B)}$ Beth, Cyril, Alex, Dan $\textbf{(C)}$ Beth, Cyril, Dan, Alex
$\textbf{(D)}$ Beth, Dan, Cyril, Alex $\textbf{(E)}$ Dan, Beth, Cyril, Alex

Problem 5

David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

David，Hikmet，Jack，Marta，Rand，Todd 和其他 6 人参加了一场 12 人的比赛。Rand 排在 Hikmet 前面 6 位，Marta 排在 Jack 后面 1 位，David 排在 Hikmet 的后面 2 位，Jack 排在 Todd 后面 2 位，Todd 排在 Rand 后面 1 位。Marta 排名第 6。排在第 8 的是谁?

$\textbf{(A)}$ David $\quad$ $\textbf{(B)}$ Hikmet $\quad$ $\textbf{(C)}$ Jack $\quad$ $\textbf{(D)}$ Rand $\quad$ $\textbf{(E)}$ Todd

Problem 6

Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

Marley 每周每天仅练习一项运动。她每周有三天跑步，但是不会有连续的两天都跑步，周一她打篮球，两天后打高尔夫球，她也游泳和打网球，但是不会在跑步或游泳的后一天打球。Marley 是周几游泳的？

$\textbf{(A)}$ Sunday $\quad$ $\textbf{(B)}$ Tuesday $\quad$ $\textbf{(C)}$ Thursday $\quad$ $\textbf{(D)}$ Friday $\quad$ $\textbf{(E)}$ Saturday

Problem 7

Consider the operation "minus the reciprocal of," defined by $a \diamond b = a - \frac{1}{b}$. What is $((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3))$?

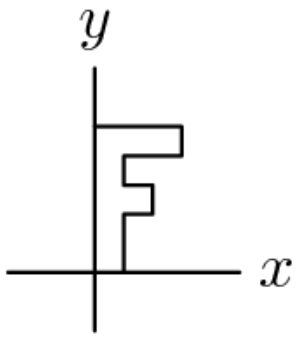
考虑操作符“减去某个数的倒数”，定义为$a \diamond b = a - \frac{1}{b}$。那么$((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3))$是多少？

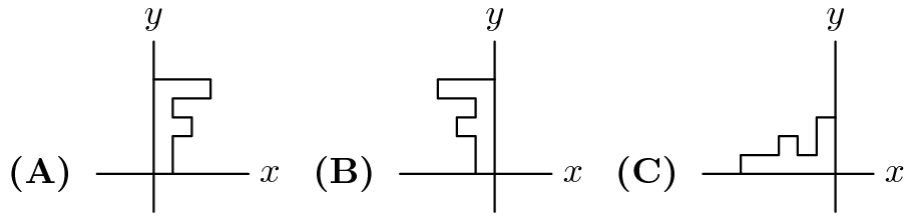
$\textbf{(A)}\ -\frac{7}{30} \qquad \textbf{(B)}\ -\frac{1}{6} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{7}{30}$

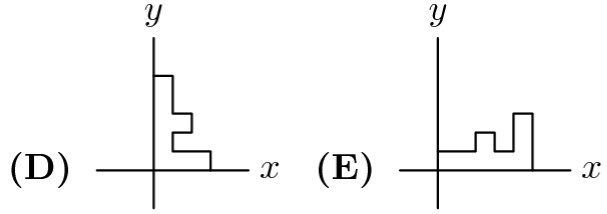
Problem 8

The letter F shown below is rotated 90° clockwise around the origin, then reflected in the y-axis, and then rotated a half turn around the origin. What is the final image?

如下所示的字母 F 先绕着原点顺时针旋转 90°，然后关于 y 轴作对称图像，最后绕着原点转半圈，哪个是最后的图形？



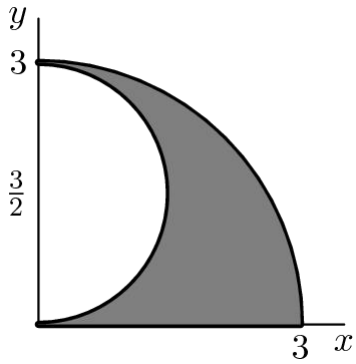




Problem 9

The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0,\frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata?

下图阴影部分区域称作鲨鱼的鱼鳍弯刀。是达芬奇研究的一个图形。它是由圆心在（0，0），半径为 3 的圆在第一象限的部分，圆心在$(0,\frac{3}{2})$，半径为$\frac{3}{2}$的圆在第一象限的部分，和连接（0，0）与（3，0）的线段所包围而成。鲨鱼的鱼鳍弯刀的面积是多少?



(A) $\frac{4\pi}{5}$ **(B)** $\frac{9\pi}{8}$ **(C)** $\frac{4\pi}{3}$ **(D)** $\frac{7\pi}{5}$ **(E)** $\frac{3\pi}{2}$

Problem 10

What is the sign and units digit of the product of all the odd negative integers strictly greater than -2015?

严格大于-2015 的所有负奇数的乘积的符号和个位数是什么？

（A） It is a negative number ending with a1．｜它是一个负数，以 1 结尾.

（B） It is a positive number ending with a 1．｜它是一个正数，以 1 结尾.

（C） It is a negative number ending with a 5．｜它是一个负数，以 5 结尾.

（D） It is a positive number ending with a 5．｜它是一个正数，以 5 结尾.

（E） It is a negative number ending with a 0．｜它是一个负数，以 0 结尾.

Problem 11

Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

在所有各个位上数字都是质数，且小于 100 的正整数中，随机选择 1 个数。所选择的数是个质数的概率是多少？

$\textbf{(A)}\ \frac{8}{99} \qquad \textbf{(B)}\ \frac{2}{5} \qquad \textbf{(C)}\ \frac{9}{20} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{9}{16}$

Problem 12

For how many integers x is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?

有多少个这样的整数的 *x*，使得点（*x*，-*x*）在圆心为（5，5），半径为 10 的圆上或其内部？

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Problem 13

The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

直线$12x + 5y = 60$与坐标轴形成一个三角形。这个三角形的三条高的长度之和为多少？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ \frac{360}{17} \qquad \textbf{(C)}\ \frac{107}{5} \qquad \textbf{(D)}\ \frac{43}{2} \qquad \textbf{(E)}\ \frac{281}{13}$

Problem 14

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?

a，*b*，*c* 是 3 个不同的 1 位数，方程$(x - a)(x - b) + (x - b)(x - c) = 0$的根的和的最大值是多少？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 15.5 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 16.5 \qquad \textbf{(E)}\ 17$

Problem 15

The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?

哈姆雷特小镇上，每匹马和 3 个人搭配，每头牛和 4 只绵羊搭配，每个人和 3 只鸭子搭配，下面哪个数不可能是哈姆雷特小镇上人、马、绵羊、牛和鸭子的总数？

$\textbf{(A)}\ 41 \qquad \textbf{(B)}\ 47 \qquad \textbf{(C)}\ 59 \qquad \textbf{(D)}\ 61 \qquad \textbf{(E)}\ 66$

Problem 16

Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?

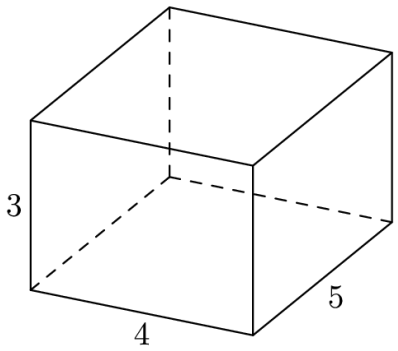
Al，Bill 和 Cal 每个人被随机分配一个 1 到 10 之间（包含 1 和 10）的整数，且每个人的数字都不一样。Al 的数正好是 Bill 的数的整数倍，同时 Bill 的数也是 Cal 的数的整数倍的概率是多少？

$\textbf{(A)}\ \frac{9}{1000} \qquad \textbf{(B)}\ \frac{1}{90} \qquad \textbf{(C)}\ \frac{1}{80} \qquad \textbf{(D)}\ \frac{1}{72} \qquad \textbf{(E)}\ \frac{2}{121}$

Problem 17

When the centers of the faces of the right rectangular prism shown below are joined to create an octahedron, what is the volume of the octahedron?

当如下图所示的长方形的每个面的中心用直线连起来形成一个八边形，这个八边形的体积是多少？



$\textbf{(A)}\ \frac{75}{12} \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 10\sqrt{2} \qquad \textbf{(E)}\ 15$

Problem 18

Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

Johann 有 64 枚标准硬币，她把这些硬币一起扔，如果有反面朝上的硬币，她就将这些反面朝上的硬币重新再扔一次。第二次扔了之后若仍然有反面朝上的硬币，她就将这些反面朝上的硬币再扔第三次，那么现在正面朝上的硬币的个数的期望值是多少？

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 56 \qquad \textbf{(E)}\ 64$

Problem 19

In $\triangle ABC, \angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

$\triangle ABC$中，$\angle C = 90^\circ$且$AB = 12$。在三角形的外部作出正方形 *ABXY* 和 *ACWZ*。点 *X*，*Y*，*Z* 和 *W* 在同一个圆上，这个三角形的周长是多少？

$\textbf{(A)}\ 12 + 9\sqrt{3} \qquad \textbf{(B)}\ 18 + 6\sqrt{3} \qquad \textbf{(C)}\ 12 + 12\sqrt{2} \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Problem 20

Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

蚂蚁 Erin 从正方体的某个给定的顶点开始出发，爬过恰好 7 条棱，必须满足她每个顶点都经过恰好一次，而且最后她发现无法沿着某条棱再回到她出发的顶点。满足这些条件的所有可能的路径有多少条？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 24$

Problem 21

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

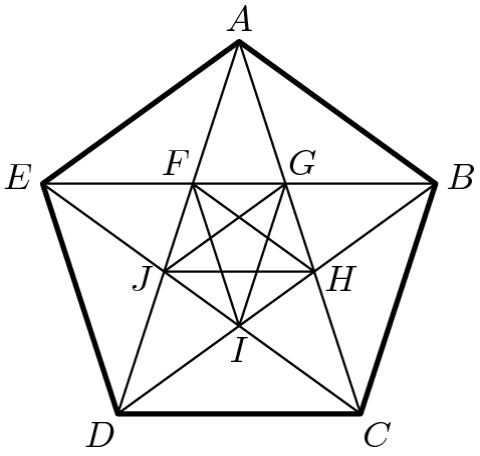
Cozy 这只猫和 Dash 这条狗在有一定数目台阶的楼梯上爬楼梯，然而，Cozy 和 Dash 都是跳着爬楼梯，而不是一次一个台阶的走。Cozy 每跳一次就爬 2 个台阶（如果最后只剩一个台阶，他就直接跳过最后这个台阶）。Dash 每跳一次就爬 5 个台阶（如果最后剩下不足 5 个台阶，他就直接一步跳过去）。假设最终到达楼梯的顶部，Dash 跳的次数比 Cozy 少了 19 次。s 表示这个楼梯所有可能的台阶数的和。那么 s 的各个位上数字之和为多少?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

Problem 22

In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?

如下图所示，$ABCDE$ 是一个正五边形，$AG=1$。那么 $FG+JH+CD$ 等于多少?



(A) 3 **(B)** $12-4\sqrt{5}$ **(C)** $\frac{5+2\sqrt{5}}{3}$ **(D)** $1+\sqrt{5}$ **(E)** $\frac{11+11\sqrt{5}}{10}$

Problem 23

Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let s denote the sum of the four least possible values of n. What is the sum of the digits of s?

n 是一个大于 4 的正整数，满足 n！的十进制表示*以 k* 个 0 结尾，$(2n)!$ 的十进制表示以 3*k* 个 0 结尾。*s* 表示 *n* 的最小的 4 个值之和，则 *s* 的各个位上数字之和是多少？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Problem 24

Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0 = (0,0)$ facing to the east and walks one unit, arriving at $p_1 = (1,0)$.
For $n = 1, 2, 3, \ldots$, right after arriving at the point p_n, if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1}, he does that. Otherwise, he walks one unit straight ahead to reach p_{n+1}. Thus the sequence of points continues $p_2 = (1,1), p_3 = (0,1), p_4 = (-1,1), p_5 = (-1,0)$, and so on in a counterclockwise spiral pattern. What is p_{2015}?

蚂蚁 Aaron 在坐标平面上依照下面的规则爬行:他从原点$p_0 = (0,0)$，出发，面朝东，向前走一个单位，到达$p_1 = (1,0)$。对于 *n*=1，2，3，在到达点p_{n+1}后，若 Aaron 可以向左转 90°，向前走一个单位到达一个以前没访问过的点，那么他就这样做。否则，他就直接向前走一步到p_{n+1}。因此接下来他经过的点是$p_2 = (1,1), p_3 = (0,1), p_4 = (-1,1), p_5 = (-1,0)$，以此类推，形成一个逆时针螺旋状，$p_{2015}$的坐标是多少？

$\textbf{(A)}\ (-22,-13) \qquad \textbf{(B)}\ (-13,-22) \qquad \textbf{(C)}\ (-13,22) \qquad \textbf{(D)}\ (13,-22) \qquad \textbf{(E)}\ (22,-13)$

Problem 25

A rectangular box measures $a \times b \times c$, where $a, b,$ and c are integers and $1 \leq a \leq b \leq c$. The volume and surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

一个长方形的盒子大小是$a \times b \times c$，其中 a，b，c 都是整数，且$1 \leq a \leq b \leq c$。盒子的体积和表面积相等。那么可能的有序对(a, b, c)有多少个？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 26$

2015 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	A	C	B	E	A	E	B	C	B	A	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	B	C	B	D	C	A	D	D	B	D	B	