2015 AMC 10B Problems/Problem 1

Problem

What is the value of $2-(-2)^{-2}$?

$\textbf{(A)}\ -2\qquad\textbf{(B)}\ \dfrac{1}{16}\qquad\textbf{(C)}\ \dfrac{7}{4}\qquad\textbf{(D)}\ \dfrac{9}{4}\qquad\textbf{(E)}\ 6$

Solution

$$2-(-2)^{-2}=2-\frac{1}{(-2)^2}=2-\frac{1}{4}=\frac{8}{4}-\frac{1}{4}=\boxed{\textbf{(C)}\ \frac{7}{4}}$$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by First Problem	Followed by Problem 2
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 2

Problem

Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

$\textbf{(A) }\text{3:10 PM}\qquad\textbf{(B) }\text{3:30 PM}\qquad\textbf{(C) }\text{4:00 PM}\qquad\textbf{(D) }\text{4:10 PM}\qquad\textbf{(E) }\text{4:30 PM}$

Solution

Marie does her work twice in 1 hour and 40 minutes. Therefore, one task should take 50 minutes to finish. 50 minutes after $2:40$ PM is $3:30$ PM, so our answer is $\boxed{\textbf{(B) }\text{3:30 PM}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 1	Followed by Problem 3
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 3

Problem

Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 14 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Solution

Let the first number be x and the second be y. We have $2x+3y=100$. We are given one of the numbers is 28. If x were to be 28, y would not be an integer, thus $y=28$. $2x+3(28)=100$, which gives $x=\boxed{\textbf{(A)}\ 8}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 2	Followed by Problem 4
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 4

Problem 4

Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of pizza they consumed?

$\textbf{(A)}\ \text{Alex, Beth, Cyril, Dan}$ $\textbf{(B)}\ \text{Beth, Cyril, Alex, Dan}$

$\textbf{(C)}\ \text{Beth, Cyril, Dan, Alex}$ $\textbf{(D)}\ \text{Beth, Dan, Cyril, Alex}$

$\textbf{(E)}\ \text{Dan, Beth, Cyril, Alex}$

Solution

Let the pizza have 60 slices, since the least common multiple of $(5, 3, 4) = 60$. Therefore, Alex ate $\frac{1}{5} \times 60 = 12$ slices, Beth ate $\frac{1}{3} \times 60 = 20$ slices, and Cyril ate $\frac{1}{4} \times 60 = 15$ slices. Dan must have eaten $60 - (12 + 20 + 15) = 13$ slices. In decreasing order, we see the answer is $\boxed{\textbf{(C)}\ \text{Beth, Cyril, Dan, Alex}}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 3	Followed by Problem 5
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 5

Problem

David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

$\textbf{(A) } \text{David} \qquad\textbf{(B) } \text{Hikmet} \qquad\textbf{(C) } \text{Jack} \qquad\textbf{(D) } \text{Rand} \qquad\textbf{(E) } \text{Todd}$

Solution

Because Marta was 6th, Jack was 5th, so Todd was 3rd. Thus, Rand was 2nd and the 8th place finisher was $\boxed{\textbf{(B) } \text{Hikmet}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 4	Followed by Problem 6
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 6

Problem

Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

$\textbf{(A)}\ \text{Sunday} \qquad \textbf{(B)}\ \text{Tuesday} \qquad \textbf{(C)}\ \text{Thursday} \qquad \textbf{(D)}\ \text{Friday} \qquad \textbf{(E)}\ \text{Saturday}$

Solution

Marley does basketball on Monday and golf on Wednesday. Since there are four consecutive days between Wednesday and Monday exclusive, she must run on Tuesday. Thus, she must also run on Thursday and Saturday or Friday and Sunday. She can't run on Thursday and Saturday because she would have to do tennis after running, which is not allowed. Thus, she runs on Friday and Sunday, so she must do tennis on Thursday, so she swims on $\boxed{\textbf{(E)}\ \text{Saturday}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 5	Followed by Problem 7
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 7

Problem

Consider the operation "minus the reciprocal of," defined by $a \diamond b = a - \frac{1}{b}$. What is $((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3))$?

$\textbf{(A) } -\frac{7}{30} \qquad \textbf{(B) } -\frac{1}{6} \qquad \textbf{(C) } 0 \qquad \textbf{(D) } \frac{1}{6} \qquad \textbf{(E) } \frac{7}{30}$

Solution

$1 \diamond 2 = 1 - \frac{1}{2} = \frac{1}{2}$, so $(1 \diamond 2) \diamond 3 = \frac{1}{2} \diamond 3 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. Also, $2 \diamond 3 = 2 - \frac{1}{3} = \frac{5}{3}$, so $1 \diamond (2 \diamond 3) = 1 - \frac{1}{5/3} = 1 - \frac{3}{5} = \frac{2}{5}$. Thus, $((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3)) = \frac{1}{6} - \frac{2}{5} = \boxed{\textbf{(A) } -\frac{7}{30}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 6	Followed by Problem 8
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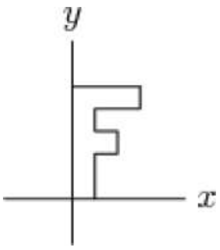
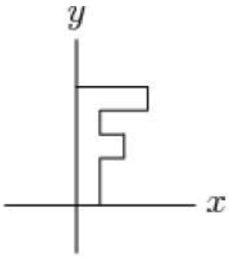
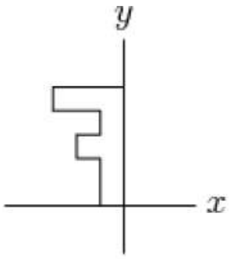
2015 AMC 10B Problems/Problem 8

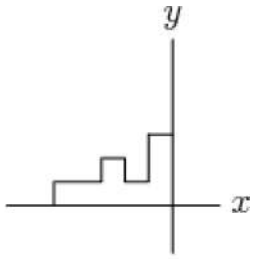
Problem

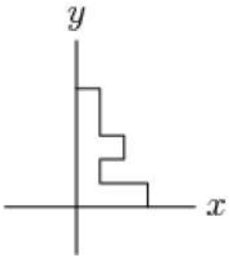
The letter F shown below is rotated 90° clockwise around the origin, then reflected in the y-axis, and then rotated a half turn around the origin. What is the final image?

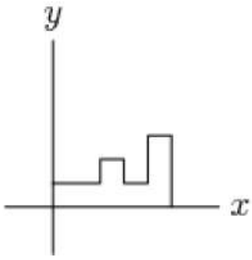


(A)


(B)


(C)


(D)


(E)


Solution

The first rotation moves the base of the F to the negative y-axis, and the stem to the positive x-axis. The reflection then moves the stem to the negative x-axis, with the base unchanged. Then the half turn moves the stem to the positive x axis and the base to the positive y-axis, choice $\boxed{\textbf{(E)}}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
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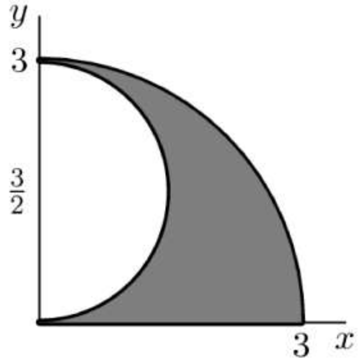
All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 9

Problem

The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0,\frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata?



$\textbf{(A) } \frac{4\pi}{5} \qquad \textbf{(B) } \frac{9\pi}{8} \qquad \textbf{(C) } \frac{4\pi}{3} \qquad \textbf{(D) } \frac{7\pi}{5} \qquad \textbf{(E) } \frac{3\pi}{2}$

Solution

The area of the shark's fin falcata is just the area of the quarter-circle minus the area of the semicircle. The quarter-circle has radius 3 so it has area $\frac{9\pi}{4}$. The semicircle has radius $\frac{3}{2}$ so it has area $\frac{9\pi}{8}$. Thus, the shaded area is $\frac{9\pi}{4}-\frac{9\pi}{8}=\boxed{\textbf{(B) } \frac{9\pi}{8}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 8	Followed by Problem 10
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Category: Prealgebra Problems

2015 AMC 10B Problems/Problem 10

Problem

What are the sign and units digit of the product of all the odd negative integers strictly greater than -2015?

Solution

Since $-5 > -2015$, the product must end with a 5.

The multiplicands are the odd negative integers from -1 to -2013. There are $\frac{2013+1}{2} = 1007$ of these numbers. Since $(-1)^{1007} = -1$, the product is negative.

Therefore, the answer must be $\boxed{\textbf{(C)}\ \text{It is a negative number ending with a 5.}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 9	Followed by Problem 11
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 11

Problem

Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

$\textbf{(A)}\,\frac{8}{99}\qquad\textbf{(B)}\,\frac{2}{5}\qquad\textbf{(C)}\,\frac{9}{20}\qquad\textbf{(D)}\,\frac{1}{2}\qquad\textbf{(E)}\,\frac{9}{16}$

Solution

The one digit prime numbers are 2, 3, 5, and 7. So there are a total of $4*4=16$ ways to choose a two digit number with both digits as primes and 4 ways to choose a one digit prime, for a total of $4+16=20$ ways. Out of these 2, 3, 5, 7, 23, 37, 53, and 73 are prime. Thus the probability is $\frac{8}{20}=\boxed{\textbf{(B)}\frac{2}{5}}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 12

Problem

For how many integers x is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Solution

The equation of the circle is $(x-5)^2+(y-5)^2=100$. Plugging in the given conditions we have $(x-5)^2+(-x-5)^2 \le 100$. Expanding gives: $x^2-10x+25+x^2+10x+25 \le 100$, which simplifies to $x^2 \le 25$ and therefore $x \le 5$ or $x \ge -5$. So x ranges from -5 to 5, for a total of $\boxed{\textbf{(A)}\ 11}$ values.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 11	Followed by Problem 13
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 13

Problem

The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ \frac{360}{17} \qquad \textbf{(C)}\ \frac{107}{5} \qquad \textbf{(D)}\ \frac{43}{2} \qquad \textbf{(E)}\ \frac{281}{13}$

Solution

We find the x-intercepts and the y-intercepts to find the intersections of the axes and the line. If $x = 0$, then $y = 12$. If y is 0, then $x = 5$. Our three vertices are $(0, 0)$, $(5, 0)$, and $(0, 12)$. Two of our altitudes are 5 and 12. Since the area of the triangle is 30, letting h be our hypotenuse, our final altitude has to be $30(2)/h$. By the Pythagorean Theorem, $h = 13$, so the sum of our altitudes is $\boxed{\textbf{(E)}\frac{281}{13}}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
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Category: Introductory Geometry Problems

2015 AMC 10B Problems/Problem 14

Contents

Problem

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

$\textbf{(A) }15\qquad\textbf{(B) }15.5\qquad\textbf{(C) }16\qquad\textbf{(D) }16.5\qquad\textbf{(E) }17$

Solution

Expanding the equation and combining like terms results in $2x^2-(a+2b+c)x+(ab+bc)=0$. By Vieta's formula the sum of the roots is $\dfrac{-[-(a+2b+c)]}{2}=\dfrac{a+2b+c}{2}$. To maximize this expression we want b to be the largest, and from there we can assign the next highest values to a and c. So let $b=9$, $a=8$, and $c=7$. Then the answer is $\dfrac{8+18+7}{2}=\boxed{\textbf{(D)}16.5}$.

Solution 2

Factoring out $(x-b)$ from the equation yields $(x-b)(2x-(a+c))=0\Rightarrow(x-b)(x-\dfrac{a+c}{2})=0$. Therefore the roots are b and $\dfrac{a+c}{2}$. Because b must be the larger root to maximize the sum of the roots, letting $a,b,$ and c be $8,9,$ and 7 respectively yields the sum $9+\dfrac{8+7}{2}=9+7.5=\boxed{\textbf{(D) }16.5}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 13	Followed by Problem 15
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All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 15

Problem

The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?

$\textbf{(A) }41\qquad\textbf{(B) }47\qquad\textbf{(C) }59\qquad\textbf{(D) }61\qquad\textbf{(E) }66$

Solution

Let the amount of people be p, horses be h, sheep be s, cows be c, and ducks be d. We know

$$3h = p$$

$$4c = s$$

$$3p = d$$

Then the total amount of people, horses, sheep, cows, and ducks may be written as $3h + h + 4c + c + 9h = 13h + 5c$. Looking through the options, we see 47 is impossible to make for integer values of h and c. So the answer is $\boxed{\textbf{(B)} 47}$.

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 14	Followed by Problem 16
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2015 AMC 10B Problems/Problem 16

Problem

Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?

Solution

We can solve this problem with a brute force approach.

- If Cal's number is 1:
  - If Bill's number is 2, Al's can be any of $4, 6, 8, 10$.
  - If Bill's number is 3, Al's can be any of $6, 9$.
  - If Bill's number is 4, Al's can be 8.
  - If Bill's number is 5, Al's can be 10.
  - Otherwise, Al's number could not be a whole number multiple of Bill's.
- If Cal's number is 2:
  - If Bill's number is 4, Al's can be 8.
  - Otherwise, Al's number could not be a whole number multiple of Bill's while Bill's number is still a whole number multiple of Cal's.
- Otherwise, Bill's number must be greater than 5, i.e. Al's number could not be a whole number multiple of Bill's.

Clearly, there are exactly 9 cases where Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's. Since there are $10 * 9 * 8$ possible permutations of the numbers Al, Bill, and Cal were assigned, the probability that this is true is $\frac{9}{10 * 9 * 8} = \boxed{\textbf{(C)}\ \frac{1}{80}}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 15	Followed by Problem 17
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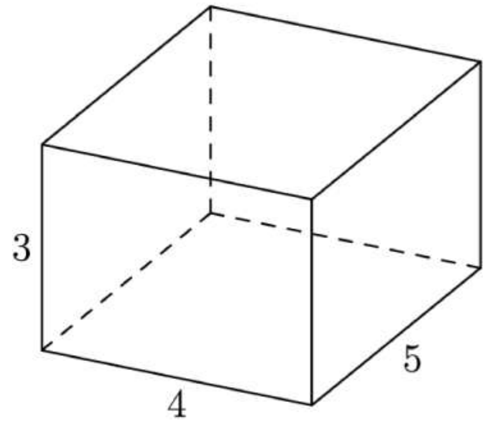
All AMC 10 Problems and Solutions	

2015 AMC 10B Problems/Problem 17

Problem

The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of this octahedron?



$\textbf{(A) } \dfrac{75}{12} \qquad \textbf{(B) } 10 \qquad \textbf{(C) } 12 \qquad \textbf{(D) } 10\sqrt2 \qquad \textbf{(E) } 15$

Solution

The octahedron is just two congruent pyramids glued together by their base. The base of one pyramid is a rhombus with diagonals 4 and 5, for an area $A=10$. The height h, of one pyramid, is $\frac{3}{2}$, so the volume of one pyramid is $\frac{Ah}{3}=5$. Thus, the octahedron has volume $2 \cdot 5 = \boxed{\textbf{(B) } 10}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
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All AMC 10 Problems and Solutions	

Category: Introductory Geometry Problems

2015 AMC 10B Problems/Problem 18

Problem

Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

$\textbf{(A) } 32 \qquad\textbf{(B) } 40 \qquad\textbf{(C) } 48 \qquad\textbf{(D) } 56 \qquad\textbf{(E) } 64$

Solution

Every time the coins are flipped, half of them are expected to turn up heads. The expected number of heads on the first flip is 32, on the second flip is is 16, and on the third flip it is 8. Adding these gives $\boxed{\textbf{(D) } 56}$

See Also

2015 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2015))	
Preceded by Problem 17	Followed by Problem 19
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2015 AMC 10B Problems/Problem 19

Problem

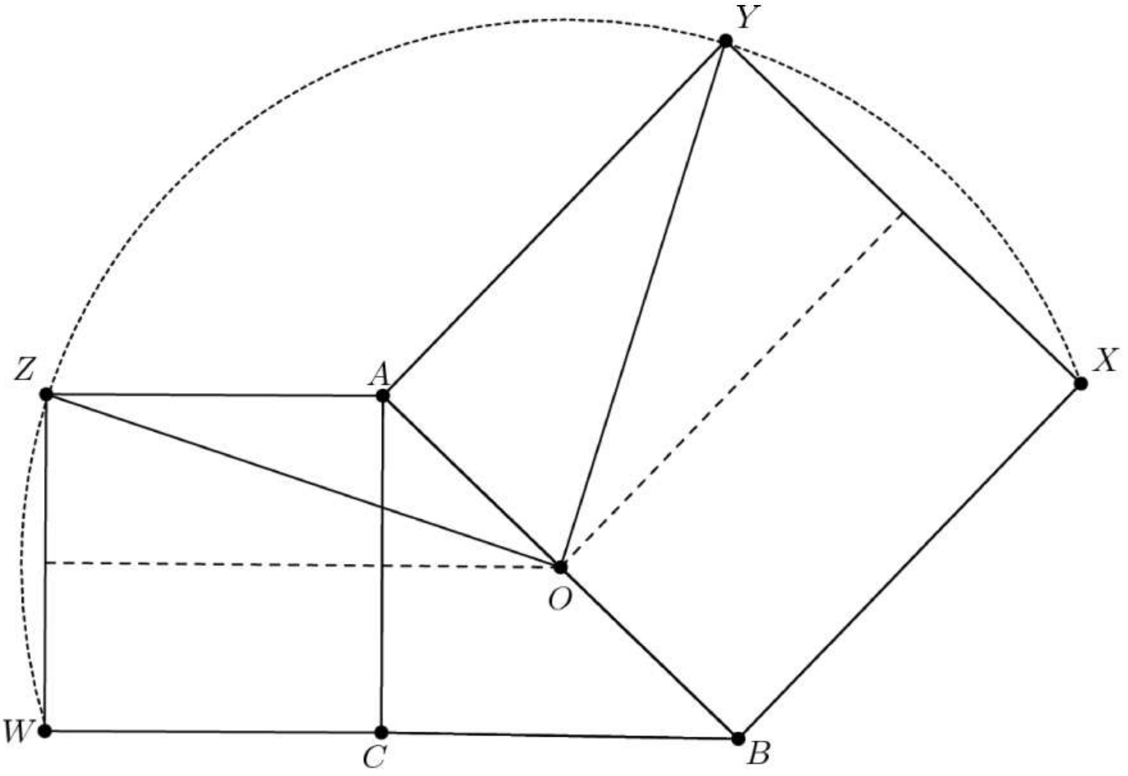
In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

$\textbf{(A)}\ 12+9\sqrt{3} \qquad \textbf{(B)}\ 18+6\sqrt{3} \qquad \textbf{(C)}\ 12+12\sqrt{2} \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Solution

The center of the circle lies on the perpendicular bisectors of both chords ZW and YX. Therefore we know the center of the circle must also be the midpoint of the hypotenuse. Let this point be O. Draw perpendiculars to ZW and YX from O, and connect OZ and OY. $OY^2 = 6^2 + 12^2 = 180$. Let $AC = a$ and $BC = b$. Then $\left(\frac{a}{2}\right)^2 + \left(a + \frac{b}{2}\right)^2 = OZ^2 = OY^2 = 180$. Simplifying this gives $\frac{a^2}{4} + \frac{b^2}{4} + a^2 + ab = 180$. But by pythagorean theorem on $\triangle ABC$, we know $a^2 + b^2 = 144$, because $AB = 12$. Thus

. So our equation simplifies further to $a^2 + ab = 144$. However $a^2 + b^2 = 144$, so $a^2 + ab = a^2 + b^2$, which means $ab = b^2$, or $a = b$. Aha! This means $\triangle ABC$ is just an isosceles right triangle, so $AC = BC = \frac{12}{\sqrt{2}} = 6\sqrt{2}$, and thus the perimeter is $\boxed{\textbf{(C)} 12 + 12\sqrt{2}}$.



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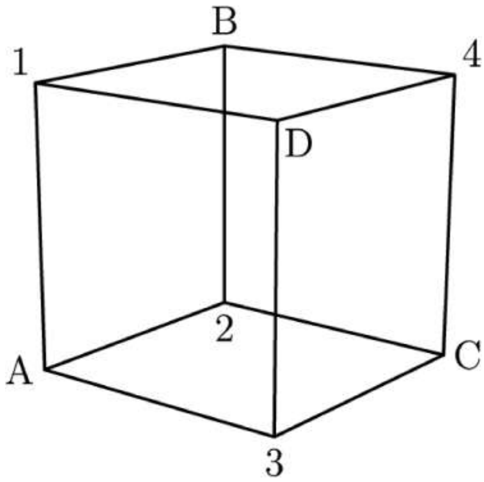
2015 AMC 10B Problems/Problem 20

Problem

Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. how many paths are there meeting these conditions? $\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 24$

Solution



We label the vertices of the cube as different letters and numbers shown above. We label these so that Erin can only crawl from a number to a letter or a letter to a number (this can be seen as a coloring argument). The starting point is labeled "A".

If we define a "move" as each time Erin crawls along a single edge from 1 vertex to another, we see that after 7 moves, Erin must be on a numbered vertex. Since this numbered vertex cannot be 1 unit away from A (since Erin cannot crawl back to A), this vertex must be 4.

Therefore, we now just need to count the number of paths from A to 4. To count this, we can work backwards. There are 3 choices for which vertex Erin was at before she moved to A, and 2 choices for which vertex Erin was at 2 moves before A. All of Erin's previous moves were forced, so the total number of legal paths from A to 4 is $3 \cdot 2 = \boxed{6}$.

See Also

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2015 AMC 10B Problems/Problem 21

Contents

Problem

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose the Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

$\textbf{(A) }9\qquad\textbf{(B) }11\qquad\textbf{(C) }12\qquad\textbf{(D) }13\qquad\textbf{(E) }15$

Solution 1

We can translate this wordy problem into this simple equation:

$$\left\lceil \frac{s}{2} \right\rceil - 19 = \left\lceil \frac{s}{5} \right\rceil$$

We will proceed to solve this equation via casework.

Case 1: $\left\lceil \frac{s}{2} \right\rceil = \frac{s}{2}$

Our equation becomes $\frac{s}{2} - 19 = \frac{s}{5} + \frac{j}{5}$, where $j \in \{0,1,2,3,4\}$ Using the fact that s is an integer, we quickly find that $j=1$ and $j=4$ yield $s=64$ and $s=66$, respectively.

Case 2: $\left\lceil \frac{s}{2} \right\rceil = \frac{s}{2} + \frac{1}{2}$

Our equation becomes $\frac{s}{2} + \frac{1}{2} - 19 = \frac{s}{5} + \frac{j}{5}$, where $j \in \{0,1,2,3,4\}$ Using the fact that s is an integer, we quickly find that $j=2$ yields $s=63$.

Summing up we get $63+64+66=193$. The sum of the digits is $\boxed{\textbf{(D) }13}$.

Solution 2

We're looking for natural numbers x such that $\left\lceil \frac{x}{5} \right\rceil + 19 = \left\lceil \frac{x}{2} \right\rceil$.

Let's call $x = 10a + b$. We now have $2a + \left\lceil \frac{b}{5} \right\rceil + 19 = 5a + \left\lceil \frac{b}{2} \right\rceil$, or

$$19 - 3a = \left\lceil \frac{b}{2} \right\rceil - \left\lceil \frac{b}{5} \right\rceil.$$

Obviously, since $b \leq 10$, this will not work for any value under 6. In addition, since obviously $\frac{b}{2} \geq \frac{b}{5}$, this will not work for any value over six, so we have $a = 6$ and $\left\lceil \frac{b}{2} \right\rceil - \left\lceil \frac{b}{5} \right\rceil = 1$.

This can be achieved when $\left\lceil \frac{b}{5} \right\rceil = 1$ and $\left\lceil \frac{b}{2} \right\rceil = 2$, or when $\left\lceil \frac{b}{5} \right\rceil = 2$ and $\left\lceil \frac{b}{2} \right\rceil = 3$.

Case One:

We have $b \leq 5$ and $3 \leq b \leq 4$, so $b = 3, 4$.

Case Two:

We have $6 \leq b \leq 9$ and $5 \leq b \leq 6$, so $b = 6$.

We then have $63 + 64 + 66 = 193$, which has a digit sum of $\boxed{13}$.

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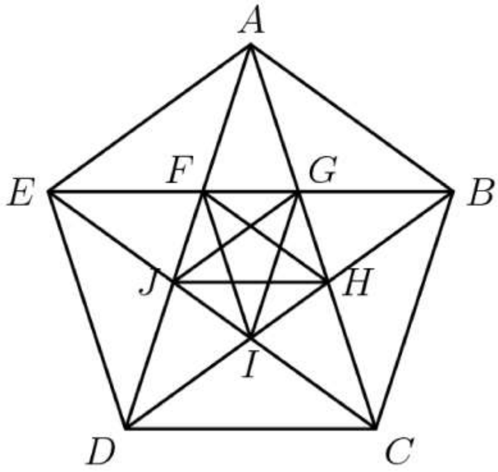
2015 AMC 10B Problems/Problem 22

Problem

In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?



$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 12 - 4\sqrt{5} \qquad \textbf{(C)}\ \dfrac{5 + 2\sqrt{5}}{3} \qquad \textbf{(D)}\ 1 + \sqrt{5} \qquad \textbf{(E)}\ \dfrac{11 + 11\sqrt{5}}{10}$

Solution

Triangle AFG is isosceles, so $AG = AF = 1$. Using the symmetry of pentagon $FGHIJ$, notice that . Therefore, $JH = AF = 1$.

Since $\triangle AJH \sim \triangle AFG$, $\dfrac{JH}{AF + FJ} = \dfrac{1}{1 + FG} = \dfrac{FG}{FI} = \dfrac{FG}{1}$. From this, we get $FG = \dfrac{\sqrt{5} - 1}{2}$.

Since $\triangle DIJ \cong \triangle AFG$, $DJ = DI = AF = 1$. Since $\triangle AFG \sim ADC$, $\dfrac{AF}{AF + FJ + JD} = \dfrac{1}{2 + FG} = \dfrac{FG}{CD} = \dfrac{\frac{\sqrt{5}-1}{2}}{CD}$. Thus, $CD = (\sqrt{5} - 1) + (\dfrac{\sqrt{5} - 1}{2})^2 = \dfrac{\sqrt{5} + 1}{2}$

Therefore, $FG + JH + CD = \dfrac{\sqrt{5} - 1}{2} + 1 + \dfrac{\sqrt{5} + 1}{2} = \boxed{\textbf{(D)}\ 1 + \sqrt{5}}$

See Also

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2015 AMC 10B Problems/Problem 23

Problem

Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let s denote the sum of the four least possible values of n. What is the sum of the digits of s?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Solution

A trailing zero requires a factor of two and a factor of five. Since factors of two occur more often than factors of five, we can focus on the factors of five. We make a chart of how many trailing zeros factorials have:

Factorial	$0!-4!$	$5!-9!$	$10!-14!$	$15!-19!$	$20!-24!$	$25!-29!$
Zeros	0	1	2	3	4	6

We first look at the case when $n!$ has 1 zero and $(2n)!$ has 3 zeros. If $n=5,6,7$, $(2n)!$ has only 2 zeros. But for $n=8,9$, $(2n)!$ has 3 zeros. Thus, $n=8$ and $n=9$ work.

Secondly, we look at the case when $n!$ has 2 zeros and $(2n)!$ has 6 zeros. If $n=10,11,12$, $(2n)!$ has only 4 zeros. But for $n=13,14$, $(2n)!$ has 6 zeros. Thus, the smallest four values of n that work are $n=8,9,13,14$, which sum to 44. The sum of the digits of 44 is $\boxed{\textbf{(B)}\ 8}$

See Also

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2015 AMC 10B Problems/Problem 24

Contents

Problem

Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0 = (0,0)$ facing to the east and walks one unit, arriving at $p_1 = (1,0)$. For $n = 1, 2, 3, \dots$, right after arriving at the point p_n, if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1}, he does that. Otherwise, he walks one unit straight ahead to reach p_{n+1}. Thus the sequence of points continues $p_2 = (1,1), p_3 = (0,1), p_4 = (-1,1), p_5 = (-1,0)$, and so on in a counterclockwise spiral pattern. What is p_{2015}?

$\textbf{(A) } (-22,-13) \qquad \textbf{(B) } (-13,-22) \qquad \textbf{(C) } (-13,22) \qquad \textbf{(D) } (13,-22) \qquad \textbf{(E) } (22,-13)$

Solution

(Used from 2015 AMC 10/12 B Math Jam)

The first thing we would do is track Aaron's footsteps:

He starts by taking 1 step East and 1 step North, ending at $(1,1)$ after 2 steps and about to head West.

Then he takes 2 steps West and 2 steps South, ending at $(-1,-1)$ after $2+4$ steps, and about to head East.

Then he takes 3 steps East and 3 steps North, ending at $(2,2)$ after $2+4+6$ steps, and about to head West.

Then he takes 4 steps West and 4 steps South, ending at $(-2,-2)$ after $2+4+6+8$ steps, and about to head East.

From this pattern, we can notice that for any integer $k \geq 1$ he's at $(-k,-k)$ after $2+4+6+...+4k$ steps, and about to head East. There are $2k$ terms in the sum, with an average value of $(2+4k)/2 = 2k+1$, so:

$$2+4+6+...+4k = 2k(2k+1)$$

If we substitute $k=22$ into the equation: $44(45) = 1980 < 2015$. So he has 35 moves to go. This makes him end up at $(-22+35,-22) = (13,-22) \implies \boxed{\textbf{(D)} (13,-22)}$

Alternate Solution

We are given that Aaron starts at $(0,0)$, and we note that his net steps follow the pattern of $+1$ in the x-direction, $+1$ in the y-direction, -2 in the x-direction, -2 in the y-direction, $+3$ in the x-direction, $+3$ in the y-direction, and so on, where we add odd and subtract even.

We want $2+4+6+8+...+2n = 2015$, but it does not work out cleanly. Instead, we get that $2+4+6+...+2(44) = 1980$, which means that there are 35 extra steps past adding -44 in the x-direction (and the final number we add in the y-direction is -44).

So $p_{2015} = (0+1-2+3-4+5...-44+35, 0+1-2+3-4+5...-44)$.

We can group $1-2+3-4+5...-44$ as $(1-2)+(3-4)+(5-6)+...+(43-44) = -22$.

Thus $p_{2015} = (13,-22)$ $\textbf{(D)}$

2015 AMC 10B Problems/Problem 25

Problem

A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 26$

Solution

The surface area is $2(ab + bc + ca)$, the volume is abc, so $2(ab + bc + ca) = abc$.

Divide both sides by $2abc$, we get that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}.$$

First consider the bound of the variable a. Since $\frac{1}{a} < \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$, we have $a > 2$, or $a \geq 3$.

Also note that $c \geq b \geq a > 0$, we have $\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$. Thus, $\frac{1}{2} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{3}{a}$, so $a \leq 6$.

So we have $a = 3, 4, 5$ or 6.

Before the casework, let's consider the possible range for b if $\frac{1}{b} + \frac{1}{c} = k > 0$.

From $\frac{1}{b} < k$, we have $b > \frac{1}{k}$. From $\frac{2}{b} \geq \frac{1}{b} + \frac{1}{c} = k$, we have $b \leq \frac{2}{k}$. Thus $\frac{1}{k} < b \leq \frac{2}{k}$

When $a = 3$, $\frac{1}{b} + \frac{1}{c} = \frac{1}{6}$, so $b = 7, 8, \cdots, 12$. The solutions we find are $(a, b, c) = (3, 7, 42), (3, 8, 24), (3, 9, 18), (3, 10, 15), (3, 12, 12)$, for a total of 5 solutions.

When $a = 4$, $\frac{1}{b} + \frac{1}{c} = \frac{1}{4}$, so $b = 5, 6, 7, 8$. The solutions we find are $(a, b, c) = (4, 5, 20), (4, 6, 12), (4, 8, 8)$, for a total of 3 solutions.

When $a = 5$, $\frac{1}{b} + \frac{1}{c} = \frac{3}{10}$, so $b = 5, 6$. The only solution in this case is $(a, b, c) = (5, 5, 10)$.

When $a = 6$, b is forced to be 6, and thus $(a, b, c) = (6, 6, 6)$.

Thus, our answer is $\boxed{\textbf{(B)}\ 10}$

See Also

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