2016 AMC 10B

Problem 1

What is the value of $\dfrac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?

求当$a=\frac{1}{2}$时，$\dfrac{2a^{-1}+\frac{a^{-1}}{2}}{a}$的值？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \dfrac{5}{2} \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 20$

Problem 2

If $n\heartsuit m=n^3m^2$, what is $\dfrac{2\heartsuit 4}{4\heartsuit 2}$?

如果$n\heartsuit m=n^3m^2$，$\dfrac{2\heartsuit 4}{4\heartsuit 2}$是多少？

$\textbf{(A)}\ \dfrac{1}{4} \qquad \textbf{(B)}\ \dfrac{1}{2} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 4$

Problem 3

Let $x=-2016$. What is the value of $\Big|\big||x|-x\big|-|x|\Big|-x$?

令$x=-2016$，求$\Big|\big||x|-x\big|-|x|\Big|-x$的值？

$\textbf{(A)}\ -2016 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2016 \qquad \textbf{(D)}\ 4032 \qquad \textbf{(E)}\ 6048$

Problem 4

Zoey read 15 books, one at a time. The first book took her 1 day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day of the week did she finish her 15th book?

Zoey 读 15 本书，一次读一本。第一本书花了 1 天时间读完，第二本书花了 2 天读完，第三本书花了 3 天读完，以此类推，每本书所花的时间都比前一本所花时间多 1 天。Zoey 在某个周一读完了第一本书，在某个周三读完了第二本书，那么在她读完第 15 本书是在周几？

$\textbf{(A)}$ Sunday $\textbf{(B)}$ Monday $\textbf{(C)}$ Wednesday $\textbf{(D)}$ Friday $\textbf{(E)}$ Saturday

Problem 5

The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins?

Amanda 的 4 个表妹的平均年龄是 8 岁，她们年龄的中位数是 5。那么 Amanda 最小和最大的表妹的年龄之和是多少?

$\textbf{(A)}\ 13$ $\textbf{(B)}\ 16$ $\textbf{(C)}\ 19$ $\textbf{(D)}\ 22$ $\textbf{(E)}\ 25$

Problem 6

Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S?

Laura 把两个三位的正整数相加。这两个数中的 6 位数字都不同。Laura 得到的和是一个三位数 S。则 S 的各个位上数字之和的最小可能值是多少？

$\textbf{(A)}\ 1$ $\textbf{(B)}\ 4$ $\textbf{(C)}\ 5$ $\textbf{(D)}\ 15$ $\textbf{(E)}\ 20$

Problem 7

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

两个锐角的度数比值是 5:4，且其中一个角的余角是另一个角的余角的 2 倍。那么这 2 个角的度数之和是多少?

$\textbf{(A)}\ 75$ $\textbf{(B)}\ 90$ $\textbf{(C)}\ 135$ $\textbf{(D)}\ 150$ $\textbf{(E)}\ 270$

Problem 8

What is the tens digit of $2015^{2016} - 2017$?

$2015^{2016} - 2017$的十位数字是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 8$

Problem 9

All three vertices of $\triangle ABC$ are lying on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\triangle ABC$的 3 个顶点位于抛物线$y = x^2$上，其中点 *A* 在原点，$\overline{BC}$和 *x* 轴平行。三角形的面积为 64，问 *BC* 的长度为多少?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 16$

Problem 10

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

一个形状为正三角形，密度均匀的薄木板边长为 3 英寸，重量为 12 盎司。第二块同样木质，相同厚度，形状也为正三角形的薄木板边长为 5 英寸。下面哪个数字最接近第二块木板的重量（单位为盎司）？

$\textbf{(A)}\ 14.0 \qquad \textbf{(B)}\ 16.0 \qquad \textbf{(C)}\ 20.0 \qquad \textbf{(D)}\ 33.3 \qquad \textbf{(E)}\ 55.6$

Problem 11

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

Carl 决定把他的长方形花园用栅栏围起来。他买了 20 个栅栏木桩，在 4 个角落各放 1 个，剩余的木桩则沿着花园的边缘均匀放置，相邻木桩之间的距离恰好是 4 码。花园的长边（包括角落）所插的木桩的数目是短边（包含角落）所插木桩数目的 2 倍。那么 Carl 的花园的面积是多少平方码?

$\textbf{(A)}\ 256 \qquad \textbf{(B)}\ 336 \qquad \textbf{(C)}\ 384 \qquad \textbf{(D)}\ 448 \qquad \textbf{(E)}\ 512$

Problem 12

Two different numbers are selected at random from $(1, 2, 3, 4, 5)$ and multiplied together. What is the probability that the product is even?

从集合｛1，2，3，4，5｝中随机选择两个不同的数字并相乘，乘积是偶数的概率是多少？

$\textbf{(A)}\ 0.2 \qquad \textbf{(B)}\ 0.4 \qquad \textbf{(C)}\ 0.5 \qquad \textbf{(D)}\ 0.7 \qquad \textbf{(E)}\ 0.8$

Problem 13

At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

Megapolis 医院某一年多胞胎的出生数据如下:总共有 1000 个新生儿是双胞胎，三胞胎，和四胞胎。三胞胎组数是四胞胎组数的 4 倍，双胞胎组数是三胞胎组数的 3 倍。这 1000 个婴儿里，有多少人是四胞胎？

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 100 \qquad \textbf{(E)}\ 160$

Problem 14

How many squares whose sides are parallel to the axis and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1$?

有多少个这样的正方形，这些正方形的边和坐标轴平行，顶点的坐标都是整数，且正方形位于直线$y = \pi x$，直线$y = -0.1$和直线 $x = 5.1$所包围的区域内？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 57$

Problem 15

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

1，2，3，4，5，6，7，8，9 这 9 个数被写入 1 个3×3的方格阵列中，每个小方格一个数字，满足如果两个数是连续的，那么这两个数所在的方格共享一条边。四个角落的数字之和为 18。中心的那个数字是多少?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 16

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

一个无限等比数列所有项之和为一个正实数 S，数列的第二项是 1。S 的最小值是多少?

$\textbf{(A)}\ \dfrac{1+\sqrt{5}}{2} \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 17

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

2，3，4，5，6，7 这 6 个数被分配给一个立方体的 6 个面。每个面一个数.对于立方体的 8 个顶点，计算出包含每个顶点的 3 个面上的数字的乘积，这 8 个乘积相加所得和的最大值是多少？

$\textbf{(A)}\ 312 \qquad \textbf{(B)}\ 343 \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 729 \qquad \textbf{(E)}\ 1680$

Problem 18

In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

有多少种方法可以把 345 写成 2 个或更多个连续正整数组成的递增数列的所有项之和?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

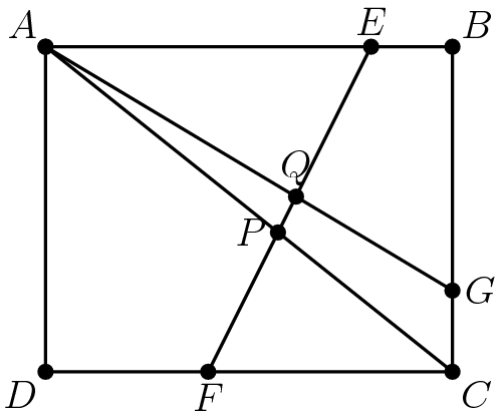
Problem 19

Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point E lies on $\overline{AB}$ so that $EB = 1$, point G lies on $\overline{BC}$ so that $CG = 1$. and point F lies on $\overline{CD}$ so that $DF = 2$.

Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at Q and P, respectively. What is the value of $\dfrac{PQ}{EF}$?

长方形 *ABCD* 的边 *AB*=5，*BC*=4。点 E 在线段$\overline{AB}$上满足 *EB*=1，点 *G* 在边$\overline{BC}$上满足 *CG*=1。

点 *F* 在边$\overline{CD}$上满足 *DF*=2。.线段$\overline{AG}$和$\overline{AC}$分别交$\overline{EF}$于 *Q* 和 *P*。$\dfrac{PQ}{EF}$的值是多少?



$\textbf{(A)}\ \dfrac{\sqrt{13}}{16} \qquad \textbf{(B)}\ \dfrac{\sqrt{2}}{13} \qquad \textbf{(C)}\ \dfrac{9}{82} \qquad \textbf{(D)}\ \dfrac{10}{91} \qquad \textbf{(E)}\ \dfrac{1}{9}$

Problem 20

A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2, 2)$ to the circle of radius 3 centered at $A'(5, 6)$. What distance does the origin $O(0, 0)$, move under this transformation?

平面的位似变换－即尺寸比例系数为正数的一种尺度变换－将圆心在 A（2，2）半径为 2 的圆，变到了圆心在$A'(5, 6)$且半径为 3 的圆。在这个位似变换下，原点 O（0，0）移动了多少距离？

(A) 0 **(B)** 3 **(C)** $\sqrt{13}$ **(D)** 4 **(E)** 5

Problem 21

What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

由方程 $x^2 + y^2 = |x| + |y|$的图像所围成的区域的面积是多少?

(A) $\pi + \sqrt{2}$ **(B)** $\pi + 2$ **(C)** $\pi + 2\sqrt{2}$ **(D)** $2\pi + \sqrt{2}$ **(E)** $2\pi + 2\sqrt{2}$

Problem 22

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

若干支队伍举行循环赛，在循环赛中，每支队伍和其他每支队伍恰好打一场比赛。每支队伍赢了 10 场比赛，输了 10 场比赛，且没有平局。有多少个这样的｛A，B，C｝三支队伍的组合，满足 A 打败了 B，B 打败了 C，C 打败了 A?

(A) 385 **(B)** 665 **(C)** 945 **(D)** 1140 **(E)** 1330

Problem 23

In regular hexagon $ABCDEF$, points W, X, Y, and Z are chosen on sides $\overline{BC}$, $\overline{CD}$, $\overline{EF}$, and $\overline{FA}$ respectively, so lines AB, ZW, YX, and ED are parallel and equally spaced. What is the ratio of the area of hexagon $WCXYFZ$ to the area of hexagon $ABCDEF$?

在正六边形 *ABCDEF* 中，点 *W*，*X*，*Y* 和 *Z* 分别在边$\overline{BC}$, $\overline{CD}$, $\overline{EF}$和$\overline{FA}$上，满足直线 *AB*，*ZW*，*YX* 和 *ED* 相互平行且间隔相等。那么六边形 *WCXYFZ* 的面积与六边形 *ABCDEF* 的面积的比是多少？

(A) $\frac{1}{3}$ **(B)** $\frac{10}{27}$ **(C)** $\frac{11}{27}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{13}{27}$

Problem 24

How many four-digit integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab < bc < cd$ form an increasing arithmetic sequence? One such number is 4692, where $a = 4$, $b = 6$, $c = 9$, and $d = 2$.

有多少个 4 位整数 *abcd*，其中，满足三个两位数 *ab*<*bc*<*cd* 形成一个递增的等其中一个这样的数是 4692，其中 *a*=4，*b*=6，*c*=9，*d*=2。

(A) 9 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 20

Problem 25

Let $f(x) = \sum_{k=2}^{10} (\lfloor kx \rfloor - k\lfloor x \rfloor)$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r.

How many distinct values does $f(x)$ assume for $x \geq 0$?

令 $f(x) = \sum_{k=2}^{10} (\lfloor kx \rfloor - k\lfloor x \rfloor)$，其中$\lfloor r \rfloor$表示取小于或者等于 *r* 的最大整数。若 *x*≥0，$f(x)$可以取多少种不同的值？

(A) 32 **(B)** 36 **(C)** 45 **(D)** 46 **(E)** infinitely many

2016 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	B	D	B	D	B	C	A	C	D	B	D	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	C	E	D	E	D	C	B	A	C	D	A	