2016 AMC12B

Problem 1

What is the value of $\dfrac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\dfrac{1}{2}$?

当$a=\dfrac{1}{2}$时，$\dfrac{2a^{-1}+\frac{a^{-1}}{2}}{a}$的值是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \dfrac{5}{2} \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 20$

Problem 2

The harmonic mean of two numbers can be calculated as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

两个数的调和平均值可以由它们乘积的 2 倍除以它们的和得到。那么 1 和 2016 的调和平均值最接近下面哪个整数？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 45 \qquad \textbf{(C)}\ 504 \qquad \textbf{(D)}\ 1008 \qquad \textbf{(E)}\ 2015$

Problem 3

Let $x=-2016$. What is the value of $\Big|\big||x|-x\big|-|x|\Big|_{-x}$?

令$x=-2016$，则$\Big|\big||x|-x\big|-|x|\Big|_{-x}$的值是多少？

$\textbf{(A)}\ -2016 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2016 \qquad \textbf{(D)}\ 4032 \qquad \textbf{(E)}\ 6048$

Problem 4

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

两个锐角的度数比值是$5:4$，且其中一个角的补角是另一个角的补角的 2 倍，那么这 2 个角的度数之和是多少?

$\textbf{(A)}\ 75 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 135 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 270$

Problem 5

The War of 1812 started with a declaration of war on Thursday, June 18, 1812. The peace treaty to end the war was signed 919 days later, on December 24, 1814. On what day of the week was the treaty signed?

1812 英美之战以 1812 年 6 月 18 日星期四的宣战开始，结束战争的和平协议的签订是在 919 天后的 1814 年 12 月 24 日，问签订协议的那天是星期几?

（A）Friday丨周五

（B）Saturday丨周六

（C）Sunday丨周日

（D）Monday丨周一

（E）Tuesday丨周二

Problem 6

All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\triangle ABC$的 3 个顶点位于抛物线$y = x^2$上，其中点 *A* 在原点，*BC* 和 *x* 轴平行.，三角形的面积为 64，问 BC 的长度为多少?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 16$

Problem 7

Josh writes the numbers $1, 2, 3, \ldots, 99, 100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

Josh 写下一列数字 1，2，3，…，99，100，他划掉 1，跳过 2，划掉 3，并继续跳过和划掉接下来的数字，直到这列数字的末尾。然后他返回到列表的开头，划掉剩下的第一个数字（2），跳过下一个数字（4），划掉 6，跳过 8，划掉 10，等等，一直到列表的末尾。Josh 重复这种方式，直到最后列表只剩 1 个数字。问这个数字是多少?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 56 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 96$

Problem 8

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

一个形状为正三角形，密度均匀的薄木板边长为 3 英寸，重量为 12 盎司，第二块同样木质，相同厚度，形状也为正三角形的薄木板边长为 5 英寸，下面哪个数字最接近第二块木板的重量（单位为盎司）？

$\textbf{(A)}\ 14.0 \qquad \textbf{(B)}\ 16.0 \qquad \textbf{(C)}\ 20.0 \qquad \textbf{(D)}\ 33.3 \qquad \textbf{(E)}\ 55.6$

Problem 9

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

Carl 决定把他的长方形花园用栅栏围起来，他买了 20 个栅栏木桩，在 4 个角落各放 1 个，剩余的木桩则沿着花园的边缘均匀放置，相邻木桩之间的距离恰好是 4 码，花园的长边（包括角落）所插的木桩的数目是短边（包含角落）所插木桩数目的 2 倍，那么 Carl 的花园的面积是多少平方码?

$\textbf{(A)}\ 256 \qquad \textbf{(B)}\ 336 \qquad \textbf{(C)}\ 384 \qquad \textbf{(D)}\ 448 \qquad \textbf{(E)}\ 512$

Problem 10

A quadrilateral has vertices $P(a,b)$, $Q(b,a)$, $R(-a,-b)$, and $S(-b,-a)$, where a and b are integers with $a>b>0$. The area of $PQRS$ is 16. What is $a+b$?

一个四边形的顶点为 P（a，b），Q（b，a），R（-a，-b），S（-b，-a），其中 a 和 b 都是整数，且$a>b>0$，四边形 $PQRS$ 的面积是 16，那么$a+b$是多少？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 13$

Problem 11

How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y=\pi x$, the line $y=-0.1$ and the line $x=5.1$?

有多少个这样的正方形，满足这样的条件：它们的边和坐标轴平行，它们的顶点的坐标都是整数，且正方形位于直线$y=\pi x$，直线$y=-0.1$和直线$x=5.1$ 所包围的区域内?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 57$

Problem 12

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

1，2，3，4，5，6，7，8，9 这 9 个数被写入 1 个3×3的方格阵列中，每个小方格一个数字，满足如果两个数是连续的，那么这两个数所在的方格共享一条边，四个角落的数字之和为 18。问中心的那个数字是多少?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 13

Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

Alice 和 Bob 的家相距 10 英里。一天 Alice 从她家向北看，看到了一架飞机。同时 Bob 从他家向西看，也看到了这架飞机。Alice 看这架飞机的仰角是 30°，Bob 的仰角是 60°。下面哪个最接近飞机的高度（单位是英里）？

$\textbf{(A)}\ 3.5 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 4.5 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 5.5$

Problem 14

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

一个无限等比数列所有项之和为一个正实数 S，数列的第二项是 1，那么 S 的最小值是多少?

$\textbf{(A)}\ \dfrac{1+\sqrt{5}}{2} \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 15

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

2，3，4，5，6，7 这 6 个数被分配给一个立方体的 6 个面。每个面一个数。根据立方体的 8 个顶点，计算出包含每个顶点的 3 个面上的数字的乘积，这 8 个乘积相加所得和的最大值是多少?

$\textbf{(A)}\ 312 \qquad \textbf{(B)}\ 343 \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 729 \qquad \textbf{(E)}\ 1680$

Problem 16

In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

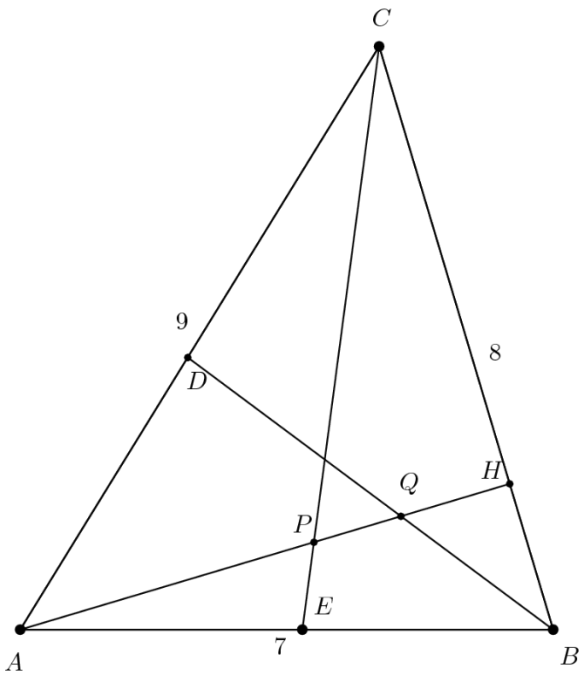
有多少种方法可以把 345 写成 2 个或更多个连续正整数组成的递增数列的所有项之和？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 17

In $\triangle ABC$ shown in the figure, $AB = 7, BC = 8, CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?

在如下图所示的$\triangle ABC$中， AB=7，BC=8，CA=9，且$\overline{AH}$是一条高。点 D 和 E 分别在边$\overline{AC}$和$\overline{AB}$上，且$\overline{BD}$和$\overline{CE}$都是角平分线，和$\overline{AH}$分别交于点 Q 和 P。那么 PQ 的长度是多少？



(A) 1 **(B)** $\frac{5}{8}\sqrt{3}$ **(C)** $\frac{4}{5}\sqrt{2}$ **(D)** $\frac{8}{15}\sqrt{5}$ **(E)** $\frac{6}{5}$

Problem 18

What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

由方程$x^2 + y^2 = |x| + |y|$的图像所围成的区域的面积是多少？

(A) $\pi + \sqrt{2}$ **(B)** $\pi + 2$ **(C)** $\pi + 2\sqrt{2}$ **(D)** $2\pi + \sqrt{2}$ **(E)** $2\pi + 2\sqrt{2}$

Problem 19

Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

Tom，Dick 和 Harry 在玩一个游戏，他们同时开始各自不断地抛一枚标准硬币，直到各自第一次得到正面朝上，那么他就停止抛硬币。问这三个人抛硬币的次数相同的概率是多少？

$\textbf{(A)}\ \frac{1}{8}\qquad \textbf{(B)}\ \frac{1}{7}\qquad \textbf{(C)}\ \frac{1}{6}\qquad \textbf{(D)}\ \frac{1}{4}\qquad \textbf{(E)}\ \frac{1}{3}$

Problem 20

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

若干支队伍举行循环赛。在循环赛中，每支队伍和其他每支队伍恰好打一场比。每支队伍赢了 10 场比赛，输了 10 场比赛，且没有平局。问有多少个这样的｛A，B，C｝三支队伍的组合，满足 A 打败了 B，B 打败了 C，C 打败了 A?

$\textbf{(A)}\ 385\qquad \textbf{(B)}\ 665\qquad \textbf{(C)}\ 945\qquad \textbf{(D)}\ 1140\qquad \textbf{(E)}\ 1330$

Problem 21

Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i = 1, 2, \ldots$, let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What

$$\sum_{\text{is}\, i=1}^{\infty} \text{Area of } \triangle DQ_iP_i \, ?$$

ABCD 是一个单位正方形。Q_1是$\overline{CD}$的中点。对于$i = 1, 2, \ldots,$，令P_i为$\overline{AQ_i}$和$\overline{BD}$的交点。Q_{i+1}是过点P_i并垂直于$\overline{CD}$的垂线的垂足，则下式的值是多少

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i \, ?$$

$\textbf{(A)}\ \frac{1}{6}\qquad \textbf{(B)}\ \frac{1}{4}\qquad \textbf{(C)}\ \frac{1}{3}\qquad \textbf{(D)}\ \frac{1}{2}\qquad \textbf{(E)}\ 1$

Problem 22

For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period of 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

n 是一个小于 1000 的正整数，已知$\frac{1}{n}$的小数表示是$0.\overline{abcdef}$，这是个循环周期为 6 的循环小数，而$\frac{1}{n+6}$的小数表示是$0.\overline{wxyz}$，这是个循环周期为 4 的循环小数。问 n 在哪个区间内？

$\textbf{(A)}\ [1,200]$ $\textbf{(B)}\ [201,400]$ $\textbf{(C)}\ [401,600]$ $\textbf{(D)}\ [601,800]$ $\textbf{(E)}\ [801,999]$

Problem 23

What is the volume of the region in three-dimensional space defined by the inequalities $|x|+|y|+|z|\le 1$ and $|x|+|y|+|z-1|\le 1$?

由不等式$|x|+|y|+|z|\le 1$和$|x|+|y|+|z-1|\le 1$定义的三维空间内区域的体积是多少？

$\textbf{(A)}\ \frac{1}{6}$ $\textbf{(B)}\ \frac{1}{4}$ $\textbf{(C)}\ \frac{1}{3}$ $\textbf{(D)}\ \frac{1}{2}$ $\textbf{(E)}\ 1$

Problem 24

There are exactly $77,000$ ordered quadruplets (a,b,c,d) such that $\gcd(a,b,c,d)=77$ and $\operatorname{lcm}(a,b,c,d)=n$. What is the smallest possible value for n?

满足$\gcd(a,b,c,d)=77$和$\operatorname{lcm}(a,b,c,d)=n$的有序四元组$(a,b,c,d)$恰好共有 77，000 个，那么 n 的最小可能值是多少?

$\textbf{(A)}\ 13,860$ $\textbf{(B)}\ 20,790$ $\textbf{(C)}\ 21,560$ $\textbf{(D)}\ 27,720$ $\textbf{(E)}\ 41,580$

Problem 25

The sequence (a_n) is defined recursively by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \geq 2$. What is the smallest positive integer k such that the product $a_1 a_2 \cdots a_k$ is an integer?

数列(a_n)定义如下：$a_0 = 1, a_1 = \sqrt[19]{2}$，且对于$n \geq 2$，有$a_n = a_{n-1}a_{n-2}^2$，使得乘积$a_1 a_2 \cdots a_k$为整数的最小整数$k$是多少？

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 21$

2016 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	A	D	C	B	C	D	D	B	A	D	C	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	D	E	D	B	B	A	B	B	A	D	A	