2016 AMC 10B Problems/Problem 1

Problem

What is the value of $\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\tfrac{1}{2}$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \frac{5}{2}\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 20$

Solution

Factorizing the numerator, $\frac{\frac{1}{a}\cdot(2+\frac{1}{2})}{a}$ then becomes $\frac{\frac{5}{2}}{a^2}$ which is equal to $\frac{5}{2}\cdot 2^2$ which is $\textbf{(D)}\ 10$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
Preceded by -	Followed by Problem 2
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2016 AMC 10B Problems/Problem 2

Problem

If $n\heartsuit m = n^3m^2$, what is $\frac{2\heartsuit 4}{4\heartsuit 2}$?

$\textbf{(A)}\ \frac{1}{4}\qquad\textbf{(B)}\ \frac{1}{2}\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ 4$

Solution

$\frac{2^3(2^2)^2}{(2^2)^3 2^2} = \frac{2^7}{2^8} = \frac{1}{2}$ which is $\textbf{(B)}$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 3

Contents

Problem

Let $x = -2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

$\textbf{(A)}\ -2016\qquad\textbf{(B)}\ 0\qquad\textbf{(C)}\ 2016\qquad\textbf{(D)}\ 4032\qquad\textbf{(E)}\ 6048$

Solution

Substituting carefully, $\Bigg\vert\Big\vert 2016-(-2016)\Big\vert-2016\Bigg\vert-(-2016)$

becomes $|4032-2016|+2016=2016+2016=4032$ which is $\textbf{(D)}$.

Solution 2

Solution by e_power_pi_times_i

Substitute $-y = x = -2016$ into the equation. Now, it is $\Bigg\vert\Big\vert |y|+y\Big\vert-|y|\Bigg\vert+y$. Since $y = 2016$, it is a positive number, so $|y| = y$. Now the equation is $\Bigg\vert\Big\vert y+y\Big\vert-y\Bigg\vert+y$. This further simplifies to $2y-y+y = 2y$, so the answer is $\textbf{(D)} 4032$

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 4

Problem

Zoey read 15 books, one at a time. The first book took her 1 day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day the week did she finish her 15th book?

$\textbf{(A)}\ \text{Sunday}\qquad\textbf{(B)}\ \text{Monday}\qquad\textbf{(C)}\ \text{Wednesday}\qquad\textbf{(D)}\ \text{Friday}\qquad\textbf{(E)}\ \text{Saturday}$

Solution

The process took $1+2+3+\ldots+13+14+15=120$ days, so the last day was 119 days after the first day. Since 119 is divisible by 7, both must have been the same day of the week, so the answer is $\textbf{(B)}\ \text{Monday}$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 5

Problem

The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins?

$\textbf{(A)}\ 13\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 19\qquad\textbf{(D)}\ 22\qquad\textbf{(E)}\ 25$

Solution

The sum of the ages of the cousins is 4 times the mean, or 32. There are an even number of cousins, so there is no single median, so 5 must be the median of the two in the middle. Therefore the sum of the ages of the two in the middle is 10. Subtracting 10 from 32 produces $\textbf{(D)}\ 22$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 6

Contents

Problem

Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 15\qquad\textbf{(E)}\ 20$

Solution

Let the two three-digit numbers she added be a and b with $a+b=S$ and $a<b$. The hundreds digits of these numbers must be at least 1 and 2, so $a\geq 102$ and $b\geq 203$, which means $S\geq 305$, so the digits of S must sum to at least 4, in which case S would have to be either 310 or 400. But b is too big for 310, so we consider the possibility $S=400$.

Say $a=100+p$ and $b=200+q$; then we just need $p+q=100$ with p and q having different digits which aren't 1 or 2. There are many solutions, but $p=3$ and $q=97$ give $103+297=400$ which proves that $\textbf{(B)}\ 4$ is attainable.

Solution 2

Ok, so for this problem, to find the 3-digit integer with the smallest sum of digits, one should make the units and tens digit add to 0. To do that, we need to make sure the digits are all distinct. For the units digit, we can have a variety of digits that work. 7 works best for the top number which makes the bottom digit 3. The tens digits need to add to 9 because of the 1 that needs to be carried from the addition of the units digits. We see that 5 and 4 work the best as we can't use 6 and 3. Finally, we use 2 and 1 for our hundreds place digits.

Adding the numbers 257 and 143 ,

We get 400 which means our answer is $\textbf{(B)}\ 4$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 7

Problem

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

$\textbf{(A)}\ 75\qquad\textbf{(B)}\ 90\qquad\textbf{(C)}\ 135\qquad\textbf{(D)}\ 150\qquad\textbf{(E)}\ 270$

Solution

We can set up a system of equations where x and y are the two acute angles. WLOG, assume that $x<y$ in order for the complement of x to be greater than the complement of y. Therefore, $5x=4y$ and $90-x=2\left(90-y\right)$. Solving for y in the first equation and substituting into the second equation yields

$$\begin{aligned} 90-x &= 2(90-1.25x) \\ 1.5x &= 90 \\ x &= 60 \end{aligned}$$

Substituting this x value back into the first equation yields $y=75$, leaving $x+y$ equal to $\boxed{\textbf{(C)}\ 135}$.

(Solution by akaashp11)

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 8

Contents

Problem

What is the tens digit of $2015^{2016} - 2017?$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 8$

Solution 1

Notice that, for $n \geq 2$, $2015^n \equiv 15^n$ is congruent to $25 \pmod{100}$ when n is even and $75 \pmod{100}$ when n is odd. (Check for yourself). Since 2016 is even, $2015^{2016} \equiv 25 \pmod{100}$ and $2015^{2016} - 2017 \equiv 25 - 17 \equiv \underline{0}8 \pmod{100}$.

So the answer is $\textbf{(A)}\ 0$.

Solution 2

In a very similar fashion, we find that $2015^{2016} \equiv 15^{2016} \pmod{100}$, which equals 225^{1008}. Next, since every power (greater than 0) of every number ending in 25 will end in 25 (which can easily be verified), we get $225^{1008} \equiv 25 \pmod{100}$. (In this way, we don't have to worry about the exponent very much.) Finally, $2017 \equiv 17 \pmod{100}$, and thus $2015^{2016} - 2017 \equiv 25 - 17 \equiv 08 \pmod{100}$, as above.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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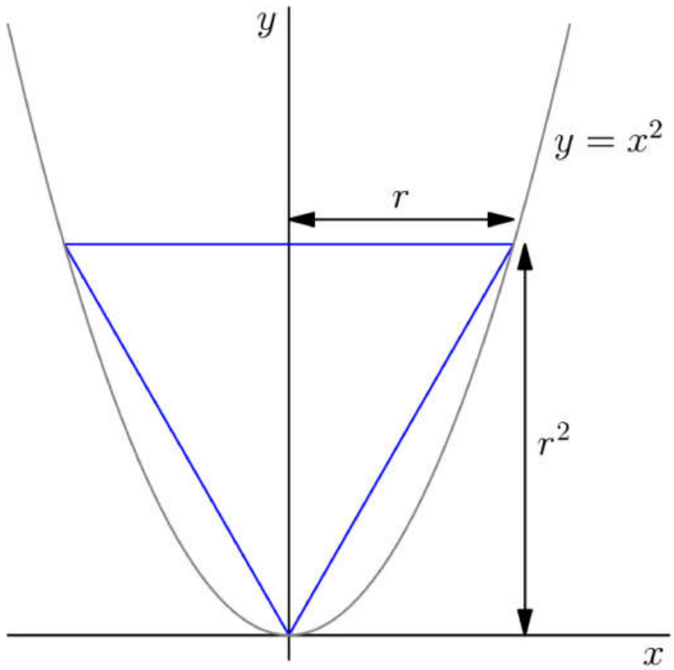
2016 AMC 10B Problems/Problem 9

Problem

All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 8\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 16$

Solution



The area of the triangle is r^3, so $r^3 = 64 \implies r = 4$, giving a total distance across the top of 8, which is answer $\textbf{(C)}$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 10

Contents

Problem

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

$\textbf{(A)}\ 14.0\qquad\textbf{(B)}\ 16.0\qquad\textbf{(C)}\ 20.0\qquad\textbf{(D)}\ 33.3\qquad\textbf{(E)}\ 55.6$

Solution 1

We can solve this problem by using similar triangles, since two equilateral triangles are always similar. We can then use

$$(\frac{3}{5})^2 = \frac{12}{x}.$$

We can then solve the equation to get $x = \frac{100}{3}$ which is closest to $\boxed{\textbf{(D)}\ 33.3}$

Solution 2

Also recall that the area of an equilateral triangle is $\frac{a^2\sqrt{3}}{4}$ so we can give a ratio as follows:

$$\frac{\frac{9\sqrt{3}}{4}}{12} = \frac{\frac{25\sqrt{3}}{4}}{x}$$

Cross multiplying and simplifying, we get $12 \cdot \frac{25}{9}$

Which is $33.\overline{3} \approx \boxed{\textbf{(D)}\ 33.3}$

- Solution by $AOPS12142015$

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 11

Contents

Problem

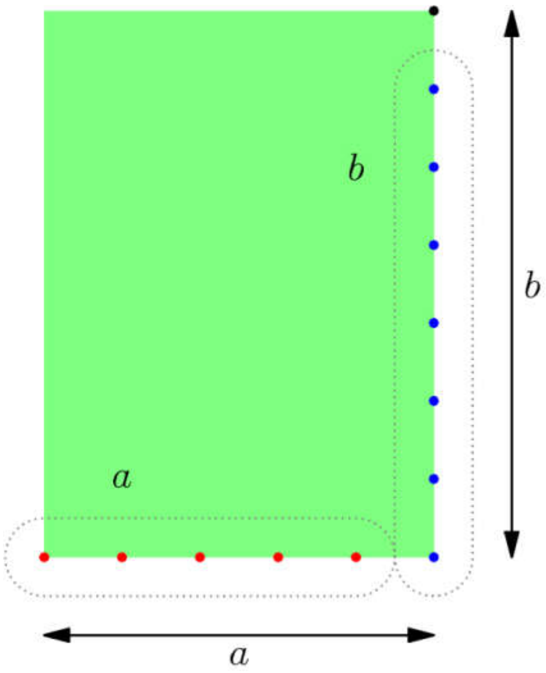
Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

$\textbf{(A)}\ 256\qquad\textbf{(B)}\ 336\qquad\textbf{(C)}\ 384\qquad\textbf{(D)}\ 448\qquad\textbf{(E)}\ 512$

Solution

If the dimensions are $a\times b$, then one side will have $a+1$ posts (including corners) and the other $b+1$.

The total number of posts is $2(a+b)=20$.



Solve the system $\begin{cases}b+1=2(a+1)\\ a+b=10\end{cases}$ to get $a=3,\ b=7$. Then the area is $4a\cdot 4b=336$ which is $\textbf{(B)}$.

Solution 2

To do this problem, we have to draw a rectangle. We also have to keep track of the fence posts. Putting a post on each corner leaves us with only 16 posts. Now there are twice as many posts on the longer side then the shorter side. From this we can see that we can put 8 posts on the longer side and 4 posts on the shorter side. On the shorter side, we have 3 spaces between the 4 posts. On the longer side, we have 7 spaces between the 8 fence posts. There are 4 yards between each post. Therefore, the answer is $12*28=336$

$\textbf{(B)}\ 336$.

2016 AMC 10B Problems/Problem 12

Problem

Two different numbers are selected at random from $\{1,2,3,4,5\}$ and multiplied together. What is the probability that the product is even?

$\textbf{(A)}\ 0.2 \qquad \textbf{(B)}\ 0.4 \qquad \textbf{(C)}\ 0.5 \qquad \textbf{(D)}\ 0.7 \qquad \textbf{(E)}\ 0.8$

Solution

The product will be even if at least one selected number is even, and odd if none are. Using complementary counting, the chance that both numbers are odd is $\frac{\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}$, so the answer is $1 - 0.3$ which is $\textbf{(D)}\ 0.7$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 13

Problem

At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

$\textbf{(A)}\ 25\qquad\textbf{(B)}\ 40\qquad\textbf{(C)}\ 64\qquad\textbf{(D)}\ 100\qquad\textbf{(E)}\ 160$

Solution

We can set up a system of equations where a is the sets of twins, b is the sets of triplets, and c is the sets of quadruplets.

$$\begin{aligned} 2a+3b+4c &= 1000 \\ b &= 4c \\ a &= 3b \end{aligned}$$

Solving for c and a in the second and third equations and substituting into the first equation yields

$$\begin{aligned} 2(3b)+3b+4(0.25b) &= 1000 \\ 6b+3b+b &= 1000 \\ b &= 100 \end{aligned}$$

Since we are trying to find the number of babies and NOT the number of sets of quadruplets, the solution is not c, but rather $4c$. Therefore, we strategically use the second initial equation to realize that $b=4c$, leaving us with the number of babies born as quadruplets equal to $\boxed{\textbf{(D)}\ 100}$.

See Also

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2016 AMC 10B Problems/Problem 14

Contents

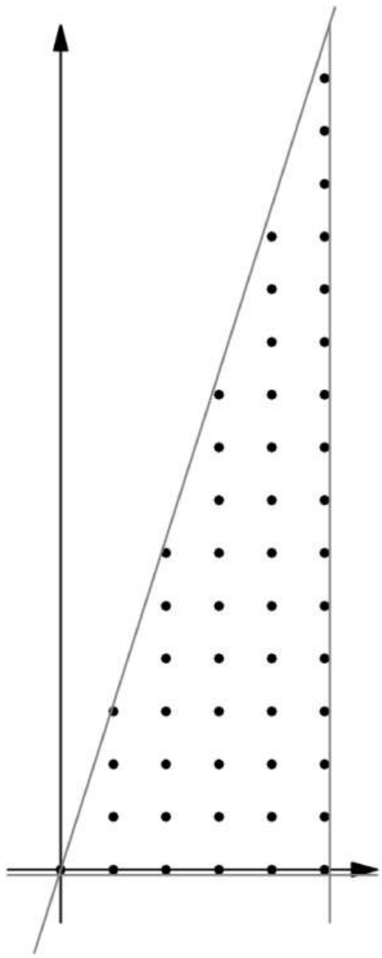
Problem

How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1?$

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 57$

Solution 1

The region is a right triangle which contains the following lattice points:
$(0,0); (1,0)-(1,3); (2,0)-(2,6); (3,0)-(3,9); (4,0)-(4,12); (5,0)-(5,15)$



Squares 1×1: Suppose that the top-right corner is (x,y), with $2\leq x\leq 5$. Then to include all other corners, we need $1\leq y\leq 3(x-1)$. This produces $3+6+9+12=30$ squares.

Squares 2×2: Here $3\leq x\leq 5$. To include all other corners, we need $2\leq y\leq 3(x-2)$. This produces $2+5+8=15$ squares.

Squares 3×3: Similarly this produces 5 squares.

No other squares will fit in the region. Therefore the answer is $\textbf{(D)}\ 50$.

Solution 2

The vertical line is just to the right of $x = 5$, the horizontal line is just under $y = 0$, and the sloped line will always be above the y value of $3x$. This means they will always miss being on a coordinate with integer coordinates so you just have to count the number of squares to the left, above, and under these lines. After counting the number of 1x1, 2x2, 3x3, squares and getting 30, 15, and 5 respectively, and we end up with $\textbf{(D)}\ 50$

Solution by Wwang

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 15

Contents

Problem

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution 1 - Trial Error

Quick testing shows that

$$3\ 2\ 1$$

$$4\ 7\ 8$$

$$5\ 6\ 9$$

is a valid solution. $3+1+5+9=18$, and the numbers follow the given condition. The center number is found to be $\boxed{7}$. — @adihaya (talk) 12:27, 21 February 2016 (EST)

Solution 2

First let the numbers be

$$1\ 8\ 7$$

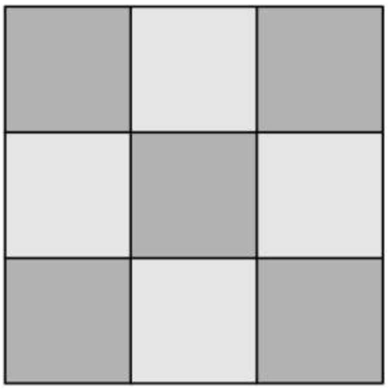
$$2\ 9\ 6$$

$$3\ 4\ 5$$

with the numbers $1-8$ around the outsides and 9 in the middle. We see that the sum of the four corner numbers is 16. If we switch 7 and 9, then the corner numbers will add up to 18 and the consecutive numbers will still be touching each other. The answer is $\boxed{7}$.

Solution 3

Consecutive numbers share an edge. That means that it is possible to walk from 1 to 9 by single steps north, south, east, or west. Consequently, the squares in the diagram with different shades have different parity:



But there are only four even numbers in the set, so the five darker squares must contain the odd numbers, which sum to $1+3+5+7+9=25$. Therefore if the sum of the numbers in the corners is 18, the number in the centre must be 7, which is answer $\textbf{(C)}$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 16

Contents

Problem

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

$\textbf{(A)}\ \frac{1+\sqrt{5}}{2} \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

The sum of an infinite geometric series is of the form:

$$S = \frac{a_1}{1-r}$$

where a_1 is the first term and r is the ratio whose absolute value is less than 1.

We know that the second term is the first term multiplied by the ratio. In other words:

$$a_1 \cdot r = 1$$
$$a_1 = \frac{1}{r}$$

Thus, the sum is the following:

$$S = \frac{\frac{1}{r}}{1-r}$$
$$S = \frac{1}{r-r^2}$$

Since we want the minimum value of this expression, we want the maximum value for the denominator, $-r^2+r$. The maximum x-value of a quadratic with negative a is $\frac{-b}{2a}$.

$$r = \frac{-(1)}{2(-1)}$$
$$r = \frac{1}{2}$$

Plugging $r = \frac{1}{2}$ into the quadratic yields:

$$S = \frac{1}{\frac{1}{2} - (\frac{1}{2})^2}$$
$$S = \frac{1}{\frac{1}{4}}$$

Therefore, the minimum sum of our infinite geometric sequence is $\boxed{\textbf{(E)}\ 4}$. (Solution by akaashp11)

Solution 2

After observation we realize that in order to minimize our sum $\frac{a}{1-r}$ with a being the reciprocal of r. The common ratio r has to be in the form of $1/x$ with x being an integer as anything more than 1 divided by x would give a larger sum than a ratio in the form of $1/x$.

The first term has to be x, so then in order to minimize the sum, we have minimize x.

The smallest possible value for x such that it is an integer that's greater than 1 is 2. So our first term is 2 and our common ratio is $1/2$. Thus the sum is $\frac{2}{1/2}$ or $\boxed{\textbf{(E)}\ 4}$. Solution 2 by No_One

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 17

Contents

Problem

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

$\textbf{(A)}\ 312 \qquad \textbf{(B)}\ 343 \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 729 \qquad \textbf{(E)}\ 1680$

Solution 1

Let us call the six sides of our cube $a, b, c, d, e,$ and f (where a is opposite d, c is opposite e, and b is opposite f. Thus, for the eight vertices, we have the following products: $abc, abe, bcd, bde, acf, cdf, cef,$ and def. Let us find the sum of these products:

$$abc + abe + bcd + bde + acf + cdf + aef + def$$

We notice b is a factor of the first four terms, and f is factor is the last four terms.

$$b(ac + ae + cd + de) + f(ac + ae + cd + de)$$

Now, we can factor even more:

$$\begin{array}{l} (b+f)(ac+ae+cd+de) \\ (b+f)(a(c+e)+d(c+e)) \\ (b+f)(a+d)(c+e) \end{array}$$

We have the product. Notice how the factors are sums of opposite faces. The best sum for this is to make $(7 + 2)$, $(6 + 3)$, and $(5 + 4)$ all factors.

$$\begin{array}{c} (7+2)(6+3)(5+4) \\ 9 * \ \ 9 \ \ * \ \ 9 \\ 729 \end{array}$$

Thus our answer is $\textbf{(D)}\ 729$.

Solution 2

We first find the factorization $(b+f)(a+d)(c+e)$ using the method in Solution 1. By using AM-GM, we get, $(b+f)(a+d)(c+e) \leq \left(\dfrac{a+b+c+d+e+f}{3}\right)^3$. To maximize, the factorization, we get the answer is $\left(\dfrac{27}{3}\right)^3 = \boxed{\textbf{(D)}\ 729}$

2016 AMC 10B Problems/Problem 18

Problem

In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 6\qquad\textbf{(E)}\ 7$

Solution

Factorize $345 = 3\cdot 5 \cdot 23$.

Suppose we take an odd number k of consecutive integers, centered on m. Then $mk = 345$ with $\tfrac{1}{2}k < m$. Looking at the factors of 345, the possible values of k are $3, 5, 15, 23$ centred on $115, 69, 23, 15$ respectively.

Suppose instead we take an even number $2k$ of consecutive integers, centred on m and $m+1$. Then $k(2m+1) = 345$ with $k \le m$. Looking again at the factors of 345, the possible values of k are $1, 3, 5$ centred on $(172,173), (57,58), (34,35)$ respectively.

Thus the answer is $\textbf{(E)}\ 7$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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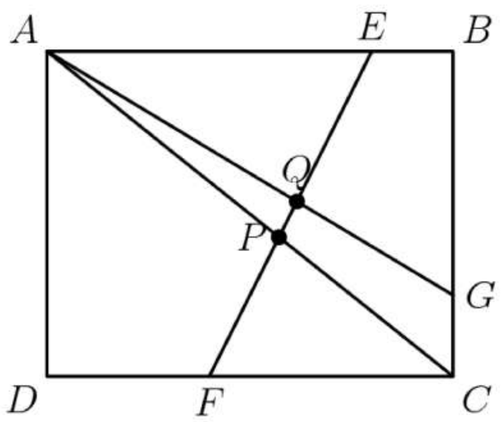
2016 AMC 10B Problems/Problem 19

Contents

Problem

Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point E lies on $\overline{AB}$ so that $EB = 1$, point G lies on $\overline{BC}$ so that $CG = 1$. and point F lies on $\overline{CD}$ so that $DF = 2$. Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at Q and P, respectively. What is the value of $\frac{PQ}{EF}$?



$\textbf{(A) } \frac{\sqrt{13}}{16} \qquad \textbf{(B) } \frac{\sqrt{2}}{13} \qquad \textbf{(C) } \frac{9}{82} \qquad \textbf{(D) } \frac{10}{91} \qquad \textbf{(E) } \frac{1}{9}$

Solution 1 (Answer Choices)

Since the opposite sides of a rectangle are parallel and $\angle APE = \angle CPF$ due to vertical angles, $\triangle APE \sim \triangle CPF$. Furthermore, the ratio between the side lengths of the two triangles is $\frac{AE}{FC} = \frac{4}{3}$. Labeling $EP = 4x$ and $FP = 3x$, we see that EF turns out to be equal to $7x$. Since the denominator of $\frac{PQ}{EF}$ must now be a multiple of 7, the only possible solution in the answer choices is $\boxed{\textbf{(D) } \frac{10}{91}}$.

Solution 2 (Coordinate Geometry)

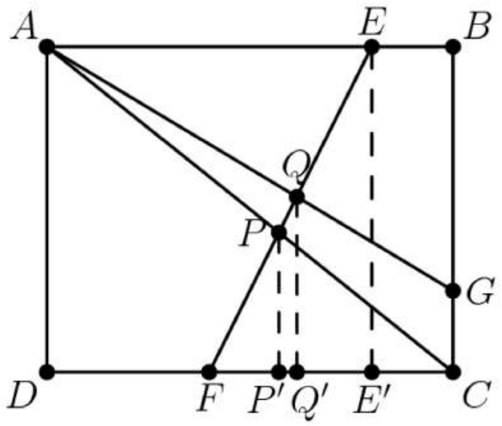
First, we will define point D as the origin. Then, we will find the equations of the following three lines: AG, AC, and EF. The slopes of these lines are $-\frac{3}{5}$, $-\frac{4}{5}$, and 2, respectively. Next, we will find the equations of AG, AC, and EF. They are as follows:

$$AG = f(x) = -\frac{3}{5}x + 4$$

$$AC = g(x) = -\frac{4}{5}x + 4$$

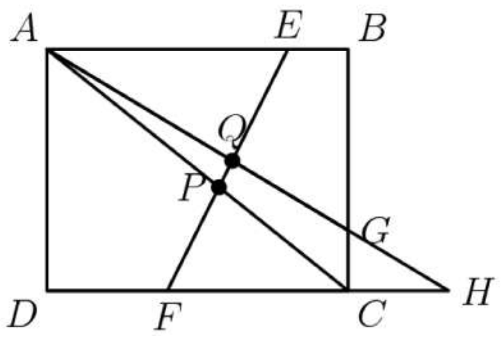
$$EF = h(x) = 2x - 4$$

After drawing in altitudes to DC from P, Q, and E, we see that $\frac{PQ}{EF} = \frac{P'Q'}{E'F}$ because of similar triangles, and so we only need to find the x-coordinates of P and Q.



Finding the intersections of AC and EF, and AG and EF gives the x-coordinates of P and Q to be $\frac{20}{7}$ and $\frac{40}{13}$. This means that $P'Q' = DQ' - DP' = \frac{20}{7} - \frac{40}{13} = \frac{20}{91}$. Now we can find $\frac{PQ}{EF} = \frac{P'Q'}{E'F} = \frac{\frac{20}{91}}{2} = \textbf{(D)}\ \frac{10}{91}$

Solution 3 (Similar Triangles)



Extend AG to intersect CD at H. Letting $x = \overline{HC}$, we have that

$$\triangle HCG \sim \triangle HDA \implies \frac{\overline{HC}}{\overline{CG}} = \frac{\overline{HD}}{\overline{AD}} \implies \frac{x}{1} = \frac{x+5}{4} \implies x = \frac{5}{3}.$$

Then, notice that $\triangle AEQ \sim \triangle HFQ$ and $\triangle AEP \sim \triangle CFP$. Thus, we see that

$$\frac{AE}{HF} = \frac{EQ}{QF} \implies \frac{EQ}{QF} = \frac{4}{3+\frac{5}{3}} = \frac{12}{14} = \frac{6}{7} \implies \frac{EQ}{EF} = \frac{6}{13}$$

and

$$\frac{AE}{CF} = \frac{EP}{FP} \implies \frac{4}{3} = \frac{EP}{FP} \implies \frac{FP}{FE} = \frac{3}{7}.$$

Thus, we see that

$$\frac{PQ}{EF} = 1 - \left(\frac{6}{13} + \frac{3}{7}\right) = 1 - \left(\frac{42+39}{91}\right) = 1 - \left(\frac{81}{91}\right) = \boxed{\textbf{(D) } \frac{10}{91}}.$$

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 20

Contents

Problem

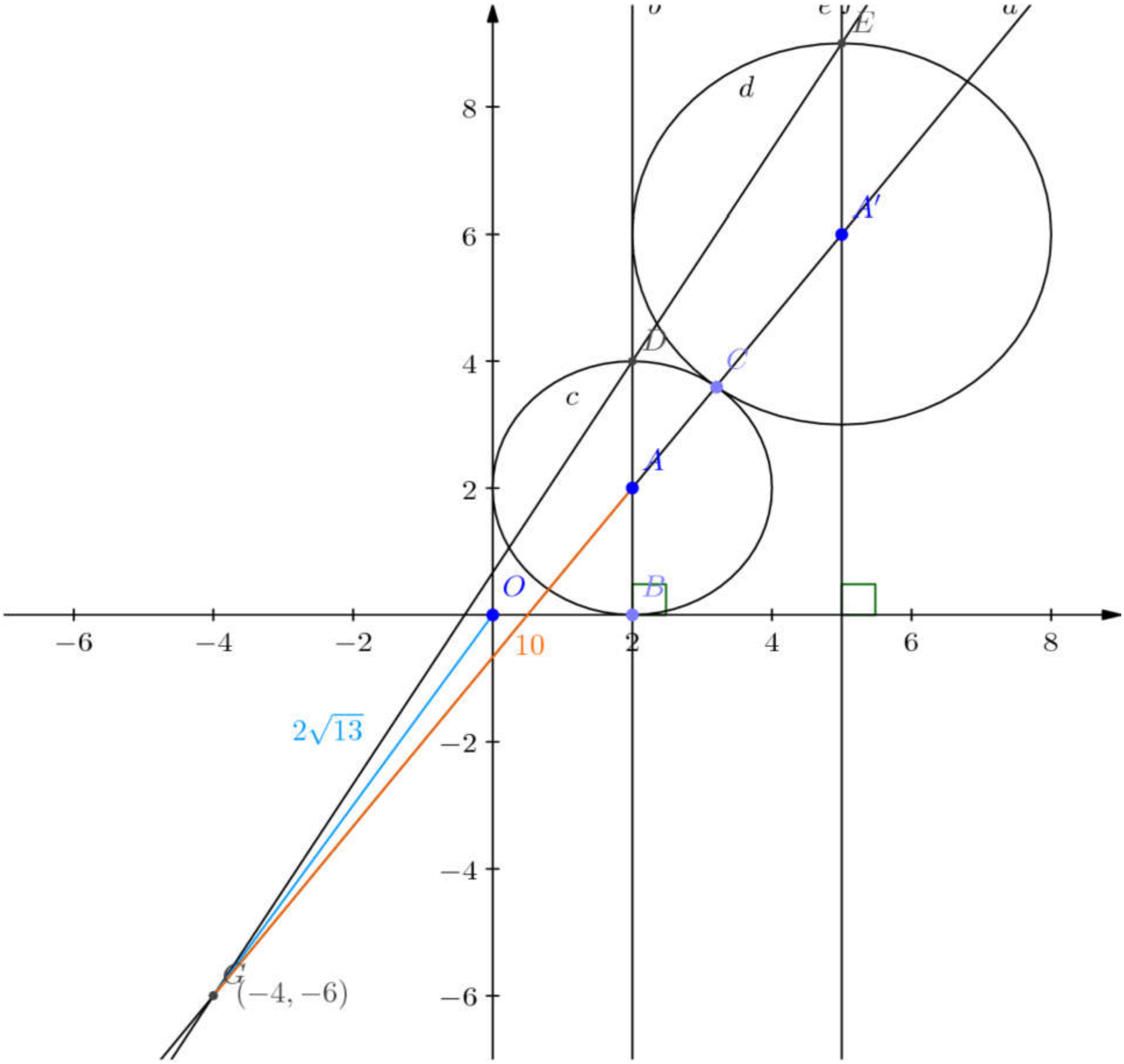
A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A'(5,6)$. What distance does the origin $O(0,0)$, move under this transformation?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ \sqrt{13}\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution 1: Algebraic

The center of dilation must lie on the line AA', which can be expressed $y=\frac{4x}{3}-\frac{2}{3}$. Also, the ratio of dilation must be equal to $\frac{3}{2}$, which is the ratio of the radii of the circles. Thus, we are looking for a point (x,y) such that $\frac{3}{2}(2-x)=5-x$ (for the x-coordinates), and $\frac{3}{2}(2-y)=6-y$. Solving these, we get $x=-4$ and $y=-6$. This means that any point (a,b) on the plane will dilate to the point $\left(\frac{3}{2}(a+4)-4,\frac{3}{2}(b+6)-6\right)$, which means that the point $(0,0)$ dilates to .
Thus, the origin moves $\sqrt{2^2+3^2}=\boxed{\sqrt{13}}$ units.

Solution 2: Geometric



Using analytic geometry, we find that the center of dilation is at $(-4, -6)$ and the coefficient/factor is 1.5. Then, we see that the origin is $2\sqrt{13}$ from the center, and will be $1.5 \times 2\sqrt{13} = 3\sqrt{13}$ from it afterwards.

Thus, it will move $3\sqrt{13} - 2\sqrt{13} = \boxed{\sqrt{13}}$. — @adihaya (talk) 15:04, 21 February 2016 (EST)

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 21

Contents

Problem

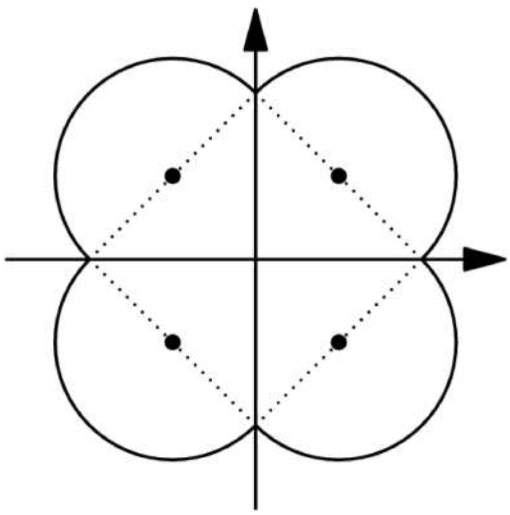
What is the area of the region enclosed by the graph of the equation $x^2+y^2=|x|+|y|$?

$\textbf{(A)}\ \pi+\sqrt{2}\qquad\textbf{(B)}\ \pi+2\qquad\textbf{(C)}\ \pi+2\sqrt{2}\qquad\textbf{(D)}\ 2\pi+\sqrt{2}\qquad\textbf{(E)}\ 2\pi+2\sqrt{2}$

Solution

WLOG note that if a point in the first quadrant satisfies the equation, so do its corresponding points in the other three quadrants. Therefore, we can assume that $x, y \geq 0$ and multiply by 4 at the end.

We can rearrange the equation to get $x^2-x+y^2-y=0 \Rightarrow (x-\frac{1}{2})^2+(y-\frac{1}{2})^2=(\frac{\sqrt{2}}{2})^2$, which describes a circle with center $\left(\frac{1}{2},\frac{1}{2}\right)$ and radius $\frac{\sqrt{2}}{2}$. It's clear we now want to find the union of four circles with overlap.



There are several ways to find the area, but note that if you connect $(0,1)$ to its other three respective points in the other three quadrants, you get a square of area 2, along with four half-circles of diameter $\sqrt{2}$, for a total area of $2+2\cdot(\frac{\sqrt{2}}{2})^2\pi=\pi+2$ which is $\textbf{(B)}$.

Solution 2

Another way to solve this problem is using cases. Though this may seem tedious, we only have to do one case. The equation for this figure is $x^2+y^2=|x|+|y|$ To make this as easy as possible, we can make both x and y positive. Simplifying the equation for x and y being positive, we get the equation x^2+y^2-x-y

Using the complete the square method, we get $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$

Therefore, the origin of this section of the shape is at $\left(\frac{1}{2},\frac{1}{2}\right)$.

Using the equation we can also see that the radius has a length of $\frac{\sqrt{2}}{2}$.

With this shape we see that this shape can be cut into a right triangle and a semicircle. The length of the hypotenuse of the triangle is $\sqrt{2}$ so using special right triangles, we see that the area of the triangle is $\frac{1}{2}$. The semicircle has the area of $\frac{1}{4}\pi$.

But this is only 1 case. There are 4 cases in total so we have to multiply $\frac{1}{2}+\frac{1}{4}\pi$ by 4.

After multiplying, our answer is:

$$\boxed{\textbf{(B)} 2+\pi}.$$

See Also

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2016 AMC 10B Problems/Problem 22

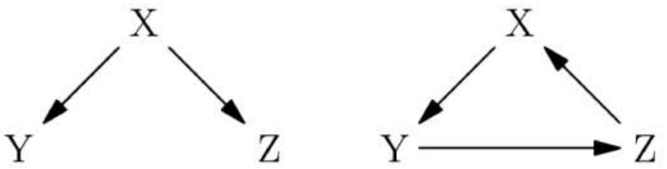
Problem

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

$\textbf{(A)}\ 385 \qquad \textbf{(B)}\ 665 \qquad \textbf{(C)}\ 945 \qquad \textbf{(D)}\ 1140 \qquad \textbf{(E)}\ 1330$

Solution

There are 21 teams. Any of the $\binom{21}{3} = 1330$ sets of three teams must either be a fork (in which one team beat both the others) or a cycle:



But we know that every team beat exactly 10 other teams, so for each possible X at the head of a fork, there are always exactly $\binom{10}{2}$ choices for Y and Z. Therefore there are $21 \cdot \binom{10}{2} = 945$ forks, and all the rest must be cycles.

Thus the answer is $1330 - 945 = 385$ which is $\textbf{(A)}$.

See Also

2016 AMC 10B Problems/Problem 23

Contents

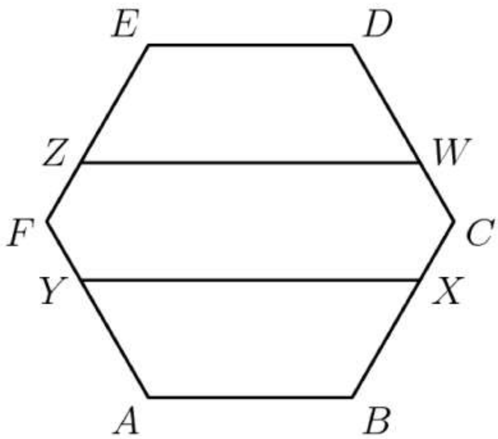
Problem

In regular hexagon $ABCDEF$, points W, X, Y, and Z are chosen on sides $\overline{BC}$, $\overline{CD}$, $\overline{EF}$, and $\overline{FA}$ respectively, so lines AB, ZW, YX, and ED are parallel and equally spaced. What is the ratio of the area of hexagon $WCXYFZ$ to the area of hexagon $ABCDEF$?

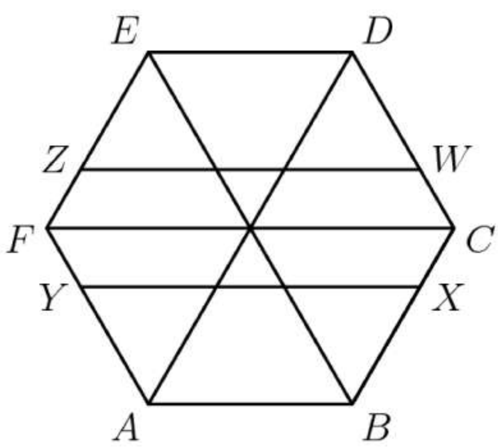
$\textbf{(A)}\ \frac{1}{3}\qquad\textbf{(B)}\ \frac{10}{27}\qquad\textbf{(C)}\ \frac{11}{27}\qquad\textbf{(D)}\ \frac{4}{9}\qquad\textbf{(E)}\ \frac{13}{27}$

Solution 1

We draw a diagram to make our work easier:



Assume that AB is of length 1. Therefore, the area of $ABCDEF$ is $\frac{3\sqrt{3}}{2}$. To find the area of $WCXYFZ$, we draw $\overline{CF}$, and find the area of the trapezoids $WCFZ$ and $CXYF$.



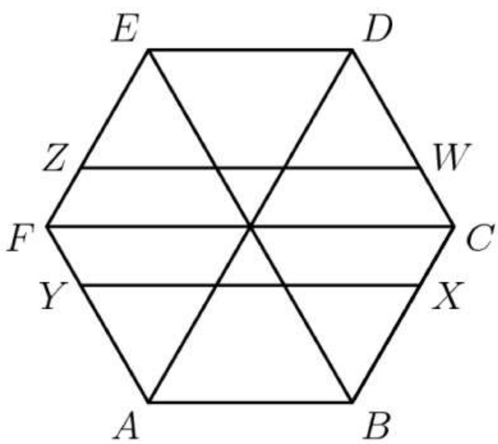
From this, we know that $CF = 2$. We also know that the combined heights of the trapezoids is $\frac{\sqrt{3}}{3}$, since $\overline{ZW}$ and $\overline{YX}$ are equally spaced, and the height of each of the trapezoids is $\frac{\sqrt{3}}{6}$. From this, we know $\overline{ZW}$ and $\overline{YX}$ are each $\frac{1}{3}$ of the way from $\overline{CF}$ to $\overline{DE}$ and $\overline{AB}$, respectively. We know that these are both equal to $\frac{5}{3}$.

We find the area of each of the trapezoids, which both happen to be $\frac{11}{6} \cdot \frac{\sqrt{3}}{6} = \frac{11\sqrt{3}}{36}$, and the combined area is $\frac{11\sqrt{3}}{18}^*$.

We find that $\frac{\frac{11\sqrt{3}}{18}}{\frac{3\sqrt{3}}{2}}$ is equal to $\frac{22}{54} = \boxed{\textbf{(C)}\ \frac{11}{27}}$.

* At this point, you can answer $\textbf{(C)}$ and move on with your test.

Solution 2 (a lot faster than Solution 1)



First, like in the first solution, split the large hexagon into 6 equilateral triangles. Each equilateral triangle can be split into three rows of smaller equilateral triangles. The first row will have one triangle, the second three, the third five. Once you have drawn these lines, it's just a matter of counting triangles. There are 22 small triangles in hexagon $ZWCXYF$, and $9 \cdot 6 = 54$ small triangles in the whole hexagon.

Thus, the answer is $\frac{22}{54} = \boxed{\textbf{(C)}\ \frac{11}{27}}$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 24

Contents

Problem

How many four-digit integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab < bc < cd$ form an increasing arithmetic sequence? One such number is 4692, where $a = 4$, $b = 6$, $c = 9$, and $d = 2$.

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 20$

Solution

The numbers are $10a + b, 10b + c,$ and $10c + d$. Note that only d can be zero, and that $a \le b \le c$.

To form the sequence, we need $(10c + d) - (10b + c) = (10b + c) - (10a + b)$. This can be rearranged as $10(c - 2b + a) = 2c - b - d$. Notice that since the left-hand side is a multiple of 10, the right-hand side can only be 0 or 10. (A value of -10 would contradict $a \le b \le c$.) Therefore we have two cases: $a + c - 2b = 1$ and $a + c - 2b = 0$.

Case 1

If $c = 9$, then $b + d = 8,\ 2b - a = 8$, so $5 \le b \le 8$. This gives $2593, 4692, 6791, 8890$. If $c = 8$, then $b + d = 6,\ 2b - a = 7$, so $4 \le b \le 6$. This gives $1482, 3581, 5680$. If $c = 7$, then $b + d = 4,\ 2b - a = 6$, so $b = 4$, giving 2470. There is no solution for $c = 6$.

Case 2

This means that the digits themselves are in arithmetic sequence. This gives $1234, 1357, 2345, 2468, 3456, 3579, 4567, 5678, 6789$.

Counting the solutions, the answer is $\textbf{(D)}\ 17$.

See Also

2016 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2016))	
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2016 AMC 10B Problems/Problem 25

Problem

Let $f(x)=\sum_{k=2}^{10}(\lfloor kx \rfloor -k \lfloor x \rfloor)$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r. How many distinct values does $f(x)$ assume for $x \ge 0$?

$\textbf{(A)}\ 32\qquad\textbf{(B)}\ 36\qquad\textbf{(C)}\ 45\qquad\textbf{(D)}\ 46\qquad\textbf{(E)}\ \text{infinitely many}$

Solution

Since $x = \lfloor x \rfloor + \{ x \}$, we have

$$f(x) = \sum_{k=2}^{10} (\lfloor k \lfloor x \rfloor + k \{ x \} \rfloor - k \lfloor x \rfloor)$$

The function can then be simplified into

$$f(x) = \sum_{k=2}^{10} (k \lfloor x \rfloor + \lfloor k \{ x \} \rfloor - k \lfloor x \rfloor)$$

which becomes

$$f(x) = \sum_{k=2}^{10} \lfloor k \{ x \} \rfloor$$

We can see that for each value of k, $\lfloor k \{ x \} \rfloor$ can equal integers from 0 to k-1.

Clearly, the value of $\lfloor k \{ x \} \rfloor$ changes only when x is equal to any of the fractions $\frac{1}{k}, \frac{2}{k} \dots \frac{k-1}{k}$.

So we want to count how many distinct fractions have the form $\frac{m}{n}$ where $n \le 10$. We can find this easily by computing

$$\sum_{k=2}^{10} \phi(k)$$

where $\phi(k)$ is the Euler Totient Function. Basically $\phi(k)$ counts the number of fractions with k as its denominator (after simplification). This comes out to be 31.

Because the value of $f(x)$ is at least 0 and can increase 31 times, there are a total of 32 different possible values of $f(x)$.

See Also