2017 AMC 12B Problems/Problem 1

Problem 1

Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Solution

Kymbrea has 30 comic books initially and every month, she adds two. This can be represented as $30+2x$ where x is the number of months elapsed. LaShawn's collection, similarly, is $10+6x$. To find when LaShawn will have twice the number of comic books as Kymbrea, we solve for x with the equation $2(2x+30)=6x+10$ and get $x=25$ $\boxed{\textbf{E}}$.

See Also

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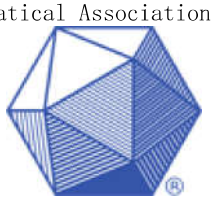

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2017 AMC 10B Problems/Problem 3

Problem

Real numbers x, y, and z satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

Solution

Notice that $y+z$ must be positive because $|z| > |y|$. Therefore the answer is $\boxed{\textbf{(E)}\ y+z}$.

The other choices:

$\textbf{(A)}$ As x grows closer to 0, x^2 decreases and thus becomes less than y.

$\textbf{(B)}$ x can be as small as possible ($x > 0$), so xz grows close to 0 as x approaches 0.

$\textbf{(C)}$ For all $-1 < y < 0$, $y > y^2$, and thus it is always negative.

$\textbf{(D)}$ The same logic as above, but when $-\frac{1}{2} < y < 0$ this time.

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2017 AMC 10B Problems/Problem 4

Problem

Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y}=-2$. What is the value of $\frac{x+3y}{3x-y}$?

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Solution

Rearranging, we find $3x+y=-2x+6y$, or $5x=5y \implies x=y$ Substituting, we can convert the second equation into $\frac{x+3x}{3x-x}=\frac{4x}{2x}=\boxed{\textbf{(D)}\ 2}$

Solution(Cheap)

Substituting each x and y with 1, we see that the given equation holds true, as $\frac{3(1)+1}{1-3(1)}=-2$. Thus, $\frac{x+3y}{3x-y}=\boxed{\textbf{(D)}\ 2}$

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2017 AMC 10B Problems/Problem 7

Problem

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Solution 1

Let's call the distance that Samia had to travel in total as $2x$, so that we can avoid fractions. We know that the length of the bike ride and how far she walked are equal, so they are both $\frac{2x}{2}$, or x.

She bikes at a rate of 17 kph, so she travels the distance she bikes in $\frac{x}{17}$ hours. She walks at a rate of 5 kph, so she travels the distance she walks in $\frac{x}{5}$ hours.

The total time is $\frac{x}{17}+\frac{x}{5}=\frac{22x}{85}$. This is equal to $\frac{44}{60}=\frac{11}{15}$ of an hour. Solving for x, we have:

$$\frac{22x}{85}=\frac{11}{15}$$

$$\frac{2x}{85}=\frac{1}{15}$$

$$30x=85$$

$$6x=17$$

$$x=\frac{17}{6}$$

Since x is the distance of how far Samia traveled by both walking and biking, and we want to know how far Samia walked to the nearest tenth, we have that Samia walked about $\boxed{\textbf{(C)}2.8}$.

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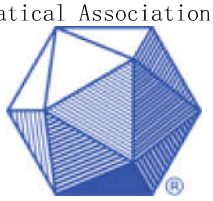

2017 AMC 12B Problems/Problem 5

Problem 5

The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartle (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

The interquartile range is defined as $Q3 - Q1$, which is $43 - 33 = 10$. 1.5 times this value is 15, so all values more than 15 below $Q1 = 33; -15 = 18$ is an outlier. The only one that fits this is 6. All values more than 15 above $Q3 = 43; +15 = 58$ are also outliers, of which there are none so there is only 1 $\boxed{\textbf{(B)}}$

See Also

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2017 AMC 12B Problems/Problem 6

Problem 6

The circle having $(0,0)$ and $(8,6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

Solution

Because the two points are on a diameter, the center must be halfway between them at the point $(4,3)$. The distance from $(0,0)$ to $(4,3)$ is 5 so the circle has radius 5. Thus, the equation of the circle is $(x-4)^2+(y-3)^2=25$.

To find the x-intercept, y must be 0, so $(x-4)^2+(0-3)^2=25$, so $(x-4)^2=16$, $x-4=4$, $x=8$.

Written by: SilverLion

See Also

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2017 AMC 12B Problems/Problem 7

Problem 7

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

$\textbf{(A)}\ \frac{\pi}{2} \qquad \textbf{(B)}\ \pi \qquad \textbf{(C)}\ 2\pi \qquad \textbf{(D)}\ 4\pi \qquad \textbf{(E)}$ The function is not periodic.

Solution

$\sin(x)$ has values $0, 1, 0, -1$ at its peaks and x-intercepts. Increase them to $0, \pi/2, 0, -\pi/2$. Then we plug them into $\cos(x)$. $\cos(0) = 1, \cos(\pi/2) = 0, \cos(0) = 1$, and $\cos(-\pi/2) = 0$. So, $\cos(sin(x))$ is $\frac{2\pi}{2} = \pi \boxed{\textbf{(B)}}$

Solution by TheUltimate123 (Eric Shen)

See Also

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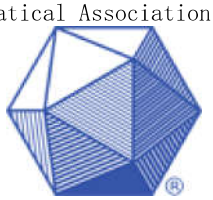

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2017 AMC 12B Problems/Problem 8

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Problem 8

The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

$\textbf{(A)}\ \frac{\sqrt{3}-1}{2} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(E)}\ \frac{\sqrt{6}-1}{2}$

Solution 1: Cross-Multiplication

Let a be the short side of the rectangle, and b be the long side of the rectangle. The diagonal, therefore, is $\sqrt{a^2+b^2}$. We can get the equation $\frac{a}{b}=\frac{b}{\sqrt{a^2+b^2}}$. Cross-multiplying, we get $a\sqrt{a^2+b^2}=b^2$. Squaring both sides of the equation, we get $a^2(a^2+b^2)=b^4$, which simplifies to $a^4+a^2b^2-b^4=0$. Solving for a quadratic in a^2, using the quadratic formula we get $a^2=\frac{-b^2\pm\sqrt{5b^4}}{2}$ which gives us $\frac{a^2}{b^2}=\frac{-1\pm\sqrt{5}}{2}$. We know that the square of the ratio must be positive (the square of any real number is positive), so the solution is $\boxed{\textbf{(C)}\frac{\sqrt{5}-1}{2}}$.

Solution by: vedadehhc

Solution 2: Substitution

Solution by HydroQuantum

Let the short side of the rectangle be a and let the long side of the rectangle be b. Then, the diagonal, according to the Pythagorean Theorem, is $\sqrt{a+b}$. Therefore, we can write the equation:

$$\frac{a}{b}=\frac{b}{\sqrt{a^2+b^2}}.$$

We are trying to find the square of the ratio of a to b. Let's let our answer, $\frac{a^2}{b^2}$, be k. Then, squaring the above equation,

$$\frac{a^2}{b^2}=k=\frac{b^2}{a^2+b^2}=\frac{b^2}{a^2}-\frac{b^2}{b^2}=\frac{1}{k}-1.$$

Thus, $k=\frac{1}{k}-1$.

Multiplying each side of the equation by k,

$$k^2=1-k.$$

Adding each side by $k-1$,

$$k^2+k-1=0.$$

Solving for k using the Quadratic Formula,

$$k = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$$

Since the ratio of lengths and diagonals of a rectangle cannot be negative, and $\sqrt{5} > 1$, the $\pm$ symbol can only take on the $+$. Therefore,

$$k = \frac{-1 + \sqrt{5}}{2} = \boxed{(\mathbf{C})\frac{\sqrt{5} - 1}{2}}.$$

See Also

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2017 AMC 12B Problems/Problem 9

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- 4 See Also

Problem 9

A circle has center $(-10,-4)$ and has radius 13. Another circle has center $(3,9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x+y=c$. What is c?

$\textbf{(A) } 3 \qquad \textbf{(B) } 3\sqrt{3} \qquad \textbf{(C) } 4\sqrt{2} \qquad \textbf{(D) } 6 \qquad \textbf{(E) } \frac{13}{2}$

Solution 1

The equations of the two circles are $(x+10)^2+(y+4)^2=169$ and $(x-3)^2+(y-9)^2=65$. Rearrange them to $(x+10)^2+(y+4)^2-169=0$ and $(x-3)^2+(y-9)^2-65=0$, respectively. Their intersection points are where these two equations gain equality. The two points lie on the line with the equation $(x+10)^2+(y+4)^2-169=(x-3)^2+(y-9)^2-65$. We can simplify this like the following.
$(x+10)^2+(y+4)^2-169=(x-3)^2+(y-9)^2-65 \rightarrow (x^2+20x+100)+(y^2+8y+16)-(x^2-6x+9)-(y^2-18y+81)=104-26x+26y+26=104 \rightarrow 26x+26y=78 \rightarrow x+y=3$
. Thus, $c=\boxed{\textbf{(A) } 3}$.

Solution by TheUltimate123 (Eric Shen)

Solution 2: Shortcut with right triangles

Note the specificity of the radii, 13 and $\sqrt{65}$, and that specificity is often deliberately added to simplify the solution to a problem.

One may recognize 13 as the hypotenuse of the $5-12-13$ right triangle and $\sqrt{65}$ as the hypotenuse of the right triangle with legs 1 and 8. We can suppose that the legs of these triangles connect the circles' centers to their intersection along the gridlines of the plane.

If we suspect that one of the intersections lies 12 units to the right of and 5 units above the center of the first circle, we find the point $(-10+12,-4+5)=(2,1)$, which is in fact 1 unit to the left of and 8 units below the center of the second circle at $(3,9)$.

Plugging $(2,1)$ into $x+y$ gives us $c=2+1=\boxed{\textbf{(A) } 3}$.

A similar solution uses the other intersection point, $(-5,8)$.

See Also

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2017 AMC 12B Problems/Problem 10

Problem

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ \frac{100}{3}\%$

Solution

Let there be 100 students. 60 of them like dancing, and 40 do not. Of those who like dancing, 20%, or 12 of them say they dislike dancing. Of those who dislike dancing, 90%, or 36 of them say they dislike it. Thus,

$$\frac{12}{12+36} = \frac{12}{48} = \frac{1}{4} = 25\% \boxed{\textbf{(D)}}$$

See Also

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2017 AMC 10B Problems/Problem 17

The following problem is from both the 2017 AMC 12B #11 and 2017 AMC 10B #17, so both problems redirect to this page.

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- 5 See Also

Problem

Call a positive integer $monotonous$ if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

$\textbf{(A) } 1024 \qquad \textbf{(B) } 1524 \qquad \textbf{(C) } 1533 \qquad \textbf{(D) } 1536 \qquad \textbf{(E) } 2048$

Solution 1

Case 1: monotonous numbers with digits in ascending order

There are $\Sigma_{n=1}^{9}\binom{9}{n}$ ways to choose n digits from the digits 1 to 9. For each of these ways, we can generate exactly one monotonous number by ordering the chosen digits in ascending order. Note that 0 is not included since it will always be a leading digit and that is not allowed. The sum is equivalent to

$$\Sigma_{n=0}^{9}\binom{9}{n}-\binom{9}{0}=2^9-1=511.$$

Case 2: monotonous numbers with digits in descending order

There are $\Sigma_{n=1}^{10}\binom{10}{n}$ ways to choose n digits from the digits 0 to 9. For each of these ways, we can generate exactly one monotonous number by ordering the chosen digits in descending order. Note that 0 is included since we are allowed to end numbers with zeros. The sum is equivalent to $\Sigma_{n=0}^{10}\binom{10}{n}-\binom{10}{0}=2^{10}-1=1023.$ We discard the number 0 since it is not positive. Thus there are 1022 here.

Since the 1-digit numbers 1 to 9 satisfy both case 1 and case 2, we have overcounted by 9. Thus there are $511+1022-9=\boxed{\textbf{B } 1524}$ monotonous numbers.

Solution 2

Like Solution 1, divide the problem into an increasing and decreasing case:

- Case 1: Monotonous numbers with digits in ascending order.

Arrange the digits 1 through 9 in increasing order, and exclude 0 because a positive integer cannot begin with 0.

To get a monotonous number, we can either include or exclude each of the remaining 9 digits, and there are $2^9=512$ ways to do this. However, we cannot exclude every digit at once, so we subtract 1 to get $512-1=511$ monotonous numbers for this case.

- Case 2: Monotonous numbers with digits in descending order.

This time, we arrange all 10 digits in decreasing order and repeat the process to find $2^{10} = 1024$ ways to include or exclude each digit. We cannot exclude every digit at once, and we cannot include only 0, so we subtract 2 to get $1024 - 2 = 1022$ monotonous numbers for this case.

At this point, we have counted all of the single-digit monotonous numbers twice, so we must subtract 9 from our total.

Thus our final answer is $511 + 1022 - 9 = \boxed{\textbf{(B)}1524}$.

Solution 3: Answer Choices

by e_power_pi_times_i

It adds in 9 at the end, but we know since it is MAA, it is probably a troll question, so we look at the answers. (C) looks likely, as it is just $(B) + 9$, but we remember that MAA is trolly so it is probably $\boxed{\textbf{(B) } 1524}$ because they would put the answer as the one that looks the most unlikely.

See Also

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2017 AMC 12B Problems/Problem 12

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Problem 12

What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ \sqrt{2}+2\sqrt{3} \qquad \textbf{(D)}\ 2\sqrt{2}+\sqrt{6} \qquad \textbf{(E)}\ (1+\sqrt{3})+(1+\sqrt{3})i$

Solution 1

The root of any polynomial of the form $z^n = a$ will have all n of it roots will have magnitude $\sqrt[n]{a}$ and be the vertices of a regular n-gon in the complex plane (This concept is known as the Roots of Unity). For the equation $z^{12} = 64$, it is easy to see $\pm\sqrt{2}$ and $\pm i\sqrt{2}$ as roots. Graphing these in the complex plane, we have four vertices of a regular dodecagon. Since the roots must be equally spaced, besides $\sqrt{2}$, there are four more roots with positive real parts lying in the first and fourth quadrants. We also know that the angle between these roots is 30°. We only have to find the real parts of the roots lying in the first quadrant, because the imaginary parts would cancel out with those from the fourth quadrant. We have two $30-60-90$ triangles (the triangles formed by connecting the origin to the roots, and dropping a perpendicular line from each root to the real-axis), both with hypotenuse $\sqrt{2}$. This means that one has base $\frac{\sqrt{2}}{2}$ and the other has base $\frac{\sqrt{6}}{2}$. Adding these and multiplying by two, we get the sum of the four roots as $\sqrt{2}+\sqrt{6}$. However, we have to add in the original solution of $\sqrt{2}$, so the answer is $\boxed{\textbf{(D)}2\sqrt{2}+\sqrt{6}}$.

Solution by vedadehhc

Solution 2

$z^{12} = 64$ has a factor of $\sqrt{2}$, so we need to remember to multiply our solution below, using the Roots of Unity. We notice that the sum of the complex parts of all these roots is 0, because the points on the complex plane are symmetric. The roots with $re(z) > 0$ are $e^{0i}, e^{\frac{\pm\pi}{6}i}$, and $e^{\frac{\pm\pi}{3}i}$ by the Roots of Unity. Their real parts are $\cos(0), \pm\cos(\frac{\pi}{6})$, and $\pm\cos(\frac{\pi}{3})$. Their sum is $1+2(\frac{\sqrt{3}}{2}+\frac{1}{2}) = 1+\sqrt{3}+1 = 2+\sqrt{3}$. But, remember to multiply by $\sqrt{2}$. The answer is $\sqrt{2}(2+\sqrt{3}) = \boxed{\textbf{(D)}\ 2\sqrt{2}+\sqrt{6}}$.

Solution by TheUltimate123 (Eric Shen)

See Also

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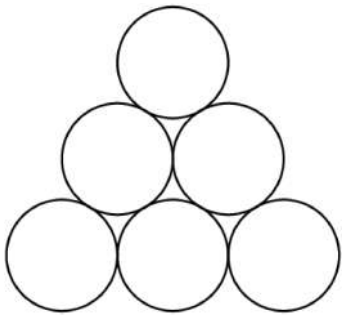
2017 AMC 12B Problems/Problem 13

Problem 13

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Solution

Looking at the answer choices, we see that the possibilities are indeed countable. Thus, we will utilize that approach in the form of two separate cases, as rotation and reflection take care of numerous possibilities. First, consider the case that the green disk is in a corner. This yields 6 possible arrangements for the 3 blue disks and 2 red disks in the remaining available slots. Now, consider the case that the green disk is on an edge. This yields 6 more possible arrangements for the 3 blue disks and 2 red disks in the remaining available slots. Thus, our answer is $6+6=\boxed{\textbf{(D)}\ 12}$

Solution by akaashp11

See Also

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2017 AMC 12B Problems/Problem 14

Problem

An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

$\textbf{(A)}\ 8\pi \qquad \textbf{(B)}\ \frac{28\pi}{3} \qquad \textbf{(C)}\ 12\pi \qquad \textbf{(D)}\ 14\pi \qquad \textbf{(E)}\ \frac{44\pi}{3}$

Solution

The top cone has radius 2 and height 4 so it has volume $\frac{1}{3}\pi(2)^2 \times 4$.

The frustum is made up by taking away a small cone of radius 1, height 4 from a large cone of radius 2, height 8, so it has volume $\frac{1}{3}\pi(2)^2 \times 8 - \frac{1}{3}\pi(1)^2 \times 4$.

Adding, we get $\frac{1}{3}\pi(16+32-4) = \boxed{\textbf{E}\ \frac{44\pi}{3}}$.

Solution by: SilverLion

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2017 AMC 12B Problems/Problem 15

Problem 15

Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

$\textbf{(A)}\ 9:1 \qquad \textbf{(B)}\ 16:1 \qquad \textbf{(C)}\ 25:1 \qquad \textbf{(D)}\ 36:1 \qquad \textbf{(E)}\ 37:1$

Solution 1: Law of Cosines

Solution by HydroQuantum

Let $AB = BC = CA = x$.

Recall The Law of Cosines. Letting $A'B' = B'C' = C'A' = y$,

$$y^2 = (3x)^2 + (x+3x)^2 - 2(3x)(x+3x)(cos120) =$$

$$(3x)^2+(4x)^2-2(3x)(4x)(cos120) = 9x^2+16x^2-24cos120 = 25x^2+12x^2 = 37x^2.$$

Since both $\triangle ABC$ and $\triangle A'B'C'$ are both equilateral triangles, they must be similar due to AA similarity. This means that $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'A'}{CA} = \frac{[\triangle A'B'C']}{[\triangle ABC]} = \frac{37}{1}$.

Therefore, our answer is $\boxed{\textbf{(E)}\ 37:1}$.

Solution 2: Inspection

Note that the height and base of $\triangle A'CC'$ are respectively 4 times and 3 times that of $\triangle ABC$. Therefore the area of $\triangle A'CC'$ is 12 times that of $\triangle ABC$.

By symmetry, $\triangle A'CC' \cong \triangle B'AA' \cong \triangle C'BB'$. Adding the areas of these three triangles and $\triangle ABC$ for the total area of $\triangle A'B'C'$ gives a ratio of $(12+12+12+1):1$, or $\boxed{\textbf{(E)}\ 37:1}$.

2017 AMC 10B Problem 19 Solution

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2017 AMC 12B Problems/Problem 16

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Problem 16

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

$\textbf{(A)}\ \frac{1}{21} \qquad \textbf{(B)}\ \frac{1}{19} \qquad \textbf{(C)}\ \frac{1}{18} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{11}{21}$

Solution

If a factor of $21!$ is odd, that means it contains no factors of 2. We can find the number of factors of two in $21!$ by counting the number multiples of 2, 4, 8, and 16 that are less than or equal to 21. After some quick counting we find that this number is $10+5+2+1=18$. If the prime factorization of $21!$ has 18 factors of 2, there are 19 choices for each divisor for how many factors of 2 should be included (0 to 18 inclusive). The probability that a randomly chosen factor is odd is the same as if the number of factors of 2 is 0 which is $\boxed{\textbf{(B)}\frac{1}{19}}$.

Solution by: vedadehhc

Solution 2

We can write $21!$ as its prime factorization:

$$21! = 2^{18} \times 3^9 \times 5^4 \times 7^3 \times 11 \times 13 \times 17$$

Each exponent of these prime numbers are one less than the number of factors at play here. This makes sense; 2^{18} is going to have 19 factors: $2^0, 2^1, 2^2, \ldots 2^{18}$, and the other exponents will behave identically.

In other words, $21!$ has $(18+1)(9+1)(4+1)(3+1)(1+1)(1+1)(1+1)$ factors.

We are looking for the probability that a randomly chosen factor of $21!$ will be odd--numbers that do not contain multiples of 2 as factors.

From our earlier observation, the only factors of $21!$ that are even are ones with at least one multiplier of 2, so our probability of finding an odd factor becomes the following:

$$P(\text{odd}) = \frac{\text{number of odd factors}}{\text{number of all factors}} = \frac{(9+1)(4+1)(3+1)(1+1)(1+1)(1+1)}{(18+1)(9+1)(4+1)(3+1)(1+1)(1+1)(1+1)} = \frac{1}{(18+1)} = \boxed{\frac{1}{19}}$$

Solution submitted by David Kim

See Also

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2017 AMC 12B Problems/Problem 17

Problem 17

A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

$\mathbf{(A)}$ The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

$\mathbf{(B)}$ The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

$\mathbf{(C)}$ The probabilities are the same.

$\mathbf{(D)}$ The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

$\mathbf{(E)}$ The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

Solution

The probability of winning Game A is the sum of the probabilities of getting three tails and getting three heads which is $\left(\frac{2}{3}\right)^3+\left(\frac{1}{3}\right)^3=\frac{8}{27}+\frac{1}{27}=\frac{9}{27}=\frac{27}{81}$. The probability of winning Game B is the sum of the probabilities of getting two heads and getting two tails squared. This gives us $\left(\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2\right)^2=\left(\frac{5}{9}\right)^2=\frac{25}{81}$. The probability of winning Game A is $\frac{27}{81}$ and the probability of winning Game B is $\frac{25}{81}$, so the answer is $\boxed{\mathbf{(D)}}$

Solution by: vedadehhc

See Also

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2017 AMC 12B Problems/Problem 18

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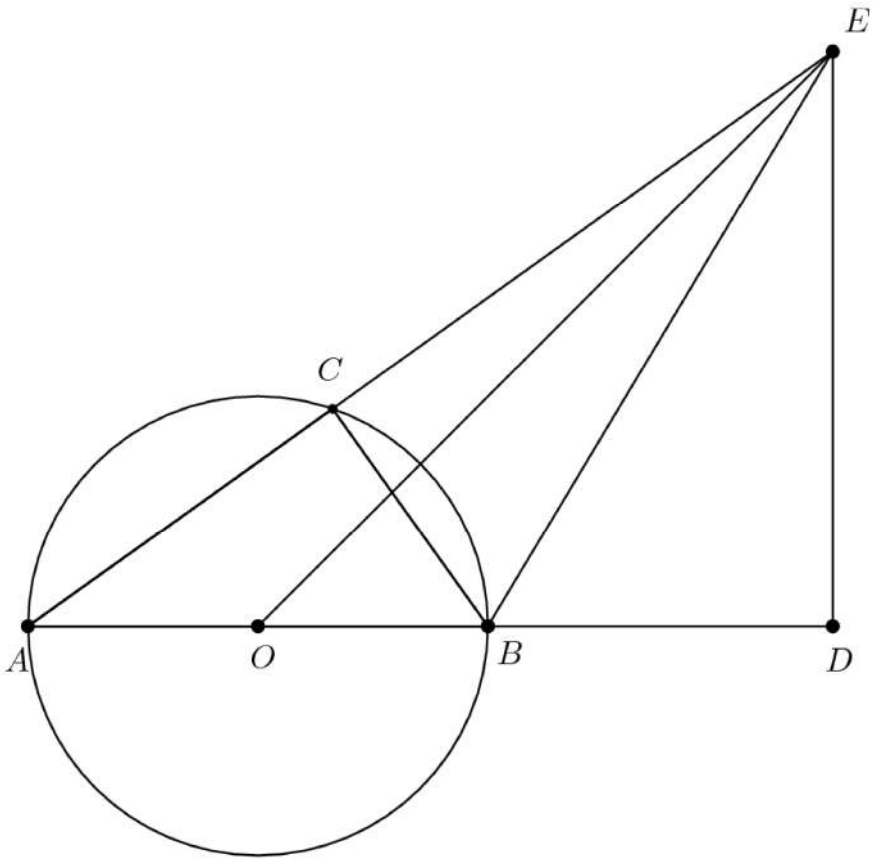
Problem

The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

$\textbf{(A)}\ \frac{120}{37} \qquad \textbf{(B)}\ \frac{140}{39} \qquad \textbf{(C)}\ \frac{145}{39} \qquad \textbf{(D)}\ \frac{140}{37} \qquad \textbf{(E)}\ \frac{120}{31}$

Solution 1



Let O be the center of the circle. Note that $EC + CA = EA = \sqrt{AD^2 + DE^2} = \sqrt{(2+2+3)^2 + 5^2} = \sqrt{74}$. However, by Power of a Point,

$$(EC)(EC + CA) = EO^2 - R^2 = (2+3)^2 + 5^2 - 2^2 = 25 + 25 - 4 = 46 \implies EC = \frac{46}{\sqrt{74}}$$

, so $AC = \sqrt{74} - \frac{46}{\sqrt{74}} = \frac{28}{\sqrt{74}}$. Now $BC = \sqrt{AB^2 - AC^2} = \sqrt{4^2 - \frac{28^2}{74}} = \sqrt{\frac{16 \cdot 74 - 28^2}{74}} = \sqrt{\frac{1184 - 784}{74}} = \frac{20}{\sqrt{74}}$. Since $\angle ACB = 90^\circ, [ABC] = \frac{1}{2} \cdot BC \cdot AC = \frac{1}{2} \cdot \frac{20}{\sqrt{74}} \cdot \frac{28}{\sqrt{74}} = \boxed{\textbf{(D)}\ \frac{140}{37}}$.

Solution 2: Similar triangles

AB is the diameter of the circle, so $\angle ACB$ is a right angle, and therefore by AAA similarity, $\triangle ACB \sim$.

Because of this, $\frac{AC}{AD} = \frac{AB}{AE} \implies \frac{AC}{2+2+3} = \frac{4}{\sqrt{7^2+5^2}}$, so $AC = \frac{28}{\sqrt{74}}$.

Likewise, $\frac{CB}{DE} = \frac{AB}{AE} \implies \frac{CB}{5} = \frac{4}{\sqrt{74}}$, so $CB = \frac{20}{\sqrt{74}}$.

Thus the area of $\triangle ABC = \frac{1}{2} \cdot \frac{28}{\sqrt{74}} \cdot \frac{20}{\sqrt{74}} = \boxed{\textbf{(D)}\ \frac{140}{37}}$.

See Also

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2017 AMC 12B Problems/Problem 19

Problem

Let $N = 123456789101112\ldots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 44$

Solution

We will consider this number $\mod 5$ and $\mod 9$. By looking at the last digit, it is obvious that the number is $\equiv 4 \mod 5$. To calculate the number $\mod 9$, note that

$$123456\cdots4344 \equiv 1+2+3+4+5+6+7+8+9+(1+0)+(1+1)+\cdots+(4+3)+(4+4) \equiv 1+2+\cdots+44 \mod 9.$$

so it is equivalent to

$$\frac{44\cdot 45}{2} = 22\cdot 45 \equiv 0 \mod 9.$$

Thus it is $0 \mod 9$ and $4 \mod 5$, so it is $(\mathbf{C}) = 9 \mod 45$.

See Also

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2017 AMC 12B Problems/Problem 20

Problem 20

Real numbers x and y are chosen independently and uniformly at random from the interval $(0,1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r ?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution

First let us take the case that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor = -1$. In this case, both x and y lie in the interval $[1/2, 1)$. The probability of this is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Similarly, in the case that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor = -2$, x and y lie in the interval $[1/4, 1/2)$, and the probability is $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$. It is easy to see that the probabilities for $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor = n$ for $-\infty < n < 0$ are the infinite geometric series that starts at $\frac{1}{4}$ and with common ratio 14. Using the formula for the sum of an infinite geometric series, we get that the probability is $\frac{\frac{1}{4}}{1-\frac{1}{4}} = \boxed{\textbf{(D)}\frac{1}{3}}$.

Solution by: vedadehhc

See Also

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2017 AMC 12B Problems/Problem 21

Problem 21

Last year, Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. Was was her score on the sixth test?

(A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

Solution 1

Let us simplify the problem. Since all of Isabella's test scores can be expressed as the sum of 90 and an integer between 1 and 10, we rewrite the problem into receiving scores between 1 and 10. Later, we can add 90 to her score to obtain the real answer.

From this point of view, the problem states that isabella's score on the seventh test was 5. We note that Isabella received 7 integer scores out of 1 to 10. Since 5 is already given as the seventh test score, the possible scores for Isabella on the other six tests are S={1,2,3,4,6,7,8,9,10}.

The average score for the seven tests must be an integer. In other words, six distinct integers must be picked from set S above, and their sum with 5 must be a multiple of 7. The interval containing the possible sums of the six numbers in S are from 23(=1+2+3+4+6+7) to 44(=4+6+7+8+9+10). We must now find multiples of 7 from the interval 28(=23+5) to 49(=44+5). There are four possibilities: 28, 35, 42, 49. However, we also note that the sum of the six numbers (besides 5) must be a multiple of 6 as well. Thus, 35 is the only valid choice.(The six numbers sum to 30.)

Thus the sum of the six numbers equals to 30. We apply the logic above in a similar way for the sum of the scores from the first test to the fifth test. The sum must be a multiple of 5. The possible interval is from 16(=1+2+3+4+6) to 40(=6+7+8+9+10). Since the sum of the five scores must be less than 30, the only possibilities are 20 and 25. However, we notice that 25 does not work because the sixth score turns out to be 5 from the calculation. Therefore, the sum of Isabella's scores from test 1 to 5 is 20. Therefore, her score on the sixth test is 10. Our final answer is 10+90=100 ==> (E)

(I would really appreciate it if someone could insert mathematical symbols to make this solution more clear.)

Solution 2

Let n be Isabella's average after 6 tests. $6n+95=0(mod7)$, so $n=4(mod7)$. The only integer between 90 and 100 that satisfies this condition is 95. Let m be Isabella's average after 5 tests, and let a be her sixth test score. $5m+a=95$, so a is a multiple of 5. Since 100 is the only choice that is a multiple of 5, the answer is 100=D.

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2017 AMC 12B Problems/Problem 22

Problem 22

Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn---one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

$\textbf{(A)}\ \frac{7}{576} \qquad \textbf{(B)}\ \frac{5}{192} \qquad \textbf{(C)}\ \frac{1}{36} \qquad \textbf{(D)}\ \frac{5}{144} \qquad \textbf{(E)}\ \frac{7}{48}$

Solution

It amounts to filling in a 4x4 matrix M. Columns C1 - C4 are the random draws each round; rows RA -RD are the coin changes of each player. Also, let <RA>=number of non-0 elements in RA.

WOLG, let C1=[1,-1,0,0]. Parity demands that <RA> and= 1 or 3.

CASE 1. <RA>=3, and <RB>=3. 3 choices for -1in RA, then everything is determined. So 3 subcases here.

Case 2. <RA>=1, and <RB>=3. 3 Choices for the -1in RA, 2 choices for the rest of 1 -1 in RB, 2 choices between C and D to fill in the remaining pair of 1 and -1. Then double for symmetrical case of <RA>=3 and <RB>=1. Total 24 subcases.

Case 3. <RA>=1, and <RB>=1. 3 Choices for -1in RA. Then (1)1 in RB goes directly under 1 in RA. Then 2 choices to assign remaining two pairs of 1, -1in RC and RD. Or (2) 1 in RB goes not directly under 1 in RA: 2 choices here. Then 2 choices to finish. Hence 3(2+2×2)=18.

In sum, 3+24+18=45 cases. Probability= 45÷(12^3)=5/192.

See Also

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2017 AMC 12B Problems/Problem 23

Problem 23

The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$?

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{24}{5} \qquad \textbf{(E)}\ 8$

Solution

First, we can define $f(x) = a(x-2)(x-3)(x-4) + x^2$, which contains points A, B, and C. Now we find that lines AB, AC, and BC are defined by the equations $y = 5x - 6$, $y = 6x - 8$, and $y = 7x - 12$ respectively. Since we want to find the x-coordinates of the intersections of these lines and $f(x)$, we set each of them to $f(x)$, and synthetically divide by the solutions we already know exist (eg. if we were looking at line AB, we would synthetically divide by the solutions $x = 2$ and $x = 3$, because we already know AB intersects the graph at A and B, which have x-coordinates of 2 and 3). After completing this process on all three lines, we get that the x-coordinates of D, E, and F are $\frac{4a-1}{a}$, $\frac{3a-1}{a}$, and $\frac{2a-1}{a}$ respectively. Adding these together, we get $\frac{9a-3}{a} = 24$ which gives us $a = -\frac{1}{5}$. Substituting this back into the original equation, we get $f(x) = -\frac{1}{5}(x-2)(x-3)(x-4) + x^2$, and

$$f(0) = -\frac{1}{5}(-2)(-3)(-4) + 0 = \boxed{\textbf{(D)}\frac{24}{5}}$$

Solution by: vedadehhc and tdeng

See Also

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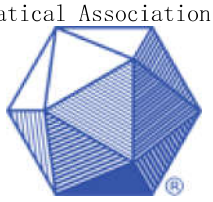

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2017 AMC 12B Problems/Problem 24

Problem

Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC$ is similar to $\triangle BCD$, and $AB > BC$. There exists a point E in the interior of $ABCD$ such that $\triangle ABC$ is similar to $\triangle CEB$ and the area of Triangle AED is 17 times the area of Triangle CEB. Find AB/BC.

$\textbf{(A)}\ 1+\sqrt{2} \qquad \textbf{(B)}\ 2+\sqrt{2} \qquad \textbf{(C)}\ \sqrt{17} \qquad \textbf{(D)}\ 2+\sqrt{5} \qquad \textbf{(E)}\ 1+2\sqrt{3}$

Solution

Let $CD = 1$, $BC = x$, and $AB = x^2$. Note that $AB/BC = x$. By the Pythagorean Theorem, $BD = \sqrt{x^2+1}$. Since $\triangle BCD \sim \triangle ABC \sim \triangle CEB$, the ratios of side lengths must be equal. Since $BC = x$, $CE = \frac{x^2}{\sqrt{x^2+1}}$ and $BE = \frac{x}{\sqrt{x^2+1}}$. Let F be a point on $\overline{BC}$ such that $\overline{EF}$ is an altitude of triangle CEB. Note that $\triangle CEB \sim \triangle CFE \sim \triangle EFB$. Therefore, $BF = \frac{x}{x^2+1}$ and $CF = \frac{x^3}{x^2+1}$. Since $\overline{CF}$ and $\overline{BF}$ form altitudes of triangles CED and BEA, respectively, the areas of these triangles can be calculated. Additionally, the area of triangle BEC can be calculated, as it is a right triangle. Solving for each of these yields:

$$[BEC] = [CED] = [BEA] = (x^3)/(2(x^2+1))$$

$$[ABCD] = [AED] + [DEC] + [CEB] + [BEA]$$

$$(AB+CD)(BC)/2 = 17*[CEB] + [CEB] + [CEB] + [CEB]$$

$$(x^3+x)/2 = (20x^3)/(2(x^2+1))$$

$$(x)(x^2+1) = 20x^3/(x^2+1)$$

$$(x^2+1)^2 = 20x^2$$

$$x^4 - 18x^2 + 1 = 0 \implies x^2 = 9 + 4sqrt(5) = 4 + 2(2sqrt(5)) + 5$$

Therefore, the answer is $\boxed{\textbf{(D)}\ 2+\sqrt{5}}$

See Also

2017 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2017))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

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2017 AMC 12B Problems/Problem 25

Problem

A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no two teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \leq n \leq 2017$, can be the number of participants?

(A) 477 **(B)** 482 **(C)** 487 **(D)** 557 **(E)** 562

Solution

Solution by Pieater314159

Let there be T teams. For each team, there are $\binom{n-5}{4}$ different subsets of 9 players including that full team, so the total number of team-(group of 9) pairs is

$$T\binom{n-5}{4}.$$

Thus, the expected value of the number of full teams in a random set of 9 players is

$$\frac{T\binom{n-5}{4}}{\binom{n}{9}}.$$

Similarly, the expected value of the number of full teams in a random set of 8 players is

$$\frac{T\binom{n-5}{3}}{\binom{n}{8}}.$$

The condition is thus equivalent to the existence of a positive integer T such that

$$\frac{T\binom{n-5}{4}}{\binom{n}{9}}\frac{T\binom{n-5}{3}}{\binom{n}{8}} = 1.$$

$$T^2\frac{(n-5)!(n-5)!8!9!(n-8)!(n-9)!}{n!n!(n-8)!(n-9)!3!4!} = 1$$

$$T^2 = \big((n)(n-1)(n-2)(n-3)(n-4)\big)^2\frac{3!4!}{8!9!}$$

$$T^2 = \big((n)(n-1)(n-2)(n-3)(n-4)\big)^2\frac{144}{7!7!8\cdot 8\cdot 9}$$

$$T^2 = \left((n)(n-1)(n-2)(n-3)(n-4)\right)^2 \frac{1}{4 \cdot 7!7!}$$

$$T = \frac{(n)(n-1)(n-2)(n-3)(n-4)}{2^5 \cdot 3^2 \cdot 5 \cdot 7}$$

Note that this is always less than $\binom{n}{5}$, so as long as T is integral, n is a possibility. Thus, we have that this is equivalent to

$$2^5 \cdot 3^2 \cdot 5 \cdot 7 \Big| (n)(n-1)(n-2)(n-3)(n-4).$$

It is obvious that 5 divides the RHS, and that 7 does iff $n \equiv 0, 1, 2, 3, 4 \mod 7$. Also, 3^2 divides it iff $n \not\equiv 5, 8 \mod 9$. One can also bash out that 2^5 divides it in 16 out of the 32 possible residues $\mod 32$.

Using all numbers from 2 to 2017, inclusive, it is clear that each possible residue $\mod 7, 9, 32$ is reached an equal number of times, so the total number of working n in that range is $5 \cdot 7 \cdot 16 = 560$. However, we must subtract the number of "working" $2 \leq n \leq 8$, which is 3. Thus, the answer is $\textbf{(D)}\ 557$.

See Also

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