# 2017 AMC 10B Problems/Problem 1

## Contents



## Problem

Mary thought of a positive two-digit number. She multiplied it by $3$ and added $11$. Then she switched the digits of the result, obtaining a number between $71$ and $75$, inclusive. What was Mary's number?

$\textbf{(A)}\ 11\qquad\textbf{(B)}\ 12\qquad\textbf{(C)}\ 13\qquad\textbf{(D)}\ 14\qquad\textbf{(E)}\ 15$

## Solution 1

Let her $12$-digit number be $x$. Multiplying by $3$ makes it a multiple of $3$, meaning that the sum of its digits is divisible by $3$. Adding on $11$ increases the sum of the digits by $1+1=2$, and reversing the digits keeps the sum of the digits the same; this means that the resulting number must be $2$ more than a multiple of $3$. There are two such numbers between $71$ and $75$: $71$ and $74.$ Now that we have narrowed down the choices, we can simply test the answers to see which one will provide a two-digit number when the steps are reversed:

For $71,$ we reverse the digits, resulting in $17.$ Subtracting $11$, we get $6.$ We can already see that dividing this by $3$ will not be a two-digit number, so $71$ does not meet our requirements.

Therefore, the answer must be the reversed steps applied to $74.$ We have the following:

$74 \rightarrow 47 \rightarrow 36 \rightarrow 12$

Therefore, our answer is $\boxed{\textbf{(B)} 12.}$

## Solution 2

Working backwards, we reverse the digits of each number from $71$~$75$ and subtract $11$ from each, so we have

$$6, 16, 26, 36, 46$$

The only numbers from this list that are divisible by $3$ are $6$ and $36$. We divide both by $3$, yielding $2$ and $12$. Since $2$ is not a two-digit number, the answer is $\boxed{\textbf{(B)}\ 12}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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| Preceded by First Problem | Followed by Problem 2 |
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# 2017 AMC 10B Problems/Problem 2

## Problem

Sofia ran $5$ laps around the $400$-meter track at her school. For each lap, she ran the first $100$ meters at an average speed of $4$ meters per second and the remaining $300$ meters at an average speed of $5$ meters per second. How much time did Sofia take running the $5$ laps?

$\textbf{(A)}\ \text{5 minutes and 35 seconds} \qquad \textbf{(B)}\ \text{6 minutes and 40 seconds} \qquad \textbf{(C)}\ \text{7 minutes and 5 seconds} \qquad \textbf{(D)}\ \text{7 minutes and 25 seconds}$
$\textbf{(E)}\ \text{8 minutes and 10 seconds}$

## Solution

If Sofia ran the first $100$ meters of each lap at $4$ meters per second and the remaining $300$ meters of each lap at $5$ meters per second, then she took $\frac{100}{4}+\frac{300}{5}=25+60=85$ seconds for each lap. Because she ran $5$ laps, she took a total of $5\cdot 85=425$ seconds, or $7$ minutes and $5$ seconds. The answer is $\boxed{\textbf{(C)}\ \text{7 minutes and 5 seconds}}$.

## See Also


| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
|---|---|
| Preceded by<br>Problem 1 | Followed by<br>Problem 3 |
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# 2017 AMC 10B Problems/Problem 3

## Problem

Real numbers $x$, $y$, and $z$ satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

## Solution

Notice that $y+z$ must be positive because $|z|>|y|$. Therefore the answer is $\boxed{\textbf{(E)}\ y+z}$.

The other choices:

$\textbf{(A)}$ As $x$ grows closer to $0$, $x^2$ decreases and thus becomes less than $y$.

$\textbf{(B)}$ $x$ can be as small as possible ($x>0$), so $xz$ grows close to $0$ as $x$ approaches $0$.

$\textbf{(C)}$ For all $-1<y<0$, $y>y^2$, and thus it is always negative.

$\textbf{(D)}$ The same logic as above, but when $-\frac{1}{2}<y<0$ this time.


| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
|---|---|
| Preceded by Problem 2 | Followed by Problem 4 |
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# 2017 AMC 10B Problems/Problem 4

## Problem

Supposed that $x$ and $y$ are nonzero real numbers such that $\frac{3x+y}{x-3y} = -2$. What is the value of $\frac{x+3y}{3x-y}$?

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

## Solution

Rearranging, we find $3x+y = -2x+6y$, or $5x = 5y \implies x = y$ Substituting, we can convert the second equation into $\frac{x+3x}{3x-x} = \frac{4x}{2x} = \boxed{\textbf{(D)}\ 2}$

## Solution(Cheap)

Substituting each $x$ and $y$ with $1$, we see that the given equation holds true, as $\frac{3(1)+1}{1-3(1)} = -2$. Thus, $\frac{x+3y}{3x-y} = \boxed{\textbf{(D)}\ 2}$

Solution by sp1729

# 2017 AMC 10B Problems/Problem 5

## Problem

Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

$\textbf{(A)}\ 10\qquad\textbf{(B)}\ 20\qquad\textbf{(C)}\ 30\qquad\textbf{(D)}\ 40\qquad\textbf{(E)}\ 50$

## Solution 1

Denote the number of blueberry and cherry jelly beans as $b$ and $c$ respectively. Then $b=2c$ and $b-10=3(c-10)$. Substituting, we have $2c-10=3c-30$, so $c=20$, $b=\boxed{\textbf{(D)}\ 40}$.

## Solution 2 (using answer choices)

From the problem, we see that 10 less than one of the answer choices must be a multiple of 3 and positive. The only answer choice satisfying this is $\boxed{\textbf{(D)}\ 40}$. We can check that 40 blueberry and 20 cherry jelly beans indeed does work.

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 6

## Problem

What is the largest number of solid $2\text{-in} \times 2\text{-in} \times 1\text{-in}$ blocks that can fit in a $3\text{-in} \times 2\text{-in} \times 3\text{-in}$ box?

$\textbf{(A)}\ 3\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 6\qquad\textbf{(E)}\ 7$

## Solution

We find that the volume of the larger block is $18$, and the volume of the smaller block is $4$. Dividing the two, we see that only a maximum of four $2$ by $2$ by $1$ blocks can fit inside the $3$ by $3$ by $2$ block. Drawing it out, we see that such a configuration is indeed possible. Therefore, the answer is $\boxed{\textbf{(B)}\ 4}$.



| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 7

## Problem

Samia set off on her bicycle to visit her friend, traveling at an average speed of $17$ kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at $5$ kilometers per hour. In all it took her $44$ minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

$\textbf{(A)}\ 2.0\qquad\textbf{(B)}\ 2.2\qquad\textbf{(C)}\ 2.8\qquad\textbf{(D)}\ 3.4\qquad\textbf{(E)}\ 4.4$

## Solution 1

Let's call the distance that Samia had to travel in total as $2x$, so that we can avoid fractions. We know that the length of the bike ride and how far she walked are equal, so they are both $\frac{2x}{2}$, or $x$.

She bikes at a rate of $17$ kph, so she travels the distance she bikes in $\frac{x}{17}$ hours. She walks at a rate of $5$ kph, so she travels the distance she walks in $\frac{x}{5}$ hours.

The total time is $\frac{x}{17}+\frac{x}{5}=\frac{22x}{85}$. This is equal to $\frac{44}{60}=\frac{11}{15}$ of an hour. Solving for $x$, we have:

$$\frac{22x}{85}=\frac{11}{15}$$

$$\frac{2x}{85}=\frac{1}{15}$$

$$30x=85$$

$$6x=17$$

$$x=\frac{17}{6}$$

Since $x$ is the distance of how far Samia traveled by both walking and biking, and we want to know how far Samia walked to the nearest tenth, we have that Samia walked about $\boxed{\textbf{(C)} 2.8}$.


| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 8

## Contents



## Problem

Points $A(11, 9)$ and $B(2, -3)$ are vertices of $\triangle ABC$ with $AB = AC$. The altitude from $A$ meets the opposite side at $D(-1, 3)$. What are the coordinates of point $C$?

$\textbf{(A)}\ (-8, 9) \qquad \textbf{(B)}\ (-4, 8) \qquad \textbf{(C)}\ (-4, 9) \qquad \textbf{(D)}\ (-2, 3) \qquad \textbf{(E)}\ (-1, 0)$

## Solution 1

Since $AB = AC$, then $\triangle ABC$ is isosceles, so $BD = CD$. Therefore, the coordinates of $C$ are $(-1-3, 3+6) = \boxed{\textbf{(C)}\ (-4, 9)}$.

## Solution 2

Calculating the equation of the line running between points $B$ and $D$, $y = -2x + 1$. The only coordinate of $C$ that is also on this line is $\boxed{\textbf{(C)}\ (-4, 9)}$.

## See Also


| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 9

## Contents



## Problem

A radio program has a quiz consisting of $3$ multiple-choice questions, each with $3$ choices. A contestant wins if he or she gets $2$ or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

$$\textbf{(A)}\ \frac{1}{27}\qquad\textbf{(B)}\ \frac{1}{9}\qquad\textbf{(C)}\ \frac{2}{9}\qquad\textbf{(D)}\ \frac{7}{27}\qquad\textbf{(E)}\ \frac{1}{2}$$

## Solution 1

There are two ways the contestant can win.

Case 1: The contestant guesses all three right. This can only happen $\frac{1}{3} * \frac{1}{3} * \frac{1}{3} = \frac{1}{27}$ of the time.

Case 2: The contestant guesses only two right. We pick one of the questions to get wrong, $3$, and this can happen $\frac{1}{3} * \frac{1}{3} * \frac{2}{3}$ of the time. Thus, $\frac{2}{27} * 3$ = $\frac{6}{27}$.

So, in total the two cases combined equals $\frac{1}{27} + \frac{6}{27}$ = $\boxed{\textbf{(D)}\ \frac{7}{27}}$.

## Solution 2 (complementary counting)

Complementary counting is good for solving the problem and checking work if you solved it using the method above.

There are two ways the contestant can lose.

Case 1: The contestant guesses zero questions correctly.

The probability of guessing incorrectly for each question is $\frac{2}{3}$. Thus, the probability of guessing all questions incorrectly is $\frac{2}{3} * \frac{2}{3} * \frac{2}{3} = \frac{8}{27}$.

Case 2: The contestant guesses one question correctly. There are 3 ways the contestant can guess one question correctly since there are 3 questions. The probability of guessing correctly is $\frac{1}{3}$ so the probability of guessing one correctly and two incorrectly is 1 2 2 4 .

The sum of the two cases is $\frac{8}{27} + \frac{4}{9} = \frac{20}{27}$. This is the complement of what we want to the answer is $1 - \frac{20}{27} = \boxed{\textbf{(D)} \frac{7}{27}}$

## See Also

# 2017 AMC 10B Problems/Problem 10

## Problem

The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is $c$?

$\textbf{(A)}\ -13\qquad\textbf{(B)}\ -8\qquad\textbf{(C)}\ 2\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 13$

## Solution

Writing each equation in slope-intercept form, we get $y = \frac{a}{2}x - \frac{1}{2}c$ and $y = -\frac{2}{b}x - \frac{c}{b}$. We observe the slope of each equation is $\frac{a}{2}$ and $-\frac{2}{b}$, respectively. Because the slope of a line perpendicular to a line with slope $m$ is $-\frac{1}{m}$, we see that $\frac{a}{2} = -\frac{1}{-\frac{2}{b}}$ because it is given that the two lines are perpendicular. This equation simplifies to $a = b$.

Because $(1, -5)$ is a solution of both equations, we deduce $a \times 1 - 2 \times -5 = c$ and $2 \times 1 + b \times -5 = -c$. Because we know that $a = b$, the equations reduce to $a + 10 = c$ and $2 - 5a = -c$. Solving this system of equations, we get $c = \boxed{\textbf{(E)}\ 13}$

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 11

## Problem

At Typico High School, $60\%$ of the students like dancing, and the rest dislike it. Of those who like dancing, $80\%$ say that they like it, and the rest say that they dislike it. Of those who dislike dancing, $90\%$ say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ 33\frac{1}{3}\%$

## Solution

$60\% \cdot 20\% = 12\%$ of the people that claim that they dislike dancing actually like it, and $40\% \cdot 90\% = 36\%$. Therefore, the answer is $\frac{12\%}{12\% + 36\%} = \boxed{\textbf{(D)}\ 25\%}$.

## See Also



| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
|---|---|
| Preceded by Problem 10 | Followed by Problem 12 |
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# 2017 AMC 10B Problems/Problem 12

## Problem

Elmer's new car gives $50\%$ percent better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is $20\%$ more expensive per liter than the gasoline his old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

$\textbf{(A)}\ 20\% \qquad \textbf{(B)}\ 26\tfrac{2}{3}\% \qquad \textbf{(C)}\ 27\tfrac{7}{9}\% \qquad \textbf{(D)}\ 33\tfrac{1}{3}\% \qquad \textbf{(E)}\ 66\tfrac{2}{3}\%$

## Solution

Suppose that his old car runs at $x$ km per liter. Then his new car runs at $\frac{3}{2}x$ km per liter, or $x$ km per $\frac{2}{3}$ of a liter. Let the cost of the old car's fuel be $c$, so the trip in the old car takes $xc$ dollars, while the trip in the new car takes $\frac{2}{3} \cdot \frac{6}{5}xc = \frac{4}{5}xc$. He saves $\frac{\frac{1}{5}xc}{xc} = \boxed{\textbf{(A)}\ 20\%}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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| Preceded by Problem 11 | Followed by Problem 13 |
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# 2017 AMC 10B Problems/Problem 13

## Problem

There are $20$ students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are $10$ students taking yoga, $13$ taking bridge, and $9$ taking painting. There are $9$ students taking at least two classes. How many students are taking all three classes?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

## Solution

By PIE, the answer is $10+13+9-9-20=\boxed{\textbf{(C)}\ 3}$.

## See Also

| | |
|---|---|
| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
| Preceded by Problem 12 | Followed by Problem 14 |
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# 2017 AMC 10B Problems/Problem 14

## Contents



## Problem

An integer $N$ is selected at random in the range $1 \leq N \leq 2020$ . What is the probability that the remainder when $N^{16}$ is divided by $5$ is $1$?

## Solution 1

By Fermat's Little Theorem, $N^{16} = (N^4)^4 \equiv 1 \pmod 5$ when N is relatively prime to 5. However, this happens with probability $\boxed{\textbf{(D)}\ \frac{4}{5}}$.

## Solution 2

Note that the patterns for the units digits repeat, so in a sense we only need to find the patterns for the digits $0-9$ . The pattern for $0$ is $0$, no matter what power, so $0$ doesn't work. Likewise, the pattern for $5$ is always $5$ . Doing the same for the rest of the digits, we find that the units digits of $1^{16}$, $2^{16}$ ,$3^{16}$, $4^{16}$ ,$6^{16}$, $7^{16}$ ,$8^{16}$ and $9^{16}$ all have the remainder of $1$ when divided by $5$, so $\boxed{\textbf{(D)}\ \frac{4}{5}}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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# 2017 AMC 10B Problems/Problem 15

## Problem

Rectangle $ABCD$ has $AB=3$ and $BC=4$. Point $E$ is the foot of the perpendicular from $B$ to diagonal $\overline{AC}$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{42}{25} \qquad \textbf{(C)}\ \frac{28}{15} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{54}{25}$

## Solution

First, note that $AC=5$ because $ABC$ is a right triangle. In addition, we have $AB\cdot BC=2[ABC]=AC\cdot BE$, so $BE=\frac{12}{5}$. Using similar triangles within $ABC$, we get that $AE=\frac{9}{5}$ and $CE=\frac{16}{5}$.

Let $F$ be the foot of the perpendicular from $E$ to $AB$. Since $EF$ and $BC$ are parallel, $\Delta AFE$ is similar to $\Delta ABC$. Therefore, we have $\frac{AF}{AB}=\frac{AE}{AB}=\frac{9}{25}$. Since $AB=3$, $AF=\frac{27}{25}$. Note that $AF$ is an altitude of $\Delta AED$ from $AD$, which has length $4$. Therefore, the area of $\Delta AED$ is $\frac{1}{2}\cdot\frac{27}{25}\cdot 4=\boxed{\textbf{(E)}\frac{54}{25}}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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| Preceded by Problem 14 | Followed by Problem 16 |
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# 2017 AMC 10B Problems/Problem 16

## Problem

How many of the base-ten numerals for the positive integers less than or equal to $2017$ contain the digit $0$?

$\textbf{(A)}\ 469\qquad\textbf{(B)}\ 471\qquad\textbf{(C)}\ 475\qquad\textbf{(D)}\ 478\qquad\textbf{(E)}\ 481$

## Solution

We can use complementary counting. There are $2017$ positive integers in total to consider, and there are $9$ one-digit integers, $9 \cdot 9 = 81$ two digit integers without a zero, $9 \cdot 9 \cdot 9$ three digit integers without a zero, and $9 \cdot 9 \cdot 9 = 1458$ four-digit integers without a zero. Therefore, the answer is $2017 - 9 - 81 - 729 - 729 = \boxed{\textbf{(A)}\ 469}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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| Preceded by Problem 15 | Followed by Problem 17 |
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# 2017 AMC 10B Problems/Problem 17

The following problem is from both the 2017 AMC 12B #11 and 2017 AMC 10B #17, so both problems redirect to this page.

## Problem

Call a positive integer $monotonous$ if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, $3$, $23578$, and $987620$ are monotonous, but $88$, $7434$, and $23557$ are not. How many monotonous positive integers are there?

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

## Solution

The number of one-digit numbers that work is $\binom{10}{1}$, and the number of two-digit integers that work is $\binom{10}{2} + \binom{9}{2}$. We use similar logic for three-digit integers, four digit integers, etc. Summing, we have $2^{10} + 2^9 - 9 - 1 - 1$, and we need to subtract another 1 for the 0 case, so the answer is $2^{10} + 2^9 - 9 - 1 - 1 - 1 = \boxed{\textbf{(B)}\ 1524}$.

## See Also

| 2017 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017)) | |
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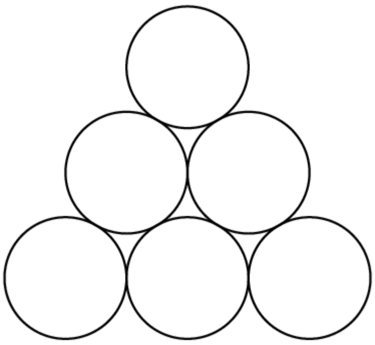
# 2017 AMC 10B Problems/Problem 18

## Contents



## Problem

In the figure below, $3$ of the $6$ disks are to be painted blue, $2$ are to be painted red, and $1$ is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



$\textbf{(A) } 6 \qquad \textbf{(B) } 8 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 15$

## Solution

### Solution 1

First we figure out the number of ways to put the $3$ blue disks. Denote the spots to put the disks as $1-6$ from left to right, top to bottom. The cases to put the blue disks are $(1,2,3),(1,2,4),(1,2,5),(1,2,6),(2,3,5),(1,4,6)$. For each of those cases we can easily figure out the number of ways for each case, so the total amount is $2+2+3+3+1+1=\boxed{\textbf{(D) } 12}$.

### Solution 2

Denote the $6$ discs as in the first solution. Ignoring reflections or rotations, there are $\binom{6}{3}*\binom{3}{2}=60$ colorings.

Now we need to count the number of fixed points under possible transformations:

1. The identity transformation. Since this doesn't change anything, there are $60$ fixed points

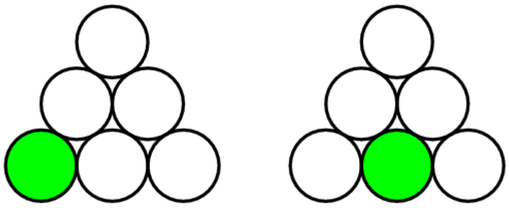
2. Reflect about a line of symmetry. There are $3$ lines of reflections. Take the line of reflection going through the centers of circles $1$ and $5$. Then, the colors of circles $2$ and $3$ must be the same, and the colors of circles $4$ and $6$ must be the same. This gives us $4$ fixed points per line of reflection

3. Rotate by $120^{\circ}$ counter clockwise or clockwise with respect to the center of the diagram. Take the clockwise case for example. There will be a fixed point in this case if the colors of circles $1$, $4$, and $6$ will be the same. Similarly, the colors of circles $2$, $3$, and $5$ will be the same. This is impossible, so this case gives us $0$ fixed points per rotation.

By Burnside's lemma, the total number of colorings is $(1*60+3*4+2*0)/(1+3+2)=\boxed{\textbf{(D) } 12}$.

## Solution 3

Note that the green disk has two possibilities; in a corner or on the side. WLOG, we can arrange these as



Take the first case. Now, we must pick two of the five remaining circles to fill in the red. There are $\binom{5}{2} = 10$ of these. However, due to reflection we must divide this by two. But, in two of these cases, the reflection is itself, so we must subtract these out before dividing by 2, and add them back afterwards, giving $\frac{10-2}{2} + 2 = 6$ arrangement in this case.

Now, look at the second case. We again must pick two of the five remaining circles, and like in the first case, two of the reflections give the same arrangement. Thus, there are also $6$ arrangements in this case.

In total, we have $6 + 6 = \boxed{\textbf{(D) } 12}$.

Solution by tdeng

## Solution 4: Burnside's Lemma

We note that the group $G$ acting on the possible colorings is $D_3 = \{e, r, r^2, s, sr, sr^2\}$, where $r$ is a $120^\circ$ rotation and $s$ is a reflection. In particular, the possible actions are the identity, the $120^\circ$ and $240^\circ$ rotations, and the three reflections.

We will calculate the number of colorings that are fixed under each action. Every coloring is fixed under the identity, so we count $\frac{6!}{3!2!1!} = 60$ fixed colorings. Note that no colorings are fixed under the rotations, since then the outer three and inner three circle must be the same color, which is impossible in our situation.

Finally, consider the reflection with a line of symmetry going through the top circle. Every fixed coloring is determined by the color of the top circle (either green or blue), and the color of the middle circles (either blue or red). Hence, there are $2 \cdot 2 = 4$ colorings fixed under this reflection action. The other two actions are symmetric, so they also have $4$ fixed colorings. Hence, by Burnside's lemma, the number of unique colorings up to reflections and rotations is

$$\frac{1}{|D_3|}(1 \cdot 60 + 2 \cdot 0 + 3 \cdot 4) = \frac{1}{6} \cdot 72 = \boxed{\textbf{(D) } 12}.$$

# See Also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# 2017 AMC 10B Problems/Problem 19

## Contents



## Problem

Let $ABC$ be an equilateral triangle. Extend side $\overline{AB}$ beyond $B$ to a point $B'$ so that $BB'=3AB$. Similarly, extend side $\overline{BC}$ beyond $C$ to a point $C'$ so that $CC'=3BC$, and extend side $\overline{CA}$ beyond $A$ to a point $A'$ so that $AA'=3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

$\textbf{(A)}\ 9:1\qquad\textbf{(B)}\ 16:1\qquad\textbf{(C)}\ 25:1\qquad\textbf{(D)}\ 36:1\qquad\textbf{(E)}\ 37:1$

## Solution

### Solution 1

Note that by symmetry, $\triangle A'B'C'$ is also equilateral. Therefore, we only need to find one of the sides of $A'B'C'$ to determine the area ratio. WLOG, let $AB=BC=CA=1$. Therefore, $BB'=3$ and $BC'=4$. Also, $\angle B'BC'=120^\circ$, so by the Law of Cosines, $B'C'=\sqrt{37}$. Therefore, the answer is $(\sqrt{37})^2:1^2=\boxed{\textbf{(E)}\ 37:1}$

### Solution 2

As mentioned in the first solution, $\triangle A'B'C'$ is equilateral. WLOG, let $AB=2$. Let $D$ be on the line passing through $AB$ such that $A'D$ is perpendicular to $AB$. Note that $\triangle A'DA$ is a 30-60-90 with right angle at $D$. Since $AA'=6$, $AD=3$ and $A'D=3\sqrt{3}$. So we know that $DB'=11$. Note that $\triangle A'DB'$ is a right triangle with right angle at $D$. So by the Pythagorean theorem, we find $A'B'=\sqrt{(3\sqrt{3})^2+11^2}=2\sqrt{37}$. Therefore, the answer is $(2\sqrt{37})^2:2^2=\boxed{\textbf{(E)}\ 37:1}$.

## See Also


| | |
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# 2017 AMC 10B Problems/Problem 20

## Problem

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

$\textbf{(A)}\ \frac{1}{21} \qquad \textbf{(B)}\ \frac{1}{19} \qquad \textbf{(C)}\ \frac{1}{18} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{11}{21}$

## Solution

We note that the only thing that affects the parity of the factor are the powers of 2. There are $10+5+2+1=18$ factors of 2 in the number. Thus, there are $18$ cases in which a factor of $21!$ would be even (have a factor of $2$ in its prime factorization), and $1$ case in which a factor of $21!$ would be odd. Therefore, the answer is $\boxed{\textbf{(B)} \frac{1}{19}}$

## See Also


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# 2017 AMC 10B Problems/Problem 21

## Problem

In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and $D$ is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscibed in $\triangle ADB$ and $\triangle ADC$?

$\textbf{(A)}\ \sqrt{5}\qquad\textbf{(B)}\ \frac{11}{4}\qquad\textbf{(C)}\ 2\sqrt{2}\qquad\textbf{(D)}\ \frac{17}{6}\qquad\textbf{(E)}\ 3$

## Solution

We note that by the converse of the Pythagorean Theorem, $\triangle ABC$ is a right triangle with a right angle at $A$. Therefore, $AD = BD = CD = 5$, and $[ADB] = [ADC] = 12$. Since $A = rs$, the inradius of $\triangle ADB$ is $\frac{12}{(5+5+6)/2} = \frac{3}{2}$, and the inradius of $\triangle ADC$ is $\frac{12}{(5+5+8)/2} = \frac{4}{3}$. Adding the two together, we have $\boxed{\textbf{(D)}\ \frac{17}{6}}$.

## See Also


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# 2017 AMC 10B Problems/Problem 22

## Contents



## Problem

The diameter $\overline{AB}$ of a circle of radius $2$ is extended to a point $D$ outside the circle so that $BD = 3$. Point $E$ is chosen so that $ED = 5$ and line $ED$ is perpendicular to line $AD$. Segment $\overline{AE}$ intersects the circle at a point $C$ between $A$ and $E$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ \frac{120}{37}\qquad\textbf{(B)}\ \frac{140}{39}\qquad\textbf{(C)}\ \frac{145}{39}\qquad\textbf{(D)}\ \frac{140}{37}\qquad\textbf{(E)}\ \frac{120}{31}$

## Solution

### Solution 1

Notice that $ADE$ and $ABC$ are right triangles. Then $AE = \sqrt{7^2+5^2} = \sqrt{74}$. $\sin DAE = \frac{5}{\sqrt{74}} = \sin BAE = \sin BAC = \frac{BC}{4}$, so $BC = \frac{20}{\sqrt{74}}$. We also find that $AC = \frac{28}{\sqrt{74}}$, and thus the area of $ABC$ is $\frac{\frac{20}{\sqrt{74}}\cdot\frac{28}{\sqrt{74}}}{2} = \frac{\frac{560}{74}}{2} = \boxed{\textbf{(D)}\ \frac{140}{37}}$.

## Solution 2

We note that $\triangle ACB$ $\triangle ADE$ by $AA$ similarity. Also, since the area of $\triangle ADE = \frac{7\cdot 5}{2} = \frac{35}{2}$ and $AE = \sqrt{74}$, $\frac{[ABC]}{[ADE]} = \frac{[ABC]}{\frac{35}{2}} = \left(\frac{4}{\sqrt{74}}\right)^2$, so the area of $\triangle ABC = \boxed{\textbf{(D)}\ \frac{140}{37}}$.

## See Also

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# 2017 AMC 10B Problems/Problem 23

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## Problem 23

Let $N = 123456789101112\dots4344$ be the $79$-digit number that is formed by writing the integers from $1$ to $44$ in order, one after the other. What is the remainder when $N$ is divided by $45$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 9\qquad\textbf{(D)}\ 18\qquad\textbf{(E)}\ 44$

## Solution

We only need to find the remainders of N when divided by 5 and 9 to determine the answer. By inspection, $N \equiv 4 \pmod{5}$. The remainder when $N$ is divided by $9$ is $1+2+3+4+ \cdots +1+0+1+1 +1+2 +\cdots + 4+3+4+4$, but since $10 \equiv 1 \pmod{9}$, we can also write this as $1+2+3 +\cdots +10+11+12+ \cdots 43 + 44 = \frac{44 \cdot 45}2 = 22 \cdot 45$, which has a remainder of 0 mod 9. Therefore, by CRT, the answer is $\boxed{\textbf{(C) } 9}$.

Note: the sum of the digits of $N$ is $270$

## Solution 2

Noting the solution above, we try to find the sum of the digits to figure out its remainder when divided by $9$. From $1$ thru $9$, the sum is $45$. $10$ thru $19$, the sum is $55$, $20$ thru $29$ is $65$, and $30$ thru $39$ is $75$. Thus the sum of the digits is $45+55+65+75+4+5+6+7+8 = 240+30 = 270$, and thus $N$ is divisible by $9$. The solution proceeds as above.

## See Also

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# 2017 AMC 10B Problems/Problem 24

## Contents



## Problem 24

The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 108 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 169$

## Solution

WLOG, let the centroid of $\triangle ABC$ be $I = (-1, -1)$. The centroid of an equilateral triangle is the same as the circumcircle. It follows that the circumcircle must hit the graph exactly three times. Therefore, $A = (1, 1)$, so $AI = BI = CI = 2\sqrt{2}$, so since $\triangle AIB$ is isosceles and $\angle AIB = 120^\circ$, then by Law of Cosines, $AB = 2\sqrt{6}$. Therefore, the area of the triangle is $\dfrac{(2\sqrt{6})^2\sqrt{3}}{4} = 6\sqrt{3}$, so the square of the area of the triangle is $\boxed{\textbf{(C)}\ 108}$.

## Solution 2

WLOG, let the centroid of $\triangle ABC$ be $G = (-1, -1)$. Then, one of the vertices must be the other vertex of the hyperbola. WLOG, let $A = (1, 1)$. Then, point $B$ must be the reflection of $C$ across the line $y = x$, so let $B = (a, 1/a)$ and $C = (1/a, a)$, where $a < -1$. Because $G$ is the centroid, the average of the $x$-coordinates of the vertices of the triangle is $-1$. So we know that $a + 1/a + 1 = -3$. Multiplying by $a$ and solving gives us $a = -2 - \sqrt{3}$. So $B = (-2 - \sqrt{3}, -2 + \sqrt{3})$ and $C = (-2 + \sqrt{3}, -2 - \sqrt{3})$. So $BC = 2\sqrt{6}$, and finding the square of the area gives us $\boxed{\textbf{(C)}\ 108}$.

## See Also

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# 2017 AMC 10B Problems/Problem 25

## Contents



## Problem

Last year Isabella took $7$ math tests and received $7$ different scores, each an integer between $91$ and $100$, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was $95$. What was her score on the sixth test?

$\textbf{(A)}\ 92 \qquad\textbf{(B)}\ 94 \qquad\textbf{(C)}\ 96 \qquad\textbf{(D)}\ 98 \qquad\textbf{(E)}\ 100$

## Solution 1

Let the sum of the scores of Isabella's first $6$ tests be $S$. Since the mean of her first $7$ scores is an integer, then $S + 95 \equiv 0 \pmod{7}$, or $S \equiv 3 \pmod{7}$. Also, $S \equiv 0 \pmod{6}$, so by CRT, $S \equiv 24 \pmod{42}$. We also know that $91 \cdot 6 \leq S \leq 100 \cdot 6$, so by inspection, $S = 570$. However, we also have that the mean of the first $5$ integers must be an integer, so the sum of the first $5$ test scores must be an multiple of $5$, which implies that the $6$th test score is $\boxed{\textbf{(E)}\ 100}$.

## Solution 2 (Cheap Solution)

By inspection, the sequences $91, 93, 92, 96, 98, 100, 95$ and $93, 91, 92, 96, 98, 100, 95$ work, so the answer is $\boxed{\textbf{(E)}\ 100}$.

## See Also

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