2021Fall AMC12A

Problem 1

What is the value of $\frac{(2112-2021)^2}{169}$?

表达式 $\frac{(2112-2021)^2}{169}$ 的值是多少?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 49 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 91$

Problem 2

Menkara has a 4×6 index card. If she shortens the length of one side of this card by 1 inch, the card would have area 18 square inches. What would the area of the card be in square inches if instead she shortens the length of the other side by 1 inch?

Menkara 有一张 4 英寸 ×6 英寸的索引卡片。如果她将这张卡片一边的长度缩短 1 英寸，则该卡片的面积变为 18 平方英寸。如果她将另一边的长度缩短 1 英寸，那么卡片的面积会是多少平方英寸?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Problem 3

Mr. Lopez has a choice of two routes to get to work. Route A is 6 miles long, and his average speed along this route is 30 miles per hour. Route B is 5 miles long, and his average speed along this route is 40 miles per hour, except for a $\frac{1}{2}$-mile stretch in a school zone where his average speed is 20 miles per hour. By how many minutes is Route B quicker than Route A?

Mr. Lopez 有两条上班路线可供选择。路线 A 长 6 英里，他沿这条路线行进的平均速度为每小时 30 英里。路线 B 长 5 英里，他沿着这条路线行进的平均速度是每小时 40 英里，除了在途经一段长为 $\frac{1}{2}$ 英里的学校附近路段时，他的平均速度是每小时 20 英里。问路线 B 比路线 A 快多少分钟?

$\textbf{(A)}\ 2\frac{3}{4} \qquad \textbf{(B)}\ 3\frac{3}{4} \qquad \textbf{(C)}\ 4\frac{1}{2} \qquad \textbf{(D)}\ 5\frac{1}{2} \qquad \textbf{(E)}\ 6\frac{3}{4}$

Problem 4

The six-digit number $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{A}$ is prime for only one digit A. What is A?

使得六位数 $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{\mathrm{A}}$ 是素数的数字 A 只有一个。问 A 是几?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

Problem 5

Elmer the emu takes 44 equal strides to walk between consecutive telephone poles on a rural road. Oscar the ostrich can cover the same distance in 12 equal leaps. The telephone poles are evenly spaced, and the 41st pole along this road is exactly one mile (5280 feet) from the first pole. How much longer, in feet, is Oscar's leap than Elmer's stride?

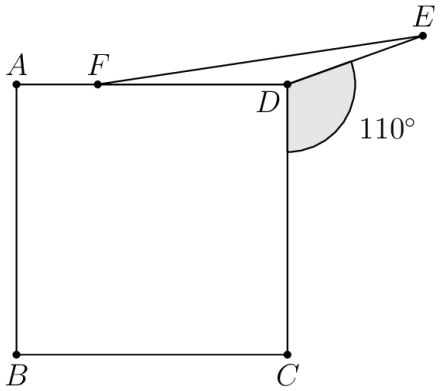
鸸鹋 Elmer 在乡村道路上的相邻电线杆之间行走需要步幅相同的 44 步。同样的距离，鸵鸟 Oscar 只需要 12 次等距的跳跃。电线杆均匀分布，从第 1 根电线杆到第 41 根电线杆的距离正好是一英里（5280 英尺）。问 Oscar 的一跳比 Elmer 的一步长多少英尺?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 15$

Problem 6

As shown in the figure below, point E lies on the opposite half-plane determined by line CD from point A so that $\angle CDE = 110^\circ$. Point F lies on $\overline{AD}$ so that $DE = DF$, and $ABCD$ is a square. What is the degree measure of $\angle AFE$?

如下图所示，点 E 位于由直线 CD 确定的与点 A 相对的半平面上，使得 $\angle CDE = 110^\circ$。点 F 位于 $\overline{AD}$ 上，使得 $DE = DF$，并且 $ABCD$ 是一个正方形。问 $\angle AFE$ 的度数是多少？



(A) 160 **(B)** 164 **(C)** 166 **(D)** 170 **(E)** 174

Problem 7

A school has 100 students and 5 teachers. In the first period, each student is taking one class, and each teacher is teaching one class. The enrollments in the classes are $50, 20, 20, 5,$ and 5. Let t be the average value obtained if a teacher is picked at random and the number of students in their class is noted. Let s be the average value obtained if a student was picked at random and the number of students in their class, including the student, is noted. What is $t - s$?

一所学校有 100 名学生和 5 名教师。在第一期，每个学生选一门课，每个老师教一门课。各课程的注册人数为 50、20、20、5 和 5。如果按随机挑选一名教师，并记录他的班级学生人数的方式进行统计，所得的平均值为 t。如果按随机挑选一名学生，并记录该学生所在的班级学生人数的方式进行统计，所获得的平均值为 s。问 $t - s$ 是多少？

(A) -18.5 **(B)** -13.5 **(C)** 0 **(D)** 13.5 **(E)** 18.5

Problem 8

Let M be the least common multiple of all the integers 10 through 30, inclusive. Let N be the least common multiple of $M, 32, 33, 34, 35, 36, 37, 38, 39,$ and 40. What is the value of $\frac{N}{M}$?

设 M 是从 10 到 30（包括 10 和 30）的所有整数的最小公倍数。设 N 是 M、32、33、34、35、36、37、38、39 和 40 的最小公倍数。问 $\frac{N}{M}$ 的值是多少？

(A) 1 **(B)** 2 **(C)** 37 **(D)** 74 **(E)** 2886

Problem 9

A right rectangular prism whose surface area and volume are numerically equal has edge lengths $\log_2 x, \log_3 x,$ and $\log_4 x$. What is x?

一个边长分别为 $\log_2 x$、$\log_3 x$ 和 $\log_4 x$ 的长方体的表面积和体积在数值上相等。问 x 是多少？

(A) $2\sqrt{6}$ **(B)** $6\sqrt{6}$ **(C)** 24 **(D)** 48 **(E)** 576

Problem 10

The base-nine representation of the number N is $27{,}006{,}000{,}052_{\text{nine}}$. What is the remainder when N is divided by 5?

数 N 的九进位制表示是 $27{,}006{,}000{,}052_9$。问 N 除以 5 的余数是多少？

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 11

Consider two concentric circles of radius 17 and 19. The larger circle has a chord, half of which lies inside the smaller circle. What is the length of the chord in the larger circle?

考虑半径为 17 和半径为 19 的两个同心圆。较大的圆有一条弦，其中的一半位于较小的圆内。问较大的圆的这条弦的长度是多少？

(A) $12\sqrt{2}$ **(B)** $10\sqrt{3}$ **(C)** $\sqrt{17 \cdot 19}$ **(D)** 18 **(E)** $8\sqrt{6}$

Problem 12

What is the number of terms with rational coefficients among the 1001 terms in the expansion of $\left(x\sqrt[3]{2}+y\sqrt{3}\right)^{1000}$?

在展开式 $\left(x\sqrt[3]{2}+y\sqrt{3}\right)^{1000}$ 的 1001 项中，有多少项的系数为有理数？

(A) 0 **(B)** 166 **(C)** 167 **(D)** 500 **(E)** 501

Problem 13

The angle bisector of the acute angle formed at the origin by the graphs of the lines $y=x$ and $y=3x$ has equation $y=kx$. What is k?

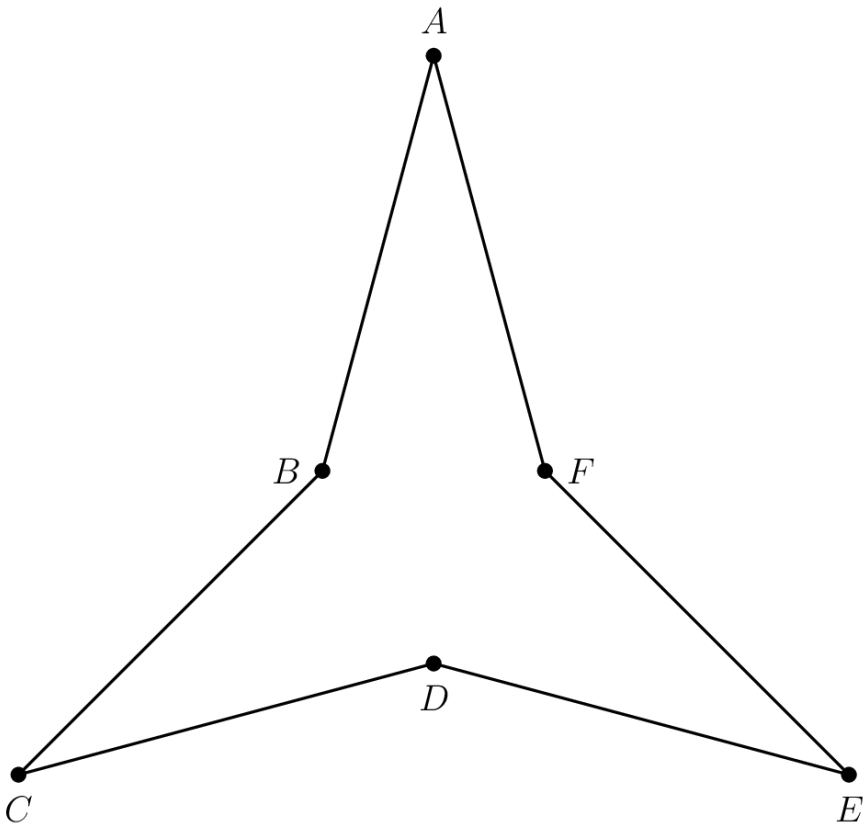
由 $y=x$ 和 $y=3x$ 图像的两条直线在原点形成的锐角的角平分线的方程是 $y=kx$。问 k 是多少？

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** $\frac{1+\sqrt{7}}{2}$ **(C)** $\frac{2+\sqrt{3}}{2}$ **(D)** 2 **(E)** $\frac{2+\sqrt{5}}{2}$

Problem 14

In the figure, equilateral hexagon $ABCDEF$ has three nonadjacent acute interior angles that each measure $30°$. The enclosed area of the hexagon is $6\sqrt{3}$. What is the perimeter of the hexagon?

在图中，等边六边形 $ABCDEF$ 有三个不相邻的均为 $30°$ 的锐角内角。由六边形所围成的区域的面积是 $6\sqrt{3}$。问六边形的周长是多少？



(A) 4 **(B)** $4\sqrt{3}$ **(C)** 12 **(D)** 18 **(E)** $12\sqrt{3}$

Problem 15

Recall that the conjugate of the complex number $w = a + bi$, where a and b are real numbers and $i = \sqrt{-1}$, is the complex number $\overline{w} = a - bi$. For any complex number z, let $f(z) = 4i\overline{z}$. The polynomial $P(z) = z^4 + 4z^3 + 3z^2 + 2z + 1$ has four complex roots: z_1, z_2, z_3, and z_4. Let $Q(z) = z^4 + Az^3 + Bz^2 + Cz + D$ be the polynomial whose roots are $f(z_1)$, $f(z_2)$, $f(z_3)$, and $f(z_4)$, where the coefficients A, B, C, and D are complex numbers. What is $B + D$?

我们知道，复数 $w = a + bi$ 的共轭是指复数 $\overline{w} = a - bi$，其中 a 和 b 是实数，而 $i = \sqrt{-1}$。对于任何复数 z，设 $f(z) = 4i\overline{z}$。多项式 $P(z) = z^4 + 4z^3 + 3z^2 + 2z + 1$ 有四个复数根：z_1、z_2、z_3 和 z_4。设 $Q(z) = z^4 + Az^3 + Bz^2 + Cz + D$ 是根为 $f(z_1)$、$f(z_2)$、$f(z_3)$ 和 $f(z_4)$ 的多项式，其中系数 A、B、C 和 D 是复数。问 $B + D$ 是多少？

$(\mathbf{A})\ -304 \qquad (\mathbf{B})\ -208 \qquad (\mathbf{C})\ 12i \qquad (\mathbf{D})\ 208 \qquad (\mathbf{E})\ 304$

Problem 16

An organization has 30 employees, 20 of whom have a brand A computer while the other 10 have a brand B computer. For security, the computers can only be connected to each other and only by cables. The cables can only connect a brand A computer to a brand B computer. Employees can communicate with each other if their computers are directly connected by a cable or by relaying messages through a series of connected computers. Initially, no computer is connected to any other. A technician arbitrarily selects one computer of each brand and installs a cable between them, provided there is not already a cable between that pair. The technician stops once every employee can communicate with each other. What is the maximum possible number of cables used?

一间公司有 30 位员工，其中 20 人各有一台 A 品牌的电脑，而另外 10 人各有一台 B 品牌的电脑。为安全起见，电脑只能与电脑相互连接，并且只能通过电缆连接。电缆只能将品牌 A 的电脑与品牌 B 的电脑相连。如果员工的电脑通过电缆直接相连或者可通过一系列相连的电脑传递信息，那么他们之间就可以互相通信。最初，电脑之间没有任何连接。技术人员任意从每个品牌的电脑中各选择一台电脑，如果这对电脑之间还没有电缆相连，那么就在它们之间安装电缆。当每位员工都可以与其他任何人互相通信时，技术人员就会停止工作。问最多可能会使用多少条电缆?

(A) 190 **(B)** 191 **(C)** 192 **(D)** 195 **(E)** 196

Problem 17

For how many ordered pairs (b, c) of positive integers does

neither $x^2 + bx + c = 0$ nor $x^2 + cx + b = 0$ have two distinct real solutions?

使得 $x^2 + bx + c = 0$ 与 $x^2 + cx + b = 0$ 都没有两个不同的实数解的有序正整数对 (b, c) 有多少个?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 16

Problem 18

Each of 20 balls is tossed independently and at random into one of 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is $\frac{p}{q}$?

20 个球中的每一个被独立并随机的扔进 5 个桶中的一个。设 p 是最终一个桶中有 3 个球，另一个桶中有 5 个球，并且其余的三个桶中各有 4 个球的概率。q 是最终每个桶中有 4 个球的概率。问 $\frac{p}{q}$ 是多少？

(A) 1 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

Problem 19

Let x be the least real number greater than 1 such that $\sin(x) = \sin(x^2)$, where the arguments are in degrees. What is x rounded up to the closest integer?

设 x 是大于 1 的最小实数，使得 $\sin x = \sin\left(x^2\right)$，其中的角度以度为单位。问向上调整所得到的与 x 最接近的整数是多少？

(A) 10 **(B)** 13 **(C)** 14 **(D)** 19 **(E)** 20

Problem 20

For each positive integer n, let $f_1(n)$ be twice the number of positive integer divisors of n, and for $j \geq 2$, let $f_j(n) = f_1(f_{j-1}(n))$. For how many values of $n \leq 50$ is $f_{50}(n) = 12$?

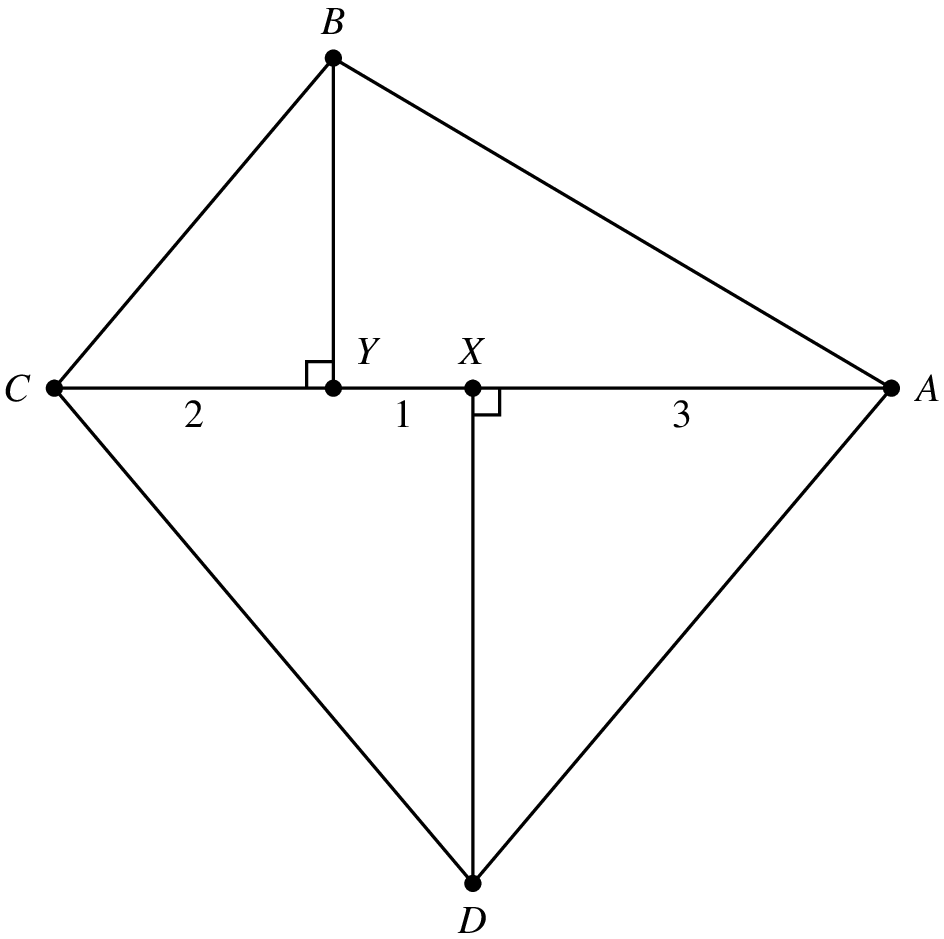
对于每个正整数 n，令 $f_1(n)$ 是 n 的正整数约数个数的两倍，并且对于 $j \geq 2$，令 $f_j(n) = f_1(f_{j-1}(n))$。对于 $n \leq 50$，满足 $f_{50}(n) = 12$ 的 n 值有多少个？

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Problem 21

Let $ABCD$ be an isosceles trapezoid with $\overline{BC} \parallel \overline{AD}$ and $AB = CD$. Points X and Y lie on diagonal $\overline{AC}$ with X between A and Y, as shown in the figure.
Suppose $\angle AXD = \angle BYC = 90^\circ$, $AX = 3$, $XY = 1$, and $YC = 2$. What is the area of $ABCD$?

设 $ABCD$ 是等腰梯形，其中 $\overline{BC} \parallel \overline{AD}$，并且 $AB = CD$。如图所示，点 X 和 Y 位于对角线 $\overline{AC}$ 上，X 位于 A 和 Y 之间。假设 $\angle AXD = \angle BYC = 90^\circ$，$AX = 3$，$XY = 1$，$YC = 2$。问 $ABCD$ 的面积是多少？



(A) 15 **(B)** $5\sqrt{11}$ **(C)** $3\sqrt{35}$ **(D)** 18 **(E)** $7\sqrt{7}$

Problem 22

Azar and Carl play a game of tic-tac-toe. Azar places an in X one of the boxes in a 3-by-3 array of boxes, then Carl places an O in one of the remaining boxes. After that, Azar places an X in one of the remaining boxes, and so on until all boxes are filled or one of the players has of their symbols in a row—horizontal, vertical, or diagonal—whichever comes first, in which case that player wins the game. Suppose the players make their moves at random, rather than trying to follow a rational strategy, and that Carl wins the game when he places his third O. How many ways can the board look after the game is over?

Azar 和 Carl 玩井字棋游戏。Azar 将 X 放在 3×3 棋盘的一个格中，然后 Carl 将 O 放置在一个空格中。之后，Azar 将 X 放置在一个空格中，依此类推，哪位玩家先把自己的 3 个符号连成了一排（可以是水平的，竖直的，或者沿着对角线），那么该玩家就赢得了游戏。游戏最终也可能是以 9 个方格都填满符号而结束。假设玩家都是随机的放置符号，而不是遵循理性的策略，并且当 Carl 放置第三个 O 时，他赢得了游戏。当游戏如此结束时，棋盘的布局有多少种？

(A) 36 **(B)** 112 **(C)** 120 **(D)** 148 **(E)** 160

Problem 23

A quadratic polynomial with real coefficients and leading coefficient 1 is called $disrespectful$ if the equation $p(p(x)) = 0$ is satisfied by exactly three real numbers. Among all the disrespectful quadratic polynomials, there is a unique such polynomial $\tilde{p}(x)$ for which the sum of the roots is maximized. What is $\tilde{p}(1)$?

如果恰好有三个实数满足方程 $p(p(x)) = 0$，那么首项系数为 1 的实系数二次多项式 $p(x)$ 被称为“不受尊重的”。在所有不受尊重的二次多项式中，有唯一的多项式 $\tilde{p}(x)$ 使得其各根之和取最大值。问 $\tilde{p}(1)$ 是多少？

(A) $\frac{5}{16}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{8}$ **(D)** 1 **(E)** $\frac{9}{8}$

Problem 24

Convex quadrilateral $ABCD$ has $AB = 18, \angle A = 60°$, and $\overline{AB} \parallel \overline{CD}$. In some order, the lengths of the four sides form an arithmetic progression, and side $\overline{AB}$ is a side of maximum length. The length of another side is a. What is the sum of all possible values of a?

在凸四边形 $ABCD$ 中，$AB = 18$，$\angle A = 60°$，并且 $\overline{AB} \parallel \overline{CD}$。按照某种顺序，四条边的长度形成一个等差数列，并且 $\overline{AB}$ 是最长的边。另一条边的长度是 a。问 a 的所有可能值的总和是多少？

(A) 24 **(B)** 42 **(C)** 60 **(D)** 66 **(E)** 84

Problem 25

Let $m \geq 5$ be an odd integer, and let $D(m)$ denote the number of quadruples (a_1, a_2, a_3, a_4) of distinct integers with $1 \leq a_i \leq m$ for all i such that m divides $a_1 + a_2 + a_3 + a_4$. There is a polynomial$q(x) = c_3x^3 + c_2x^2 + c_1x + c_0$such that $D(m) = q(m)$ for all odd integers $m \geq 5$. What is c_1?

设 $m \geq 5$ 是奇数，令 $D(m)$ 表示对于所有 i，$1 \leq a_i \leq m$，并且 $a_1 + a_2 + a_3 + a_4$ 能被 m 整除的不同整数组成的四元组 (a_1, a_2, a_3, a_4) 的个数。存在多项式 $q(x) = c_3x^3 + c_2x^2 + c_1x + c_0$，使得 $D(m) = q(m)$ 对于所有奇数 $m \geq 5$ 成立。问 c_1 是多少？

(A) -6 **(B)** -1 **(C)** 4 **(D)** 6 **(E)** 11

2021Fall AMC12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	B	E	B	D	B	D	E	D	E	C	A
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	D	B	B	E	B	D	C	D	A	E	E	