1. Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet?

   A 城与 B 城相距 45 英里．Alicia 住在 A 城，Beth 住在 B 城．Alicia 以每小时 18 英里的速度骑自行车前往 B 城．Beth 同时出发，以每小时 12 英里的速度骑自行车前往 A 城．问当他们相遇时，距离 A 城有多少英里？

   (A) 20

   (B) 24

   (C) 25

   (D) 26

   (E) 27

2. The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

   一个大比萨饼的 $\frac{1}{3}$ 与 $3\frac{1}{2}$ 杯橙片合在一起的重量与一个大比萨饼的 $\frac{3}{4}$ 与 $\frac{1}{2}$ 杯橙片合在一起的重量相同．一杯橙片的重量是 $\frac{1}{4}$ 磅．问一个大比萨饼的重量是多少磅？

   (A) $1\frac{4}{5}$

   (B) 2

   (C) $2\frac{2}{5}$

   (D) 3

   (E) $3\frac{3}{5}$

3. How many positive perfect squares less than 2023 are divisible by 5 ?

   在小于 2023 的正的完全平方数中，有多少个数可以被 5 整除?

   (A) 8

   (B) 9

   (C) 10

   (D) 11

   (E) 12

4. How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

   $8^5 \cdot 5^{10} \cdot 15^5$ 的十进制表示中有多少个数字?

   (A) 14

   (B) 15

   (C) 16

   (D) 17

   (E) 18

5. Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3 ?

   Janet 先后抛掷了标准的 6 面骰子 4 次，并逐次记录她掷出数的总和. 问在某次抛掷后，她掷出数的总和等于 3 的概率是多少?

   (A) $\frac{2}{9}$

   (B) $\frac{49}{216}$

   (C) $\frac{25}{108}$

   (D) $\frac{17}{72}$

   (E) $\frac{13}{54}$

6. Points A and B lie on the graph of $y = \log_2 x$. The midpoint of $\overline{AB}$ is $(6, 2)$. What is the positive difference between the x-coordinates of A and B?

   点 A 和 B 位于 $y = \log_2 x$ 的图像上. $\overline{AB}$ 的中点是 $(6, 2)$. 问 A 和 B 的 x 坐标之差（大减小）是多少?

   (A) $2\sqrt{11}$

(B) $4\sqrt{3}$

(C) 8

(D) $4\sqrt{5}$

(E) 9

7. A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 does each digit appear an even number of times in the 8-digit display for that date?

   电子显示器将当前日期显示为 8 位整数，其中包括表示年份的 4 位数字，后面跟着表示月份的 2 位数字，接下来是表示日子的 2 位数字．例如，今年美国的植树节显示为 20230428．问 2023 年有多少个日期，使得每个数字在该日期的 8 位显示中都出现偶数次?

   (A) 5

   (B) 6

   (C) 7

   (D) 8

   (E) 9

8. Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?

   Maureen 一直关注着她本学期各次测验成绩的平均分. 如果 Maureen 在下一次测验中获得 11 分, 那么她的平均分将增加 1 分. 如果她在接下来的三次测验中每次都获得 11 分, 那么她的平均分将增加 2 分. 问她目前的各次测验成绩的平均分是多少分?

   (A) 4

   (B) 5

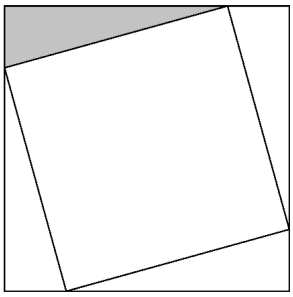
   (C) 6

   (D) 7

   (E) 8

9. A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?

   如下图所示, 面积为 2 的正方形内接于面积为 3 的正方形, 并由此形成了四个全等的三角形. 问阴影直角三角形中较短的直角边与较长的直角边之比是多少?



   (A) $\frac{1}{5}$

   (B) $\frac{1}{4}$

   (C) $2-\sqrt{3}$

   (D) $\sqrt{3}-\sqrt{2}$

   (E) $\sqrt{2}-1$

10. Positive real numbers x and y satisfy $y^3 = x^2$ and $(y - x)^2 = 4y^2$. What is $x + y$?

正实数 x 和 y 满足 $y^3 = x^2$ 和 $(y - x)^2 = 4y^2$. 问 $x + y$ 是多少?

(A) 12

(B) 18

(C) 24

(D) 36

(E) 42

11. What is the degree measure of the acute angle formed by lines with slopes 2 and $\frac{1}{3}$?

斜率为 2 和 $\frac{1}{3}$ 的两条直线形成的锐角是多少度?

(A) 30

(B) 37.5

(C) 45

(D) 52.5

(E) 60

12. What is the value of the below expression?

下面表达式的值是多少?

$$2^3 - 1^3 + 4^3 - 3^3 + 6^3 - 5^3 + \cdots + 18^3 - 17^3$$

(A) 2023

(B) 2679

(C) 2941

(D) 3159

(E) 3235

13. In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

在乒乓球锦标赛中，每位参赛者与其他参赛者恰好比赛一场．尽管右手执拍的参赛者人数是左手执拍的参赛者人数的两倍，但左手执拍的参赛者获胜的场次比右手执拍的参赛者获胜的场次多 40%．（比赛没有平局，也没有参赛者既是右手执拍又是左手执拍．）问一共进行了多少场比赛？

(A) 15

(B) 36

(C) 45

(D) 48

(E) 66

14. How many complex numbers satisfy the equation $z^5 = \overline{z}$, where $\overline{z}$ is the conjugate of the complex number z ?

令 $\overline{z}$ 表示复数 z 的共轭，问满足方程 $z^5 = \overline{z}$ 的复数有多少个？

(A) 2

(B) 3

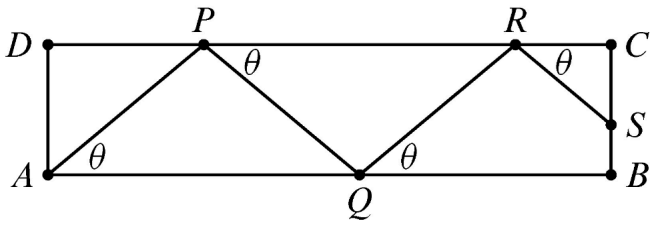
(C) 5

(D) 6

(E) 7

15. Usain is walking for exercise by zigzagging across a 100-meter by 30-meter rectangular field, beginning at point A and ending on the segment $\overline{BC}$. He wants to increase the distance walked by zigzagging as shown in the figure below ($APQRS$). What angle $\theta = \angle PAB = \angle QPC = \angle RQB = \cdots$ will produce a length that is 120 meters? (The figure is not drawn to scale. Do not assume that the zigzag path has exactly four segments as shown; it could be more or fewer.)

Usain 沿折线步行穿过长 100 米、宽 30 米的矩形场地，从 A 点开始，到 $\overline{BC}$ 边结束．下图中的 $APQRS$ 是折线路径的示意图，他想尽量增加步行的距离．当整个路径的长度为 120 米时，角度 $\theta = \angle PAB = \angle QPC = \angle RQB = \cdots$ 是怎样的？（图形未按比例绘制．不要假设折线路径正好有如图所示的四段，它的段数可能更多或更少．）



(A) $\arccos \frac{5}{6}$

(B) $\arccos \frac{4}{5}$

(C) $\arccos \frac{3}{10}$

(D) $\arcsin \frac{4}{5}$

(E) $\arcsin \frac{5}{6}$

16. Consider the set of complex numbers z satisfying $\left|1+z+z^2\right|=4$. The maximum value of the imaginary part of z can be written in the form $\frac{\sqrt{m}}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

考虑满足 $\left|1+z+z^2\right|=4$ 的复数 z 组成的集合. z 的虚部的最大值可以写成 $\frac{\sqrt{m}}{n}$ 的形式，其中 m 和 n 是互质的正整数. 问 $m+n$ 是多少?

(A) 20

(B) 21

(C) 22

(D) 23

(E) 24

17. Flora the frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, Flora leaps a positive integer distance m with probability $\frac{1}{2^m}$. What is the probability that Flora will eventually land at 10 ?

青蛙 Flora 从数轴上的 0 开始，向右进行一系列跳跃. 对于任何一次跳跃，它均与之前的跳跃相互独立，Flora 跳跃正整数距离 m 的概率是 $\frac{1}{2^m}$. 问 Flora 会落到 10 的概率是多少?

(A) $\frac{5}{512}$

(B) $\frac{45}{1024}$

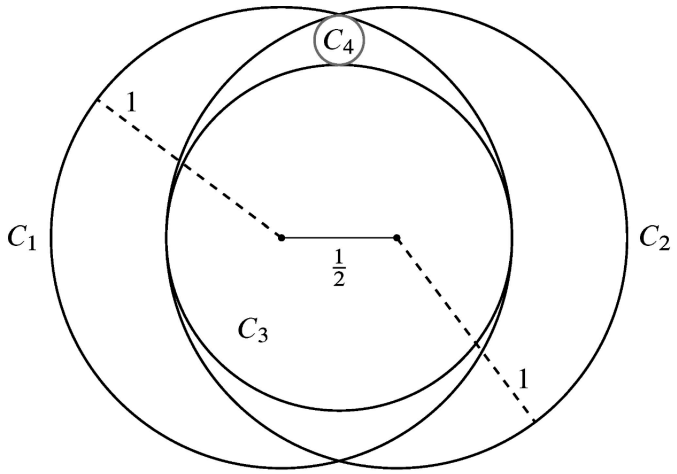
(C) $\frac{127}{1024}$

(D) $\frac{511}{1024}$

(E) $\frac{1}{2}$

18. Circles C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4 ?

圆 C_1 和 C_2 的半径均为 1，圆心之间的距离为 $\frac{1}{2}$. 圆 C_3 是与 C_1 和 C_2 内切的最大圆. 圆 C_4 与 C_1 和 C_2 内切，并且与 C_3 外切. 问 C_4 的半径是多少?



(A) $\frac{1}{14}$

(B) $\frac{1}{12}$

(C) $\frac{1}{10}$

(D) $\frac{3}{28}$

(E) $\frac{1}{9}$

19. What is the product of all solutions to the below equation?

以下的方程所有解的乘积是多少?

$$\log_{7x} 2023 \cdot \log_{289x} 2023 = \log_{2023x} 2023$$

(A) $\left(\log_{2023} 7 \cdot \log_{2023} 289\right)^2$

(B) $\log_{2023} 7 \cdot \log_{2023} 289$

(C) 1

(D) $\log_7 2023 \cdot \log_{289} 2023$

(E) $\left(\log_7 2023 \cdot \log_{289} 2023\right)^2$

20. Rows 1, 2, 3, 4, and 5 of a triangular array of integers are shown below. Each row after the first row is formed by placing a 1 at each end of the row, and each interior entry is 1 greater than the sum of the two numbers diagonally above it in the previous row. What is the units digit of the sum of the 2023 numbers in the 2023rd row?

整数三角形数阵的第 1，2，3，4，5 行如下图所示．第一行之后的每一行都是通过在把 1 放在行的两端，并让中间的每个数等于前一行与它沿斜线相邻的两个数的和再加 1 而形成的．问第 2023 行的 2023 个数的总和的个位数字是多少?

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 3 & & 1 & & & \\
 & & 1 & & 5 & & 5 & & 1 & & \\
 & 1 & & 7 & & 11 & & 7 & & 1 &
\end{array}$$

(A) 1

(B) 3

(C) 5

(D) 7

(E) 9

21. If A and B are vertices of a polyhedron, define the *distance* $d(A,B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A,B)=1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A,B)=2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q,R)>d(R,S)$?

设 A 和 B 是多面体的顶点，“距离”$d(A,B)$ 定义为由多面体的边来形成连接 A 与 B 的通路所需的最少边数. 例如，若 $\overline{AB}$ 是多面体的边，则 $d(A,B)=1$，但如果 $\overline{AC}$ 和 $\overline{CB}$ 是多面体的边，并且 $\overline{AB}$ 不是，那么 $d(A,B)=2$. 令 Q，R，S 为随机选择的正二十面体（由 20 个等边三角形组成的正多面体）的不同顶点. 问 $d(Q,R)>d(R,S)$ 的概率是多少?

(A) $\frac{7}{22}$

(B) $\frac{1}{3}$

(C) $\frac{3}{8}$

(D) $\frac{5}{12}$

(E) $\frac{1}{2}$

22. Let f be the unique function defined on the positive integers such that

$$\sum_{d|n} d\cdot f\left(\frac{n}{d}\right)=1$$

for all positive integers n, where the sum is taken over all positive divisors of n. What is $f(2023)$?

令 f 为定义在正整数上的唯一函数，使得对于所有正整数 n，

$$\sum_{d|n} d\cdot f\left(\frac{n}{d}\right)=1$$

其中求和取遍 n 的所有正约数. 问 $f(2023)$ 是多少?

(A) -1536

(B) 96

(C) 108

(D) 116

(E) 144

23. How many ordered pairs of positive real numbers (a,b) satisfy the below equation?

满足下列方程的有序正实数对 (a,b) 有多少个?

$$(1+2a)(2+2b)(2a+b)=32ab$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) an infinite number | 无穷多个

24. Let K be the number of sequences $A_1, A_2, \ldots, A_n$ such that n is a positive integer less than or equal to 10, each A_i is a subset of $\{1,2,3,\ldots,10\}$, and A_{i-1} is a subset of A_i for each i between 2 and n, inclusive. For example, $\{\}$,$\{5,7\}$,$\{2,5,7\}$,$\{2,5,7\}$,$\{2,5,6,7,9\}$ is one such sequence, with $n=5$. What is the remainder when K is divided by 10 ?

考虑序列 A_1，A_2，$\cdots$，A_n，其中 n 是小于或等于 10 的正整数，每个 A_i 是 $\{1,2,3,\cdots,10\}$ 的子集，并且对于 2 与 n 之间（包括 2 和 n）的每个 i，A_{i-1} 是 A_i 的子集. 例如，$\{\}$，$\{5,7\}$，$\{2,5,7\}$，$\{2,5,7\}$，$\{2,5,6,7,9\}$ 就是 $n=5$ 时一个这样的序列. 设 K 是所有这样的序列的数量，问 K 除以 10 的余数是多少?

(A) 1

(B) 3

(C) 5

(D) 7

(E) 9

25. There is a unique sequence of integers $a_1, a_2, a_3, \cdots, a_{2023}$ such that the below equation holds whenever $\tan 2023x$ is defined. What is a_{2023} ?

存在唯一的整数序列 a_1，a_2，a_3，$\cdots$，a_{2023}，使得以下等式在 $\tan 2023x$ 有定义时都成立．问 a_{2023} 是多少?

$$\tan 2023x = \frac{a_1 \tan x + a_3 \tan^3 x + a_5 \tan^5 x + \cdots + a_{2023} \tan^{2023} x}{1 + a_2 \tan^2 x + a_4 \tan^4 x + \cdots + a_{2022} \tan^{2022} x}$$

(A) -2023

(B) -2022

(C) -1

(D) 1

(E) 2023

Answers.

EAAEB DEDCD CDBEA BEDCC ABBCC