1. Mrs. Jones is pouring orange juice into four identical glasses for her four sons. She fills the first three glasses completely but runs out of juice when the fourth glass is only $\frac{1}{3}$ full. What fraction of a glass must Mrs. Jones pour from each of the first three glasses into the fourth glass so that all four glasses will have the same amount of juice?

   Jones 女士将橙汁倒入四个相同的玻璃杯中给她的四个儿子．她将前三杯装满后，第四杯装到 $\frac{1}{3}$ 满时，果汁就没有了．要使得所有四个玻璃杯中有相同数量的橙汁，Jones 女士必须将前三个玻璃杯的每一个中橙汁的几分之几倒入第四个玻璃杯?

   (A) $\frac{1}{12}$

   (B) $\frac{1}{4}$

   (C) $\frac{1}{6}$

   (D) $\frac{1}{8}$

   (E) $\frac{2}{9}$

2. Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20% on every pair of shoes. Carlos also knew that he had to pay a 7.5% sales tax on the discounted price. He had $43. What is the original (before discount) price of the most expensive shoes he could afford to buy?

   Carlos 去一家体育用品店买跑鞋．跑鞋正在促销，每双鞋降价 20%．Carlos 还知道他必须按照折扣价的 7.5% 缴付消费税．他有 $43．问他能买得起的最贵的跑鞋的原价（即折扣前）是多少?

   (A) $46

   (B) $50

   (C) $48

   (D) $47

   (E) $49

3. A $3-4-5$ right triangle is inscribed in circle A, and a $5-12-13$ right triangle is inscribed in circle B. What is the ratio of the area of circle A to the area of circle B ?

   三边长为 3，4，5 的直角三角形内接于圆 A，三边长为 5，12，13 的直角三角形内接于圆 B．问圆 A 的面积与圆 B 的面积之比是多少?

   (A) $\frac{9}{25}$

   (B) $\frac{1}{9}$

   (C) $\frac{1}{5}$

   (D) $\frac{25}{169}$

   (E) $\frac{4}{25}$

4. Jackson's paintbrush makes a narrow strip with a width of 6.5 millimeters. Jackson has enough paint to make a strip 25 meters long. How many square centimeters of paper could Jackson cover with paint?

   Jackson 的画笔可以画出宽度为 6.5 毫米的窄条．Jackson 有足够的油漆来制作 25 米长的窄条．问 Jackson 可以用颜料覆盖多少平方厘米的纸?

   (A) $162,500$

   (B) 162.5

   (C) $1,625$

   (D) $1,625,000$

   (E) $16,250$

5. You are playing a game. A 2×1 rectangle covers two adjacent squares (oriented either horizontally or vertically) of a 3×3 grid of squares, but you are not told which two squares are covered. Your goal is to find at least one square that is covered by the rectangle. A "turn" consists of you guessing a square, after which you are told whether that square is covered by the hidden rectangle. What is the minimum number of turns you need to ensure that at least one of your guessed squares is covered by the rectangle?

   你在玩下述的游戏．一个 2×1 矩形覆盖了 3×3 方格表中的两个相邻方格（依水平或竖直方向），但你不知道是哪两个方格被覆盖了．你的目标是找到至少一个被矩形覆盖的方格．一个"回合"是指，你猜测是某个方格，然后被告知这个方格是否被矩形覆盖．为确保至少猜中一个被矩形覆盖的方格，你最少需要多少个回合?

   (A) 3

   (B) 5

   (C) 4

   (D) 8

   (E) 6

6. When the roots of the below polynomial are removed from the real number line, what remains is the union of 11 disjoint open intervals. On how many of these intervals is $P(x)$ positive?

   当把下面的多项式的根从实数轴上移除后，剩下的是 11 个不相交的开区间的并集．问在其中的多少个区间上，$P(x)$ 为正值?

   $$P(x) = (x-1)^1(x-2)^2(x-3)^3 \cdots (x-10)^{10}$$

   (A) 3

   (B) 7

   (C) 6

   (D) 4

   (E) 5

7. For how many integers n does the below expression represent a real number, where log denotes the base 10 logarithm?

   使得下面的表达式代表实数的整数 n 有多少个？这里 log 表示以 10 为底的对数.

$$\sqrt{\frac{\log\left(n^2\right)-(\log n)^2}{\log n-3}}$$

   (A) 900

   (B) 2

   (C) 902

   (D) 2

   (E) 901

8. How many nonempty subsets B of $\{0,1,2,3,\ldots,12\}$ have the property that the number of elements in B is equal to the least element of B ? For example, $B=\{4,6,8,11\}$ satisfies the condition.

   考虑 $\{0,1,2,3,\ldots,12\}$ 的非空子集 B，满足 B 中的元素个数等于 B 中的最小元素. 问这样的子集 B 有多少个？例如，$B=\{4,6,8,11\}$ 就满足此条件.

   (A) 256

   (B) 136

   (C) 108

   (D) 144

   (E) 156

9. What is the area of the region in the coordinate plane defined by the below inequality?

坐标平面中由下列不等式定义的区域的面积是多少?

$$||x|-1|+||y|-1| \leq 1\,?$$

(A) 2

(B) 8

(C) 4

(D) 15

(E) 12

10. In the xy-plane, a circle of radius 4 with center on the positive x-axis is tangent to the y-axis at the origin, and a circle of radius 10 with center on the positive y-axis is tangent to the x-axis at the origin. What is the slope of the line passing through the two points at which these circles intersect?

在 xy 平面上，一个半径为 4、圆心位于正 x 轴的圆与 y 轴在原点处相切，另一个半径为 10、圆心位于正 y 轴的圆与 x 轴在原点处相切. 问通过这两个圆的两个交点的直线的斜率是多少?

(A) $\frac{2}{7}$

(B) $\frac{3}{7}$

(C) $\frac{2}{\sqrt{29}}$

(D) $\frac{1}{\sqrt{29}}$

(E) $\frac{2}{5}$

11. What is the maximum area of an isosceles trapezoid that has legs of length 1 and one base twice as long as the other?

在等腰梯形中，两腰的长度是 1，一条底边的长度是另一条底边的两倍，问它的面积最大是多少？

(A) $\frac{5}{4}$

(B) $\frac{8}{7}$

(C) $\frac{5\sqrt{2}}{4}$

(D) $\frac{3}{2}$

(E) $\frac{3\sqrt{3}}{4}$

12. For complex numbers $u = a + bi$ and $v = c + di$ (where $i = \sqrt{-1}$), define the binary operation

$$u \otimes v = ac + bdi .$$

Suppose z is a complex number such that $z \otimes z = z^2 + 40$. What is $|z|$?

对于复数 $u = a + bi$ 和 $v = c + di$（其中 $i = \sqrt{-1}$），定义二元运算

$$u \otimes v = ac + bdi .$$

假设 z 是一个复数，使得 $z \otimes z = z^2 + 40$．问 $|z|$ 是多少？

(A) 2

(B) 5

(C) $\sqrt{5}$

(D) $\sqrt{10}$

(E) $5\sqrt{2}$

13. A rectangular box $\mathcal{P}$ has distinct edge lengths $a, b,$ and c. The sum of the lengths of all 12 edges of $\mathcal{P}$ is 13 , the sum of the areas of all 6 faces of $\mathcal{P}$ is $\frac{11}{2}$, and the volume of $\mathcal{P}$ is $\frac{1}{2}$. What is the length of the longest interior diagonal connecting two vertices of $\mathcal{P}$?

一个长方体盒子 $\mathcal{P}$ 的三条不同的棱长为 a, b, c. $\mathcal{P}$ 的所有 12 条棱的长度之和为 13，$\mathcal{P}$ 的所有 6 个面的面积之和为 $\frac{11}{2}$，并且 $\mathcal{P}$ 的体积为 $\frac{1}{2}$. 问连接 $\mathcal{P}$ 的两个顶点，且在其内部的对角线最长是多少?

(A) 2

(B) $\frac{3}{8}$

(C) $\frac{9}{8}$

(D) $\frac{9}{4}$

(E) $\frac{3}{2}$

14. For how many ordered pairs (a, b) of integers does the polynomial $x^3 + ax^2 + bx + 6$ have 3 distinct integer roots?

使得多项式 $x^3 + ax^2 + bx + 6$ 有 3 个不同的整数根的有序整数对 (a, b) 有多少个?

(A) 5

(B) 6

(C) 8

(D) 7

(E) 4

15. Suppose that a, b, and c are positive integers such that

$$\frac{a}{14} + \frac{b}{15} = \frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a,14) = 1$ or $\gcd(b,15) = 1$ or both, then $\gcd(c,210) = 1$.

II. If $\gcd(c,210) = 1$, then $\gcd(a,14) = 1$ or $\gcd(b,15) = 1$ or both.

III. $\gcd(c,210) = 1$ if and only if $\gcd(a,14) = 1$ and $\gcd(b,15) = 1$.

假设 a，b，c 是正整数，满足

$$\frac{a}{14} + \frac{b}{15} = \frac{c}{210}\ .$$

问以下哪些陈述一定是正确的？

I. 如果 $\gcd(a,14) = 1$，或者 $\gcd(b,15) = 1$，或者两式都成立，那么 $\gcd(c,210) = 1$.

II. 如果 $\gcd(c,210) = 1$，那么 $\gcd(a,14) = 1$，或者 $\gcd(b,15) = 1$，或者两式都成立.

III. $\gcd(c,210) = 1$，当且仅当 $\gcd(a,14) = 1$，并且 $\gcd(b,15) = 1$.

这里 gcd 表示最大公约数.

(A) I, II, and III | I，II 和 III

(B) I only | 只有 I

(C) I and II only | 只有 I 和 II

(D) III only | 只有 III

(E) II and III only | 只有 II 和 III

16. In the state of Coinland, coins have values of 6, 10 , and 15 cents. Suppose x is the value in cents of the most expensive item in Coinland that cannot be purchased using these coins with exact change. What is the sum of the digits of x ?

在 Coinland 州，硬币的面值为 6 美分，10 美分，15 美分．在无法使用这些硬币来恰好购买的 Coinland 州的物品中，假设价值最贵的是 x 美分．问 x 的各位数字之和是多少?

(A) 8

(B) 10

(C) 7

(D) 11

(E) 9

17. The three side lengths of a triangle are in arithmetic progression with shortest side of length 6. One of the interior angles measures 120°. What is the area of the triangle?

三角形的三条边的长度呈等差数列，最短的边长为 6．三角形中的一个内角为 120°．问此三角形的面积是多少?

(A) $12\sqrt{3}$

(B) $8\sqrt{6}$

(C) $14\sqrt{2}$

(D) $20\sqrt{2}$

(E) $15\sqrt{3}$

18. Last academic year Yolanda and Zelda took different courses that did not necessarily administer the same number of quizzes during each of the two semesters. Yolanda's average on all the quizzes she took during the first semester was 3 points higher than Zelda's average on all the quizzes she took during the first semester. Yolanda's average on all the quizzes she took during the second semester was 18 points higher than her average for the first semester and was again 3 points higher than Zelda's average on all the quizzes Zelda took during her second semester. Which one of the following statements cannot possibly be true?

上个学年，Yolanda 和 Zelda 修读了不同的课程，在两个学期中的每个学期，这些课程的测验数量不一定相同．Yolanda 第一学期所有测验的平均分比 Zelda 第一学期所有测验的平均分高 3 分．Yolanda 第二学期所有测验的平均分比她自己第一学期所有测验的平均分高 18 分，并且比 Zelda 第二学期所有测验的平均分仍然高 3 分．问以下哪一项不可能是正确的？

(A) Yolanda's quiz average for the academic year was 22 points higher than Zelda's.

Yolanda 本学年的测验平均分比 Zelda 高 22 分．

(B) Zelda's quiz average for the academic year was higher than Yolanda's.

Zelda 本学年的测验平均分高于 Yolanda．

(C) Yolanda's quiz average for the academic year was 3 points higher than Zelda's.

Yolanda 本学年的测验平均分比 Zelda 高 3 分．

(D) Zelda's quiz average for the academic year equaled Yolanda's.

Zelda 本学年的测验平均分与 Yolanda 相同．

(E) If Zelda had scored 3 points higher on each quiz she took, then she would have had the same average for the academic year as Yolanda.

如果 Zelda 每次测验的得分都高 3 分，那么她本学年的测验平均分将与 Yolanda 相同．

19. Each of 2023 balls is randomly placed into one of 3 bins. Which of the following is closest to the probability that each of the bins will contain an odd number of balls?

将 2023 个球中的每个球都随机放入 3 个箱子中的一个．问以下哪一项最接近每个箱子中都有奇数个球的概率？

(A) $\frac{2}{3}$

(B) $\frac{3}{10}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

(E) $\frac{1}{4}$

20. Cyrus the frog sits on a flat surface. He jumps, landing 2 feet away. He then chooses a direction at random and again jumps 2 feet. What is the probability that after the second jump Cyrus lands within 1 foot of his starting position?

青蛙 Cyrus 坐在平坦的表面上．他跳了一次，落在 2 英尺外．然后他随机选择一个方向并再次跳跃 2 英尺．问第二次跳跃后，Cyrus 的落点距离起始位置在 1 英尺以内的概率是多少？

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

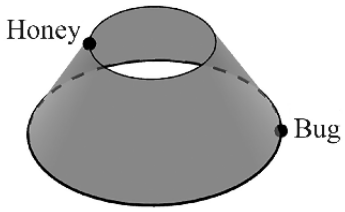
(C) $\frac{\sqrt{3}}{8}$

(D) $\frac{\arctan \frac{1}{2}}{\pi}$

(E) $\frac{2 \arcsin \frac{1}{4}}{\pi}$

21. A lampshade is made in the form of the lateral surface of the frustum of a right circular cone. The height of the frustum is $3\sqrt{3}$ inches, its top diameter is 6 inches, and its bottom diameter is 12 inches. A bug is at the bottom of the lampshade and there is a glob of honey on the top edge of the lampshade at the spot farthest from the bug. The bug wants to crawl to the honey, but it must stay on the surface of the lampshade. What is the length in inches of its shortest path to the honey?

一个灯罩的外形为截头锥体（又称圆台）的侧面．截头锥体的高度为 $3\sqrt{3}$ 英寸，顶部直径为 6 英寸，底部直径为 12 英寸．灯罩底部有一只虫子，距离虫子最远的灯罩上缘有一滴蜂蜜．虫子想爬到蜂蜜处，但它必须保持在灯罩的表面移动．问到达蜂蜜处的最短路径的长度是多少英寸?



Bug ｜虫子　　Honey ｜蜂蜜

(A) $6+3\pi$

(B) $6+6\pi$

(C) $6\sqrt{3}$

(D) $6\sqrt{5}$

(E) $6\sqrt{3}+\pi$

22. A real-valued function f has the property that for all real numbers a and b,

$$f(a+b)+f(a-b)=2f(a)f(b).$$

Which one of the following *cannot* be the value of $f(1)$?

实值函数 f 具有以下的性质：对于所有的实数 a 和 b，

$$f(a+b)+f(a-b)=2f(a)f(b).$$

问以下哪一项不可能是 $f(1)$ 的值?

(A) 0

(B) 1

(C) -1

(D) 2

(E) -2

23. When a standard 6-sided die is rolled n times, the product of the n numbers rolled can be any of 936 possible values. What is n ?

当抛掷一个标准 6 面骰子 n 次时，所得到的 n 个数的乘积有 936 个可能值. 问 n 是多少?

(A) 6

(B) 8

(C) 9

(D) 10

(E) 11

24. Suppose that $a, b, c,$ and d are positive integers satisfying all of the following relations. What is $\gcd(a,b,c,d)$?

假设 a，b，c，d 是满足以下所有关系式的正整数．问 $\gcd(a,b,c,d)$ 是多少？这里，gcd 表示最大公约数，lcm 表示最小公倍数．

$$\begin{aligned} abcd &= 2^6 \cdot 3^9 \cdot 5^7 \\ \operatorname{lcm}(a,b) &= 2^3 \cdot 3^2 \cdot 5^3 \\ \operatorname{lcm}(a,c) &= 2^3 \cdot 3^3 \cdot 5^3 \\ \operatorname{lcm}(a,d) &= 2^3 \cdot 3^3 \cdot 5^3 \\ \operatorname{lcm}(b,c) &= 2^1 \cdot 3^3 \cdot 5^2 \\ \operatorname{lcm}(b,d) &= 2^2 \cdot 3^3 \cdot 5^2 \\ \operatorname{lcm}(c,d) &= 2^2 \cdot 3^3 \cdot 5^2 \end{aligned}$$

(A) 30

(B) 45

(C) 3

(D) 15

(E) 6

25. A regular pentagon with area $1+\sqrt{5}$ is printed on paper and cut out. All five vertices are folded to the center of the pentagon, creating a smaller pentagon. What is the area of the new pentagon?

在纸上打印并剪下一个面积为 $1+\sqrt{5}$ 的正五边形．将所有的五个顶点都折叠到五边形的中心，形成一个较小的五边形．问新五边形的面积是多少？

(A) $4-\sqrt{5}$

(B) $\sqrt{5}-1$

(C) $8-3\sqrt{5}$

(D) $\dfrac{\sqrt{5}+1}{2}$

(E) $\dfrac{2+\sqrt{5}}{3}$

Answers.

CBDCC CEDBE DEDAE DEAEE EEECB