2025 AMC 10A

1.

Andy and Betsy both live in Mathville. Andy leaves Mathville on his bicycle at 1:30, traveling due north at a steady 7 miles per hour. Betsy leaves on her bicycle from the same point at2:30, traveling due east at a steady 11 miles per hour. At what time will they be exactly the same distance from their common starting point?

Andy 和 Betsy 都住在数学村。Andy 于 1:30 骑自行车离开数学村，向正北方以每小时 7 英里的恒定速度行进.Besty 于 2:30 从同一地点骑自行车出发，向正东方以每小时 11 英里的恒定速度行进。问何时他们与共同出发点的距离会完全相同?

(A) 3:30 (B) 3:45 (C) 4:00 (D) 4:15 (E) 4:30

2.

A box contains 10 pounds of a nut mix thatis 50 percent peanuts, 20 percent cashews, and 30 percent almonds. A second nut mix containing 20 percent peanuts, 40 percent cashews, and 40 percent almonds is added to the box resulting in a new nut mix that is 40 percent peanuts. How many pounds of almonds are now in the box?

一个盒子里装有 10 磅的混合坚果，其中花生占 50%,腰果占 20%，杏仁占 30%。后来在盒子中加入了另一种混合坚果，其中花生占 20%,腰果占 40%，杏仁占 40%，而最终形成的新混合坚果中花生占 40%。问现在盒子中共有多少磅杏仁?

(A) 3.5 (B) 4 (C) 4.5 (D) 5 (E) 6

3.

How many isosceles triangles are there with positive area whose side lengths are all positive integers and whose longest side has length 2025?

各边长均为正整数，最长边的长度为 2025,且面积为正的等腰三角形共有多少个?

(A) 2025 (B) 2026 (C) 3012 (D) 3037 (E) 4050

4.

A team of students is going to compete against a team of teachers in a trivia contest. The total number of students and teachers is 15.Ash, a cousin of one of the students, wants to join the contest. If Ash plays with the students, the average age on that team will increase from 12 to 14. If Ash plays with the teachers, the average age on that team will decrease from 55 to 52. How old is Ash?

一组学生将与一组教师进行知识比赛。学生和教师的总人数为 15 人。Ash 是其中一名学生的表亲，他也想参加比赛.如果 Ash 加入学生队，该队的平均年龄将从 12 岁增加到 14 岁；如果 Ash 加入教师队，该队的平均年龄将从 55 岁减少到 52 岁.问 Ash 的年龄是多少岁?

(A) 28 (B) 29 (C) 30 (D) 32 (E) 33

5.

Consider the below sequence of positive integers. What is the 2025th termin this sequence?

观察下面的正整数数列.问该数列的第 2025 项是多少?

1, 1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, ...

(A) 5 (B) 15 (C) 16 (D) 44 (E) 45

6.

In an equilateral triangle each interior angle is trisected by a pair of rays. The intersection of the interiors of the middle 20°-angle at each vertex is the interior of a convex hexagon. What is the degree measure of the largest angle of this hexagon?

在等边三角形中，每个内角都被两条射线三等分.在每个顶点处，取中间那个20°角的内部区域，这三个区域的公共部分是一个凸六边形的内部.问该六边形最大内角的度数是多少?

(A) 100 (B) 110 (C) 120 (D) 130 (E) 140

7.

Suppose a and b are real numbers. When the polynomial x^3+x^2+ax+b is divided by $x-1$, the

remainder is 5. When the polynomial is divided by $x-2$, the remainder is 7. What is $b-a$?

设 a 与 b 为实数.当多项式 x^3+x^2+ax+b 除以 $x-1$ 时，余数为 5；当该多项式除以 $x-2$ 时，余数为 7。问 $b-a$ 是多少?

(A) 19 (B) 20 (C) 21 (D) 22 (E) 23

8.

Agnes writes the following four statements on a blank piece of paper.

- At least one of these statements is true.
- At least two of these statements are true.
- At least two of these statements are false.
- At least one of these statements is false.

Each statement is either true or false.How many true statements did Agneswrite on the paper?

Agnes 在白纸上写出以下四个论断:

- 这些论断中至少有一个是真的.
- 这些论断中至少有两个是真的.
- 这些论断中至少有两个是假的.
- 这些论断中至少有一个是假的.

每个论断非真即假，问 Agnes 在纸上写的真论断有几个?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9.

Let $f(x)=100x^3-300x^2+200x$. For how many real numbers a does thegraph of $y=f(x-a)$ pass through the point (1, 25)?

设 $f(x)=100x^3-300x^2+200x$,问使得函数 $y=f(x-a)$的图像经过点(1, 25)的实数 a 有多少个?

(A) 1 (B) 2 (C) 3

(D) 4 (E) more than 4 | 多于 4 个

10.

The sequence 1, x, y, z is arithmetic. The sequence 1, p, q, z is geometric. Both sequences are strictly increasing and contain only integers, and z isas small as possible. What is the value of $x+y+z+p+q$?

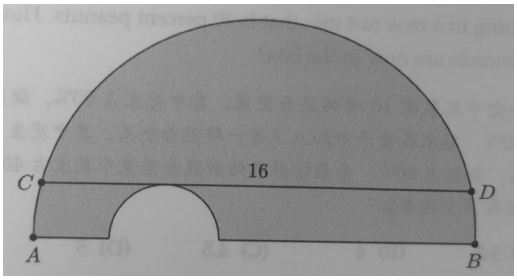
数列 $1, x, y, z$ 为等差数列，数列 $1, p, q, z$ 为等比数列.两个数列均严格递增，仅包含整数，且 z 尽可能小.问 $x+y+z+p+q$ 的值是多少?

(A) 66 (B) 91 (C) 103 (D) 132 (E) 149

11.

A semicircle has diameter $\overline{AB}$ and chord $\overline{CD}$ oflength 16 parallel to $\overline{AB}$. A smaller semicircle with diameter on $\overline{AB}$ and tangent to $\overline{CD}$ is cut from the larger semicircle, as shown below. What is the area of the resulting figure, shown shaded?

一个半圆的直径为$\overline{AB}$，与$\overline{AB}$平行的弦$\overline{CD}$的长度为 16。如图所示，从这个大半圆中挖去一个小半圆，该小半圆的直径在$\overline{AB}$上，且与弦$\overline{CD}$相切。问由此所得的图形(即阴影部分)的面积是多少?



(A) 16π (B) 24π (C) 32π (D) 48π (E) 64π

12.

Carlos uses a 4-digit passcode to unlock his computer.In his passcode, exactly one digit is even, exactly one (possibly diferent) digit is prime, and no digit is 0. How many 4-digit passcodes satisfy these conditions?

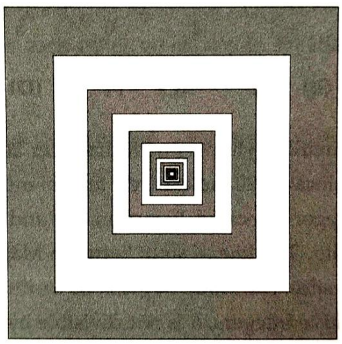
Carlos 使用一个 4 位数密码解锁电脑，在他的密码中，恰好有一个数字是偶数，恰好有一个数字(可能与前者不同)是质数，且没有数字是 0.问符合这些条件的 4 位数密码有多少个?

(A) 176 (B) 192 (C) 432 (D) 464 (E) 608

13.

In the figure below,the outside square contains infinitely many squares,each of them with the same center and sides parallel to the outside square.The ratio of the side length of a square to the side length of the next innersquare is k, where $0<k<1$.The spaces between squares are alternately shaded,as shown in the figure(which is not necessarily drawn to scale).The area of the shaded portion of the figure is 80% of the area of the originalsquare.What is k?

在下图中，最外面的正方形内包含无限多个正方形，每个正方形都有相同的中心，且边与最外面正方形的边平行。每个正方形与其相邻的内部正方形的边长之比为 k，其中 $0<k<1$。如图所示，正方形之间的区域交替涂色(图形未必按比例绘制)。若图形中阴影部分的面积是原正方形面积的 80%，问 k 是多少?



(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

14.

There are three jars.Each of three coins is placed in one of the three jars, chosen at random and independently of the placements of the other coins. What is the expected number of coins in a jar with the least coins?

有三个罐子，将三枚硬币分别放入这三个罐子中，每枚硬币放入哪个罐子是随机选择的，且与其他硬币的放置相互独立.问拥有最少硬币的罐子中硬币数量的期望值是多少?

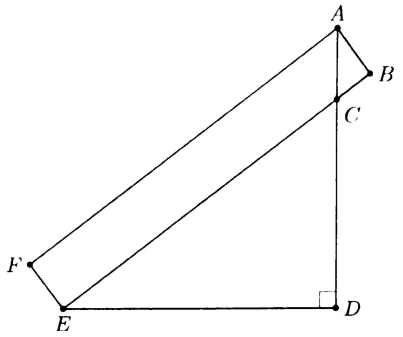
(A) $\frac{1}{27}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 1

15.

In the figure below, $ABEF$ is a rectangle, $\overline{AD} \perp \overline{DE}$, $AF=7$, $AB=1$, and $AD=5$. What is the area of

$\triangle ABC$?

在下图中，$ABEF$ 为矩形，$\overline{AD} \perp \overline{DE}$，且 $AF=7, AB=1, AD=5$。问 $\triangle ABC$ 的面积是多少?



(A) $\frac{3}{8}$ (B) $\frac{4}{9}$ (C) $\frac{1}{8}\sqrt{13}$ (D) $\frac{7}{15}$ (E) $\frac{1}{8}\sqrt{15}$

16.

Six chairs are arranged around a round table. Two students and two teachers randomly select four of the chairs to sit in. What is the probability that the two students will sit in two adjacent chairs and the two teachers will also sit in two adjacent chairs?

六把椅子围绕一张圆桌摆放.两名学生和两名教师随机选择其中的四把椅子就座.问两名学生坐在相邻椅子上，且两名教师也坐在相邻椅子上的概率是多少?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{3}{13}$ (E) $\frac{1}{4}$

17.

Let N be the unique positive integer such that dividing 273436 by N leavesa remainder of 16 and dividing 272760 by N leaves a remainder of 15. Whatis the largest digit of N?

设 N 为唯一的正整数，使得 273436 除以 N 的余数为 16，且 272760 除以 N 的余数为 15。问组成 N 的最大数字是多少?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

18.

The *harmonic mean* of a collection of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers in the collection. For example,the harmonic mean of 4, 4, and 5 is

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)}=\frac{30}{7}.$$

What is the harmonic mean of all the real roots of the 4050th degree polynomial

$$\prod_{k=1}^{2025}(kx^2-4x-3)$$
$$=(x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

一组数的调和平均数是指该组数倒数的算术平均数的倒数。例如，4, 4, 5 的调和平均数为

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)}=\frac{30}{7}.$$

问 4050 次多项式

$$\prod_{k=1}^{2025}(kx^2-4x-3)$$
$$=(x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

所有实根的调和平均数是多少?

(A) $-\frac{5}{3}$ (B) $-\frac{3}{2}$ (C) $-\frac{6}{5}$ (D) $-\frac{5}{6}$ (E) $-\frac{2}{3}$

19.

An array of numbers is constructed beginning with the numbers −1 3 1 in the top row. Each adjacent pair of numbers is summed to produce a number in the next row. Each row will begin and end with the numbers −1 and 1, respectively. The first three rows are shown below. If the process continues, one of the rows will sum to 12,288. In that row, what is the thirdnumber from the left?

数阵的第一行为−1 3 1。从第二行开始，每行的第一个数和最后一个数分别为−1 和 1，中间的数由上一行相邻两数之和构成。数阵的前三行如下图所示.若继续按照这样的规则构建，某一行中的各数总和将等于 12,288。问在该行中，从左数的第三个数是多少?

$$\begin{array}{ccccccccc} & & -1 & & 3 & & 1 & & \\ & -1 & & 2 & & 4 & & 1 & \\ -1 & & 1 & & 6 & & 5 & & 1 \end{array}$$

(A) −29 (B) −21 (C) −14 (D) −8 (E) −3

20.

A silo (right circular cylinder) with diameter 20 meters stands in a field. MacDonald is located 20 meters west and 15 meters south of the center of the silo. McGregor is located 20 meters east and $g>0$ meters south of thecenter of the silo. The line of sight between MacDonald and McGregor is tangent to the silo. The value of g can be written as $\frac{a\sqrt{b}-c}{d}$, where a, b, c, and d are positive integers, b is not divisible by the square of any prime, and d is relatively prime to the greatest common divisor of a and c. Whatis $a+b+c+d$?

田野中有一个直径为 20 米的直圆柱形的筒仓，MacDonald 位于筒仓中心以西 20 米，以南 15 米处；McGregor 位于筒仓中心以东 20 米，以南 $g>0$ 米处。MacDonald 与 McGregor 之间的视线与筒仓相切、g 的值可表示为$\frac{a\sqrt{b}-c}{d}$，其中 a, b, c, d 均为正整数，b 不被任何质数的平方整除，且 a 和 c 的最大公约数与 d 互质，问 $a+b+c+d$ 是多少?

(A) 119 (B) 120 (C) 121 (D) 122 (E) 123

21.

A set of numbers is called *sum-fre* if whenever x and y are distinct elements of the set, $x+y$ is not an element of the set. For example,{2, 4, 5}and theempty set are sum-free, but {1, 4, 5} is not. What is the greatest possible number of elements in a sum-free subset of {1, 2, 3, ..., 20}?

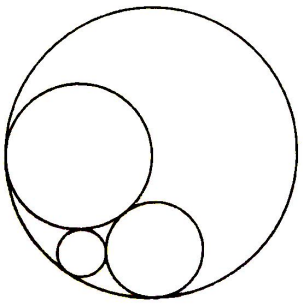
若一个数集满足: 对于其中任意两个不同的元素 $x, y, x+y$ 都不在该集中，则称此数集称为“无和集”。例如，{2, 4, 5}与空集均为无和集，但{1, 4, 5}则不是.问在集合{1, 2, 3, …, 20}的所有无和子集中，元素数目的最大可能值是多少?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

22.

Inside a large circle of radius r, there are three smaller circles, whose radii are 1, 2, and 3 respectively, all internally tangent to the large circle and externally tangent to each other, as shown in the diagram below. What is r?

如下图所示，在一个半径为 r 的圆内，有三个圆与它内切，这三个圆的半径分别为 1, 2, 3, 同时它们彼此之间互相外切。问 r 是多少?



(A) $\frac{11}{2}$ (B) $\frac{35}{6}$ (C) 6 (D) $\frac{25}{4}$ (E) $\frac{19}{3}$

23.

Triangle $\triangle ABC$ has side lengths AB=80, BC=45, and AC=75. The bisector of $\angle B$ intersects the altitude to side $\overline{AB}$ at point P, and intersects the side $\overline{AC}$ at point Q. What is $BP+CQ$?

在$\triangle ABC$ 中，AB=80, BC=45, AC=75。$\angle B$ 的角平分线与边$\overline{AB}$上的高相交于点 P，与边$\overline{AC}$相交于点 Q。问 $BP+CQ$ 的长度是多少?

(A) 44 (B) 45 (C) 46 (D) 47 (E) 48

24.

Call a positive integer fair if no digit is used more than once, it has no 0s, and no digit is adjacent to two greater digits. For example, 78, 320, and 12463 are fair, but 512, 9024 and 34321 are not fair. How many fair positive integers are there?

如果一个正整数中没有数字重复出现，且没有任何一个数字的两个相邻数字都比它大，那么称其为“平衡数”。例如，78，320，12463 均为平衡数，但 512，9024，34321 则不是。注意任何正整数的首位数字不能是 0。问共有多少个平衡的正整数?

(A) 2,584 (B) 9,841 (C) 17,711 (D) 19,682 (E) 29,524

25.

A point P is chosen at random inside square $ABCD$. The probability that $\overline{AP}$ is neither the shortest nor the longest side of $\triangle APB$ can be written as $\frac{a+b\pi-c\sqrt{d}}{e}$, where a, b, c, d, and e are positive integers, gcd(a, b, c, e)=1, and d is not divisible by the square of any prime. What is $a+b+c+d+e$?

在正方形 $ABCD$ 内随机选取一点 P。$\overline{AP}$既不是$\triangle APB$ 的最短边，也不是最长边的概率可以表示为$\frac{a+b\pi-c\sqrt{d}}{e}$，其中 a, b, c, d, e 均为正整数，a, b, c, e 的最大公约数是 1，且 d 不被任何质数的平方整除，问 $a+b+c+d+e$ 是多少？

(A) 25 (B) 26 (C) 27 (D) 28 (E) 29