2025 AMC 12A

1.

Andy and Betsy both live in Mathville. Andy leaves Mathville on his bicycle at 1:30, traveling due north at a steady 7 miles per hour. Betsy leaves on her bicycle from the same point at 2:30, traveling due east at a steady 11miles per hour. At what time will they be exactly the same distance from their common starting point?

Andy 和 Betsy 都住在数学村.Andy 于 1:30 骑自行车离开数学村，向正北方以每小时 7 英里的恒定速度行进.Besty 于 2:30 从同一地点骑自行车出发，向正东方以每小时 11 英里的恒定速度行进.问何时他们与共同出发点的距离会完全相同?

(A) 3:30 (B) 3:45 (C) 4:00 (D) 4:15 (E) 4:30

2.

Abox contains 10 pounds of a nut mix that is 50 percent peanuts,20 percent cashews, and 30 percent almonds.A second nut mix containing 20 percent peanuts,40 percent cashews, and 40 percent almonds is added to the box resulting in a new nut mix that is 40 percent peanuts. How many pounds of almonds are now in the box?

一个盒子里装有 10 磅的混合坚果，其中花生占 50%,腰果占 20%，杏仁占 30%。在盒子中后来加入了另一种混合坚果，其中花生占 20%，腰果占 40%，杏仁占 40%,而最终形成的新混合坚果中花生占 40%.问现在盒子中共有多少磅杏仁?

(A) 3.5 (B) 4 (C) 4.5 (D) 5 (E) 6

3.

A team of students is going to compete against a team of teachers in a trivia contest. The total number of students and teachers is 15. Ash, a cousin of one of the students, wants to join the contest. If Ash plays with the students, the average age on that team will increase from 12 to 14. If Ash plays with the teachers,the average age on that team will decrease from 55 to 52. How old is Ash?

一组学生将与一组教师进行知识比赛，学生和教师的总人数为 15 人。Ash 是其中一名学生的表亲，他也想参加比赛。如果 Ash 加入学生队，该队的平均年龄将从 12 岁增加到 14 岁；

如果 Ash 加入教师队，该队的平均年龄将从 55 岁减少到 52 岁。问 Ash 的年龄是多少岁?

(A) 28 (B) 29 (C) 30 (D) 32 (E) 33

4.

In a certain alien world, the maximum running speed v of an organism is dependent on its number of toes n and number of eyes m. The relationshipcan be expressed as $v=kn^am^b$ centimeters per hour, where k, a, and b are integer constants. In a population where all organisms have 5 toes, $\log v=4+2\log m$; and in a population where all organisms have 25 eyes, $\log v=4+4\log n$, where the logarithms are base 10. What is $k+a+b$?

在某个外星世界中，生物的最大奔跑速度 v 取决于它的脚趾数量 n 和眼睛数量 m、在速度以厘米每小时为单位时，两者的关系可表示为 $v=kn^am^b$，其中 k, a, b 均为整数常数、所有生物都有 5 根脚趾的种群满足 $\log v=4+2\log m$；所有生物都有 25 只眼睛的种群满足 $\log v=4+4\log n$，这里的对数均以 10 为底.问 $k+a+b$ 的值是多少?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

5.

Agnes writes the following four statements on a blank piece of paper.

- At least one of these statements is true.
- At least two of these statements are true.
- At least two of these statements are false.
- Atleast one of these statements is false.

Each statement is either true or false.How many true statements did Agneswrite on the paper?

Agnes 在白纸上写出以下四个论断:

- 这些论断中至少有一个是真的.
- 这些论断中至少有两个是真的.
- 这些论断中至少有两个是假的.
- 这些论断中至少有一个是假的.

每个论断非真即假.问 Agnes 在纸上写的真论断有几个?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6.

In the figure below,the outside square contains infinitely many squares,each of them with the same center and sides parallel to the outside square.The ratio of the side length of a square to the side length of the next innersquare is k, where $0<k<1$.The spaces between squares are alternately shaded,as shown in the figure(which is not necessarily drawn to scale).The area of the shaded portion of the figure is 80% of the area of the originalsquare.What is k?

在下图中，最外面的正方形内包含无限多个正方形，每个正方形都有相同的中心，且边与最外面正方形的边平行。每个正方形与其相邻的内部正方形的边长之比为 k，其中 $0<k<1$。如图所示,正方形之间的区域交替涂色(图形未必按比例绘制).若图形中阴影部分的面积是原正方形面积的 80%，问 k 是多少?



(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

7.

Six chairs are arranged around a round table. Two students and two teachers randomly select four of the chairs to sit in. What is the probability that the two students will sit in two adjacent chairs and the two teachers will also sit in two adjacent chairs?

六把椅子围绕一张圆桌摆放.两名学生和两名教师随机选择其中的四把椅子就座.问两名学生坐在相邻椅子上，且两名教师也坐在相邻椅子上的概率是多少?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{3}{13}$ (E) $\frac{1}{4}$

8.

Let w be the complex number $2+i$, where $i=\sqrt{-1}$. What real number r has the property that ri, w, and w^2 are three colinear points on the plane?

设 w 为复数 $2+i$,其中 $i=\sqrt{-1}$。问哪个实数 r 可以使得 ri, w 和 w^2 在复平面上是共线的三点?

(A) -5 (B) -4 (C) -3 (D) -2 (E) -1

9.

Pentagon $ABCDE$ is inscribed in a circle,and $\angle BEC=\angle CED=30°$. Let $\overline{AC}$ and $\overline{BD}$ intersect at point F, and suppose that $AB=9$ and $AD=24$. What is BF?

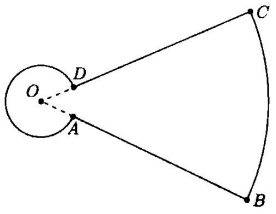
在圆内接五边形 $ABCDE$ 中,$\angle BEC=\angle CED=30°$。设$\overline{AC}$与$\overline{BD}$相交于点 F,已知 $AB=9$ 且 $AD=24$。问 BF 的长度是多少?

(A) $\frac{57}{11}$ (B) $\frac{59}{11}$ (C) $\frac{60}{11}$ (D) $\frac{61}{11}$ (E) $\frac{63}{11}$

10.

In the figure shown below,major arc $\overset{\frown}{AD}$ and minor arc $\overset{\frown}{BC}$ have the same center O. Also, A lies between O and B, and D lies between O and C. Major arc $\overset{\frown}{AD}$,minor arc $\overset{\frown}{BC}$, and each of the two segments $\overline{AB}$ and $\overline{CD}$ have length 2π.What is the distance from O to A?

在下图中，优弧 $\overset{\frown}{AD}$ 与劣弧 $\overset{\frown}{BC}$ 有相同的圆心 O。并且，点 A 位于 O 与 B 之间，点 D 位于 O 与 C 之间。优弧 $\overset{\frown}{AD}$，劣弧 $\overset{\frown}{BC}$ 以及线段 $\overline{AB}$ 和 $\overline{CD}$ 的长度均为 2π。问 O 到 A 的距离是多少?



(A) 1 (B) $1-\pi+\sqrt{1+\pi^2}$ (C) $\frac{1}{2}\pi$

(D) $\frac{1}{2}\sqrt{1+\pi^2}$ (E) 2

11.

The *harmonic mean* of a collection of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers in the collection. For example,the harmonic mean of 4, 4, and 5 is

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)}=\frac{30}{7}.$$

What is the harmonic mean of all the real roots of the 4050th degree polynomial

$$\prod_{k=1}^{2025}(kx^2-4x-3)$$
$$=(x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

一组数的调和平均数是指该组数倒数的算术平均数的倒数。例如，4, 4, 5 的调和平均数为

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)}=\frac{30}{7}.$$

问 4050 次多项式

$$\prod_{k=1}^{2025}(kx^2-4x-3)$$
$$=(x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

所有实根的调和平均数是多少?

(A) $-\frac{5}{3}$ (B) $-\frac{3}{2}$ (C) $-\frac{6}{5}$ (D) $-\frac{5}{6}$ (E) $-\frac{2}{3}$

12.

The orthocenter of a triangle is the concurrent intersection of the three (possibly extended) altitudes. What is the sum of the coordinates of the orthocenter of the triangle whose vertices are $A(2, 31)$, $B(8, 27)$, and $C(18, 27)$?

三角形的垂心是指其三条高(可能需延长)的交点.已知某三角形的三个顶点分别为 $A(2, 31)$, $B(8, 27)$, $C(18, 27)$,问该三角形垂心的坐标之和是多少?

(A) 5 (B) 17 (C) $10+4\sqrt{17}+2\sqrt{13}$

(D) $\frac{113}{3}$ (E) 54

13.

Let C={1, 2, 3,⋯, 13}. Let N be the greatest integer such that there exists a subset of C with N elements that does not contain five consecutive integers. Suppose N integers are chosen at random from C without replacement. What is the probability that the chosen elements do not include fiveconsecutive integers?

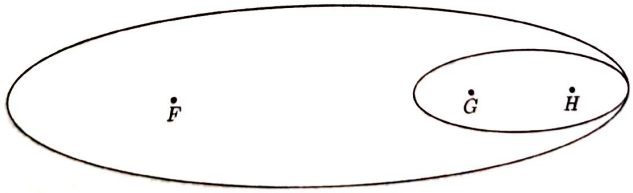
设集合 C={1, 2, 3, ..., 13}。设 N 为最大的整数，使得存在 C 的一个子集，该子集包含 N 个元素且不含五个连续的整数。现从 C 中无放回地随机选取 N 个整数，问选出的元素中不含五个连续整数的概率是多少?

(A) $\frac{3}{130}$ (B) $\frac{3}{143}$ (C) $\frac{5}{143}$ (D) $\frac{1}{26}$ (E) $\frac{5}{78}$

14.

Points F, G, and H are collinear with G between F and H. The ellipse with foci at G and H is internally tangent to the ellipse with foci at F and G, as shown below.The two ellipses have the same eccentricity e, and the ratioof their areas is 2025. (Recall that the eccentricity of an ellipse is $e=\frac{c}{a}$, where c is the distance from the center to a focus,and $2a$ is the length of the major axis.) What is e?

点 F, G, H 共线，且 G 位于 F 与 H 之间。如图所示，以 G 和 H 为焦点的椭圆与以 F 和 G 为焦点的椭圆内切.这两个椭圆有相同的离心率 e，且它们的面积比为 2025。(注意，椭圆的离心率 $e=\frac{c}{a}$，其中 c 是椭圆中心到一个焦点的距离，$2a$ 是椭圆长轴的长度.)问 e 是多少?



(A) $\frac{3}{5}$ (B) $\frac{16}{25}$ (C) $\frac{4}{5}$ (D) $\frac{22}{23}$ (E) $\frac{44}{45}$

15.

Aset of numbers is called *sum-fre* if whenever x and y are distinct elements of the set, $x+y$ is not an element of the set. For example,{2, 4, 5}and theempty set are sum-free, but {1, 4, 5} is not.

What is the greatest possible number of elements in a sum-free subset of {1, 2, 3, ..., 20}?

若一个数集满足: 对于其中任意两个不同的元素 $x, y, x+y$ 都不在该集中, 则称此数集称为"无和集"。例如，{2, 4, 5}与空集均为无和集，但{1, 4, 5}则不是.问在集合{1, 2, 3, ⋯, 20}的所有无和子集中，元素数目的最大可能值是多少?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

16.

Triangle $\triangle ABC$ has side lengths AB=80, BC=45, and AC=75. The bisector of $\angle B$ intersects the altitude to side $\overline{AB}$ at point P, and intersects the side $\overline{AC}$ at point Q.What is $BP+CQ$?

在$\triangle ABC$ 中，AB=80, BC=45, AC=75。$\angle B$ 的角平分线与边$\overline{AB}$上的高相交于点 P，与边$\overline{AC}$相交于点 Q。问 $BP+CQ$ 的长度是多少?

(A) 44 (B) 45 (C) 46 (D) 47 (E) 48

17.

The polynomial $(z+i)(z+2i)(z+3i)+10$ has three roots in the complex plane, where $i=\sqrt{-1}$. What is the area of the triangle formed by these roots?

多项式$(z+i)(z+2i)(z+3i)+10$ 在复平面内有三个根，其中 $i=\sqrt{-1}$。问由这三个根构成的三角形面积是多少?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

18.

How many ordered triples$(x, y, z)$of distinct nonnegative integers less than or equal to 9 satisfy $xy>z$, $zx>y$, and $yz>x$?

在由不同的非负整数组成,且每个数都不大于9的有序三元组(x, y, z)中,同时满足$xy>z$, $zx>y$, $yz>x$ 的有多少个?

(A) 54 (B) 120 (C) 186 (D) 300 (E) 336

19.

Let a, b, and c bethe rots of the polynomial x^3+kx-1. What is the sum the below expression?

设 a, b, c 为多项式 x^3+kx-1 的三个根。问下面和式的值是多少?

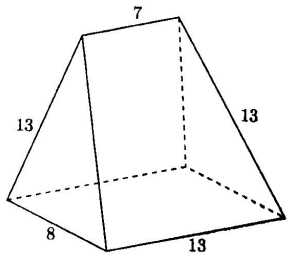
$$a^3b^2+a^2b^3+b^3c^2+b^2c^3+c^3a^2+c^2a^3$$

(A) $-k$ (B) $-k+1$ (C) 1 (D) $k-1$ (E) k

20.

The base of the pentahedron shown below is a 13×8 rectangle, and its lateral faces are two isosceles triangles with base of length 8 and congruent sides of length 13 and two isosceles trapezoids with bases of lengths 7and 13 and nonparallel sides of length 13. What is the volume of the pentahedron?

五面体如下图所示，其底面是一个 13×8 的矩形，侧面由两个等腰三角形和两个等腰梯形组成.其中等腰三角形的底边长为 8,且两条相等的腰长为 13；等腰梯形的两条底边长分别为 7 和 13，且非平行的腰长为 13。问该五面体的体积是多少?



(A) 416 (B) 520 (C) 528 (D) 676 (E) 832

21.

There real numbers are chosen independently and uniformly at random between 0 and 1. What is the probability that the greatest of these three numbers is greater than the sum of the other two numbers? (In other words, if the chosen numbers are $a \geq b \geq c$, then $a>b+c$.)

从 0 到 1 之间独立且随机的选取三个实数。问这三个数中最大的数大于另外两个数之和的概率是多少？（也就是说，如果选取的数是 $a \geq b \geq c$，那么需满足 $a>b+c$。

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

22.

There is a unique ordered triple (a, k, m) of nonnegative integers such that

$$\frac{4^a+4^{a+k}+4^{a+2k}+\cdots+4^{a+mk}}{2^a+2^{a+k}+2^{a+2k}+\cdots+2^{a+mk}}=964.$$

What is $a+k+m$?

存在唯一的由非负整数组成的有序三元组(a, k, m)，使得

$$\frac{4^a+4^{a+k}+4^{a+2k}+\cdots+4^{a+mk}}{2^a+2^{a+k}+2^{a+2k}+\cdots+2^{a+mk}}=964.$$

问 $a+k+m$ 的值是多少？

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

23.

Call a positive integer fair if no digit is used more than once, and no digit is adjacent to two greater digits. For example, 78, 320, and 12463 are fair, but 512, 9024 and 34321 are not fair. Note that the leading digit of any positive integer cannot be 0. How many fair positive integers are there?

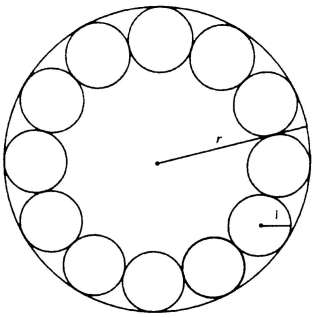
如果一个正整数中没有数字重复出现，且没有任何一个数字的两个相邻数字都比它大，那么称其为“平衡数”。例如，78，320，12463 均为平衡数，但 512，9024，34321 则不是。注意任何正整数的首位数字不能是 0。问共有多少个平衡的正整数?

(A) 2,584 (B) 9,841 (C) 17,711 (D) 19,682 (E) 29,524

24.

Inside the boundary of a large circle with radius r, 12 circles of radius 1 are arranged around it as shown. These 12 circles are all internally tangent to the large circle, and every two adjacent circles are externally tangent toeach other. Then r can be written as $\sqrt{a}+\sqrt{b}+c$,where a, b, and c areintegers.What is $a+b+c$?

在一个半径为 r 的大圆的边界内，环绕着 12 个半径为 1 的圆，如图所示，这 12 个圆都与大圆内切，而相邻的两个圆互相外切。此时 r 可表示为$\sqrt{a}+\sqrt{b}+c$ 的形式，其中 a, b, c 均为整数. 问 $a+b+c$ 是多少?



(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

25.

Polynomials $P(x)$ and $Q(x)$ each have degree 3 and leading coefficient 1, and their roots are all elements of {1, 2, 3, 4, 5}.The function $f(x)=\frac{P(x)}{Q(x)}$ has the property that there exist real numbers $a<b<c<d$ such that theset of all real numbers x such that $f(x)\leqslant 0$ consists of the closed interval $[a, b]$ together with the open interval (c, d).How many different possible graphs can such a function $f(x)$ have?

多项式$P(x)$与$Q(x)$的次数均为3且首项系数均为1,它们的根均为集合{1, 2, 3, 4, 5}中的元素. 函数$f(x)=\frac{P(x)}{Q(x)}$具有如下性质：存在实数$a<b<c<d$，使得满足$f(x)\leqslant 0$的所有实数x构成的集合是闭区间$[a, b]$与开区间(c, d)的并集，问这样的函数$f(x)$的图像有多少种不同的可能?

(A) 8 (B) 9 (C) 11 (D) 12 (E) 13