MAA AMC AMERICAN MATHEMATICS COMPETITIONS

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AMC 12 B

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The problems and solutions for this AMC 12 B were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

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INSTRUCTIONS | 说明

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
   在未收到监考老师开考指示前，请不要翻开此封面。
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
   这是一套包括25道选择题的测试。每道题目只有一个正确答案。
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet and appear sufficiently dark will be scored.
   请将每道题目的答案用＃2铅笔标注在答题卡上。请注意检查涂写的黑色圆圈的准确性，用橡皮完全擦掉错误的答案。只有在答题卡上规范填涂且颜色足够深的答案才会被计分。
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
   评分：每道题目答对得6分， 不答得1.5分，答错得0分。
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, computing devices, protractors, or graph paper are allowed. No problems in the competition will require the use of a calculator.
   只能使用空白的草稿纸、尺子、圆规和橡皮作为辅助工具。不允许的器具包括计算器、智能手表、手机、计算设备、量角器或坐标纸。竞赛中没有任何问题必须要使用计算器。
6. Figures are not necessarily drawn to scale.
   图形不一定按比例绘制。
7. Please bubble in carefully the mobile phone number used for registration. This mobile phone number will serve as your unique identification number. Please double-check it for accuracy, as the organizing committee will not accept appeals for scores invalidated due to a wrong number.
   请在答题卡上填涂报名所用手机号码，该手机号码将作为考生唯一识别标志，请务必认真填写，由于号码错误导致成绩无效，组委会不接受此类问题申诉。
8. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
   监考老师宣布开始后，你将有75分钟的时间完成测试。
9. All problems are provided with a Chinese translation for reference. In case of any discrepancy between the Chinese and English versions, the English version shall prevail.
   所有题目提供中文翻译作为参考。如果中英文两种版本之间存在差异，以英文版本为准。

1. The instructions on a 454-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 23 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

   一袋重量为 454 克的咖啡豆的使用说明上写着，冲泡一大杯手冲咖啡需要 23 克咖啡豆．问依照标准，用这袋咖啡豆最多可以冲泡多少杯大杯咖啡？

   (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

2. The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

   七进制的两位数 $\underline{a}\,\underline{b}$ 的值等于九进制的两位数 $\underline{b}\,\underline{a}$ 的值．问 $a+b$ 是多少？

   (A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 14

3. Jerry wrote down the ones digit of each of the first 2025 positive squares: $1, 4, 9, 6, 5, 6, \ldots$. What is the sum of all the numbers Jerry wrote down?

   Jerry 写下了前 2025 个正整数平方的个位数字：1，4，9，6，5，6，....问 Jerry 写下的所有数的总和是多少？

   (A) 9025 **(B)** 9070 **(C)** 9090 **(D)** 9115 **(E)** 9160

4. What is the value of $i(1-i)(2-i)(3-i)$, where $i=\sqrt{-1}$?

   已知 $i=\sqrt{-1}$，问 $i(1-i)(2-i)(3-i)$ 的值是多少？

   (A) $6-5i$ **(B)** $-10i$ **(C)** $10i$ **(D)** -10 **(E)** 10

5. Positive integers x and y satisfy the equation $57x+22y=400$. What is the least possible value of $x+y$?

   正整数 x 与 y 满足方程 $57x+22y=400$，问 $x+y$ 的最小可能值是多少？

   (A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

6. Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$\underline{A}\,\underline{B}\,\underline{B}.\underline{B}\,\underline{A}$, where A and B are digits and $A\neq 0$." After a pause, Max says, "That was a good price." What is the cost of each shirt in dollars? (Round to the nearest integer.)

   Emmy 对 Max 说："我今天订了 36 件数学社的运动衫．"Max 问："每件运动衫多少钱？"Emmy 回答说："我给你一个提示．总成本是 $\$\underline{A}\,\underline{B}\,\underline{B}.\underline{B}\,\underline{A}$，其中 A 和 B 是数字且 $A\neq 0$．"停顿了一下后，Max 说："这个价格很不错．"问每件运动衫的价格是多少美元？（四舍五入到整数．）

   (A) 11 **(B)** 14 **(C)** 15 **(D)** 17 **(E)** 18

7. What is the value of

$$\sum_{n=4}^{255}\frac{\log_2\left(1+\frac{1}{n}\right)}{(\log_2 n)(\log_2(n+1))}\,?$$

   问和式

$$\sum_{n=4}^{255}\frac{\log_2\left(1+\frac{1}{n}\right)}{(\log_2 n)(\log_2(n+1))}\,?$$

   的值是多少？

   (A) $\frac{3}{8}$ **(B)** $\frac{1}{2}-\frac{1}{\log_2 255}$ **(C)** $\frac{1}{2}$

   (D) $\frac{3}{4}$ **(E)** $\frac{7}{8}$

8. There are integers a and b such that the polynomial x^3-6x^2+ax+b has $4-\sqrt{5}$ as a root. What is $a+b$?

   存在整数 a 与 b，使得 $4-\sqrt{5}$ 是多项式 x^3-6x^2+ax+b 的根，问 $a+b$ 是多少？

   (A) 13 **(B)** 17 **(C)** 20 **(D)** 30 **(E)** 68

9. What is the tens digit of 6^{7^6} ?

问 6^{7^6} 的十位数字是多少?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

10. In a right triangle, the lengths of the two legs are in the ratio $1:2$. The altitude to the hypotenuse is divided into two segments of lengths $x<y$ by the median to the shorter leg of the triangle. What is the value of the ratio $\frac{x}{x+y}$?

一个直角三角形的两条直角边的长度之比是 $1:2$，斜边上的高被该三角形较短直角边上的中线分成两条线段，长度分别为 $x<y$. 问比值 $\frac{x}{x+y}$ 是多少?

(A) $\frac{3}{7}$ **(B)** $\frac{\sqrt{5}}{6}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{11}$ **(E)** $\frac{\sqrt{5}}{5}$

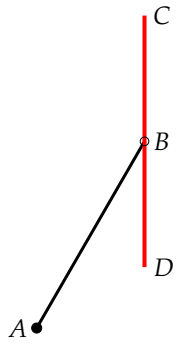
11. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

九名身高各不相同的运动员参加篮球队选拔. 他们依次从装有 3 条蓝色腕带，3 条红色腕带，3 条绿色腕带的袋子中随机抽取一条腕带，且抽取后不放回. 随后他们按腕带颜色被分成蓝队，红队，绿队. 每队中身高最高的成员被任命为队长. 问三名队长是三名最高的运动员的概率是多少?

(A) $\frac{2}{9}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{9}{28}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

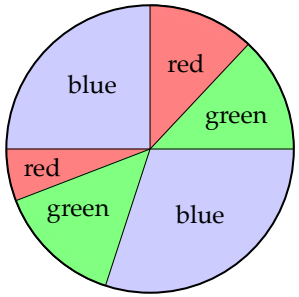
12. The windshield wiper on the driver's side of a large bus is depicted below. Arm $\overline{AB}$ pivots back and forth around point A, sweeping out an arc of 60°, symmetric about the vertical line through A. The wiper blade $\overline{CD}$ is attached to B at its midpoint and stays vertical as the arm moves. The arm is 3 feet long, and the wiper blade is 3.5 feet tall. What is the area of windshield cleaned by the wiper, in square feet, to the nearest hundredth? (Assume that the windshield is a flat vertical surface.)

大型巴士驾驶员一侧的挡风玻璃雨刷器如下图所示. 摆臂 $\overline{AB}$ 以点 A 为轴心来回摆动，扫过一个 60° 的圆弧，并且该圆弧关于通过 A 的竖直线对称. 雨刷片 $\overline{CD}$ 在其中点 B 处与摆臂相连，并在摆臂转动过程中始终保持垂直. 已知摆臂长 3 英尺，雨刷片高 3.5 英尺. 问雨刷所能清洁的挡风玻璃的面积是多少平方英尺?（答案精确到百分位，假设挡风玻璃是平坦的垂直表面.）



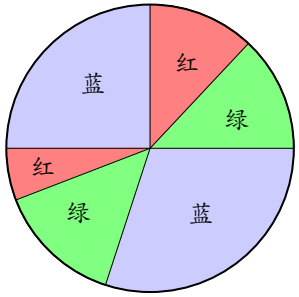
(A) 9.68 **(B)** 10.14 **(C)** 10.50 **(D)** 11.32 **(E)** 12.00

13. A circle has been divided into 6 sectors of 6 different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

一个圆被分成了 6 个大小不同的扇形. 然后将其中 2 个扇形涂成红色, 2 个涂成绿色, 2 个涂成蓝色, 使得任意两个相邻的扇形颜色都不同. 下图展示了一种这样的着色方式.



问共有多少种不同的着色方式?

(A) 12 **(B)** 16 **(C)** 18 **(D)** 24 **(E)** 28

14. Consider a decreasing sequence of n positive integers $x_1 > x_2 > x_3 > \cdots > x_n$ that satisfies the following two conditions:

- The average (arithmetic mean) of the first 2 terms in the sequence is 2025.
- For all $3 \leq k \leq n$, the average of the first k terms in the sequence is 1 less than the average of the first $k-1$ terms in the sequence.

What is the greatest possible value of n?

考虑由 n 个正整数组成的递减数列 $x_1 > x_2 > x_3 > \cdots > x_n$, 它满足以下两个条件:

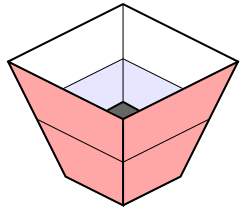
- 数列前 2 项的平均值（算术平均值）是 2025.
- 对于所有 $3 \leq k \leq n$, 数列前 k 项的平均值比前 $k-1$ 项的平均值小 1.

问 n 的最大可能值是多少?

(A) 1013 **(B)** 1014 **(C)** 1016 **(D)** 2016 **(E)** 2025

15. A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids. How many more minutes will it take to fill the remainder of the container?

如图所示, 容器的底部是 1×1 的正方形, 顶部是 3×3 的开放正方形, 还有四个全等的梯形侧面. 开始时容器是空的, 用一根水管以恒定速率向容器中注水, 需要 35 分钟才能将水注满至梯形侧面中线的高度. 问还需要多少分钟才能将容器的剩余部分注满?



(A) 70 **(B)** 85 **(C)** 90 **(D)** 95 **(E)** 105

16. An analog clock starts at midnight and runs for 2025 minutes before stopping. What is the tangent of the acute angle between the hour hand and the minute hand when the clock stops?

指针式时钟从午夜开始运行，在经过 2025 分钟后停止．问当时钟停止时，时针与分针所成锐角的正切值是多少？

(A) 0 **(B)** $\sqrt{2}-1$ **(C)** $2-\sqrt{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $3-\sqrt{2}$

17. Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

对于 3×3 方格表中的 9 个方格，每个方格都要涂上红色、蓝色或黄色，且需满足以下条件：每个红色方格都与至少一个蓝色方格有公共边，每个蓝色方格都与至少一个黄色方格有公共边，每个黄色方格都与至少一个红色方格有公共边．通过旋转、反射或者它们的组合可以互相得到的涂色方式视为相同的，问共有多少种不同的涂色方式？

(A) 3 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

18. Awnik repeatedly plays a game that has a probability of winning of $\frac{1}{2}$. The outcomes of the games are independent. What is the expected value of the number of games he will play until he has won twice or consecutively lost twice ?

Awnik 反复进行一项每局获胜概率为 $\frac{1}{2}$ 的游戏，各局游戏的结果互相独立．他在累计获胜两局或者连续输过两局时停止．问他所进行的游戏局数的期望值是多少？

(A) $\frac{21}{8}$ **(B)** $\frac{5}{2}$ **(C)** 3 **(D)** $\frac{7}{2}$ **(E)** $\frac{29}{8}$

19. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

考虑有 141 行与 91 列的矩形方格表，每个方格中可填入两个数．Horace 与 Vera 分别将从 1 到 $141 \times 91 = 12{,}831$ 的所有数填入方格中．Horace 沿水平方向填写：他在第 1 行从左到右依序填入 1 至 91，在第 2 行从左到右依序填入 92 至 182，并以此类推直至填完第 141 行．Vera 沿垂直方向填写：她在第 1 列从上到下依序填入 1 至 141，在第 2 列从上到下依序填入 142 至 282，并以此类推直至填完第 91 列．问有多少个方格中两人填入的数相同？

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 19

20. Two non-congruent triangles have the same area. Each triangle has sides of length 6 and 7, and the third side of each triangle has integer length. What is the sum of the lengths of the third sides?

两个不全等的三角形面积相同．每个三角形都有两条边，长度分别为 6 和 7，且每个三角形的第三条边的长度均为整数．问这两个第三条边的长度之和是多少？

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

21. What is the greatest possible area of the triangle in the complex plane with vertices $(1+i)z$, $(1-i)z$, and $2z^2$, where z is a complex number satisfying $|4z-2| = 1$?

在复数平面中，有一个三角形的顶点分别为 $(1+i)z$，$(1-i)z$，$2z^2$，其中 z 是满足 $|4z-2| = 1$ 的复数．问此三角形的面积最大可能是多少？

(A) $\frac{9}{32}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{3}{4}$ **(E)** 1

22. A frog hops around the circle as shown below according to the following rules.

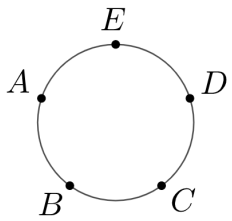
- It starts at A, and can move around only within positions A, B, C, D and E.
- If it is at A, B, C or D, then in the next step it moves to each of the two adjacent positions with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.
- If it reaches E, it will forever stay at that position.

What is the probability that the frog finally reaches E?

一只青蛙绕着下图所示的圆圈跳跃，其规则如下：

- 它从 A 出发，只在 A，B，C，D，E 这五个位置之间移动.
- 若它处于 A，B，C 或 D，它下一步移动到两个相邻位置中每一个的概率是 $\frac{1}{4}$，消失的概率是 $\frac{1}{2}$.
- 若它到达 E，则会永远停留在此位置.

问青蛙最终能到达 E 的概率是多少?



(A) $\frac{1}{4}$ **(B)** $\frac{3}{11}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

23. Let S be the set of all integers $z > 1$ such that for all pairs of nonnegative integers (x, y) with $x < y < z$, the remainder when $2025x$ is divided by z is less than the remainder when $2025y$ is divided by z. How many elements are there in S?

设 S 为所有大于 1 的整数 z 所构成的集合，且对于所有满足 $x < y < z$ 的非负整数对 (x, y)，$2025x$ 除以 z 的余数均小于 $2025y$ 除以 z 的余数. 问集合 S 中有多少个元素?

(A) 11 **(B)** 12 **(C)** 14 **(D)** 15 **(E)** 16

24. How many real numbers satisfy the equation $\csc(20\pi x) = \log_x 20$?

问有多少个实数满足方程 $\csc(20\pi x) = \log_x 20$?

(A) 199 **(B)** 200 **(C)** 398 **(D)** 399 **(E)** 400

25. Three concentric circles have radii 3, 5, and 7. An equilateral triangle with side length s has one vertex on each circle. What is largest possible value of s^2?

三个同心圆的半径分别为 3，5，7. 一个边长为 s 的正三角形，其三个顶点分别位于三个圆上. 问 s^2 的最大可能值是多少?

(A) 19 **(B)** 34 **(C)** 58 **(D)** 64 **(E)** 74