MAA AMC AMERICAN MATHEMATICS COMPETITIONS

MAA American Mathematics Competitions
44th Annual

AIME II

American Invitational Mathematics Examination II
Thursday, February 12, 2026

INSTRUCTIONS | 说明

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
   在未收到监考老师开考指示前，请不要翻开此封面。
2. This is a 15-question competition. All answers are integers ranging from 000 to 999, inclusive.
   这是一套包括15道题目的竞赛，每道题目的答案都是从000到999的整数（包括000和999）。
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
   请将每道题目的答案用＃2铅笔标注在答题卡上。请注意检查涂写的黑色圆圈的准确性，用橡皮完全擦掉错误的答案。只有恰当标注在答题卡上的答案才会被评分。
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
   评分：每道题目答对得1分， 不答得0分，答错得0分。
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, protractors, and graph paper.
   只能使用空白的草稿纸、尺子、圆规和橡皮作为辅助工具。不允许的器具包括计算器、智能手表、手机、计算设备、量角器、坐标纸。
6. Figures are not necessarily drawn to scale.
   图形不一定按比例绘制。
7. Please bubble in carefully the mobile phone number used for registration. This mobile phone number will be used as your unique identification number. The wrongly filled mobile phone number could lead to invalid test score, and such consequence has to be borne by the participant.
   请在答题卡上填涂报名所用手机号码，该手机号码将作为考生唯一识别标志，请务必认真填写，由于号码错误导致成绩无效，后果自负。
8. You will have 3 hours to complete the competition once your competition manager tells you to begin.
   监考老师宣布开始后，你将有3小时的时间完成竞赛。
9. All the problems are written in both English and Chinese. In case of any discrepancy between the two versions, the English version shall prevail.
   所有的问题同时以英文和中文表述。如果两种版本之间存在差异，以英文版本为准。

1. Find the sum of the 11th terms of all arithmetic sequences of integers that have first term equal to 4 and include both 24 and 34 as terms.

考虑首项为 4，且包含 24 和 34 的各项均为整数的等差数列，求所有这样的数列的第 11 项之和.

2. Let $ABCDE$ be a nonconvex pentagon with internal angles $\angle A = \angle E = 90^\circ$ and $\angle B = \angle D = 45^\circ$. Suppose that $DE < AB$, $AE = 20$, $BC = 14\sqrt{2}$, and points B, C, and D lie on the same side of line AE. Suppose further that AB is an integer with $AB < 2026$ and the area of pentagon $ABCDE$ is an integer multiple of 12 . Find the number of possible values of AB.

设 $ABCDE$ 为一个非凸五边形，其内角 $\angle A = \angle E = 90^\circ$，且 $\angle B = \angle D = 45^\circ$. 假设 $DE < AB$，$AE = 20$，$BC = 14\sqrt{2}$，且点 B，C，D 位于直线 AE 的同侧. 如果 AB 的长度是小于 2026 的整数，且五边形 $ABCDE$ 的面积是 12 的倍数. 求 AB 的长度的可能数值的个数.

3. For each positive integer n let $f(n)$ be the value of the base-ten numeral n viewed in base b, where b is the least integer greater than the greatest digit in n. For example, if $n = 72$, then $b = 8$, and 72 as a numeral in base 8 equals $7 \cdot 8 + 2 = 58$; therefore $f(72) = 58$. Find the number of positive integers n less than 1000 such that $f(n) = n$.

对于每个正整数 n，令 $f(n)$ 为将十进制数 n 视为 b 进制数时的值，其中 b 是比 n 的表示中最大数字还大的最小整数. 例如，如果 $n = 72$，则 $b = 8$，且 72 作为 8 进制数等于 $7 \cdot 8 + 2 = 58$，因此 $f(72) = 58$. 求小于 1000 且满足 $f(n) = n$ 的正整数 n 的个数.

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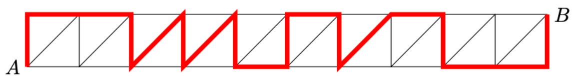
4. The figure below shows a grid of 10 squares in a row. Each square has a diagonal connecting its lower left vertex to its upper right vertex. A bug moves along the line segments from vertex to vertex, never traversing the same segment twice and never moving from right to left along a horizontal or diagonal segment. Let N be the number of paths the bug can take from the lower left corner (A) to the upper right corner (B). One such path from A to B is shown by the thick line segments in the figure. Find the sum of the digits of N.

下图显示的是一个由 10 个正方形排成一行的方格表. 每个正方形都有一条连接其左下顶点和右上顶点的对角线. 一只小虫可以沿图中的线段从一个顶点移动到另一个顶点，但不能重复经过同一条线段，并且在水平线或对角线上不能自右向左移动. 设 N 为小虫从左下角 (A) 到右上角 (B) 符合要求的不同路径的数目. 图中的粗线段显示了从 A 到 B 的一条可行路径. 求 N 的各位数字之和.



5. An urn contains n marbles. Each marble is either red or blue, and there are at least 7 marbles of each color. When 7 marbles are drawn randomly from the urn without replacement, the probability that exactly 4 of them are red equals the probability that exactly 5 of them are red. Find the sum of the four least values of n for which this is possible.

一个瓮中装有 n 个弹珠. 每个弹珠要么是红色，要么是蓝色，且每种颜色的弹珠至少有 7 个. 当从瓮中随机无放回地取出 7 个弹珠时，恰好取出 4 个红色弹珠的概率等于恰好取出 5 个红色弹珠的概率. 求此种情形可能发生的 n 的四个最小可能值之和.

6. Find the sum of all real numbers r such that there is at least one point where the circle with radius r centered at $(4,39)$ is tangent to the parabola with equation $2y = x^2 - 8x + 12$.

已知半径为 r，且圆心在 $(4,39)$ 的圆与方程为 $2y = x^2 - 8x + 12$ 的抛物线至少有一个切点，求所有这样的实数 r 的总和.

7. A standard fair six-sided die is rolled repeatedly. Each time the die reads 1 or 2, Alice gets a coin; each time it reads 3 or 4, Bob gets a coin; and each time it reads 5 or 6, Carol gets a coin. The probability that Alice and Bob each receive at least two coins before Carol receives any coins can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

一枚标准且公允的六面骰子被重复投掷.每当掷出点数为 1 或 2 时，Alice 得到一枚硬币；每当掷出点数为 3 或 4 时，Bob 得到一枚硬币；每当掷出点数为 5 或 6 时，Carol 得到一枚硬币．Alice 和 Bob 在 Carol 获得任何硬币之前各获得至少两枚硬币的概率可以表示为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数．求 $m+n$.

8. Isosceles triangle $\triangle ABC$ has $AB = BC$. Let I be the incenter of $\triangle ABC$. The perimeters of $\triangle ABC$ and $\triangle AIC$ are in the ratio $27:7$, and all the sides of both triangles have integer lengths. Find the minimum possible value of AB.

等腰三角形 $\triangle ABC$ 满足 $AB = BC$．设 I 为 $\triangle ABC$ 的内心．$\triangle ABC$ 和 $\triangle AIC$ 的周长之比为 $27:7$，且两个三角形的所有边长均为整数．求线段 AB 长度的最小可能值.

9. Let S denote the value of the infinite sum

$$\frac{1}{9} + \frac{1}{99} + \frac{1}{999} + \frac{1}{9999} + \cdots$$

Find the remainder when the greatest integer less than or equal to $10^{99}S$ is divided by 1000 .

令 S 表示无穷级数

$$\frac{1}{9} + \frac{1}{99} + \frac{1}{999} + \frac{1}{9999} + \cdots$$

的和．求小于或等于 $10^{99}S$ 的最大整数除以 1000 的余数.

10. Let $\triangle ABC$ be a triangle with D on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Let ω be the circle that passes through A and is tangent to segment $\overline{BC}$ at D. Let $E \neq A$ and $F \neq A$ be the intersections of ω with segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that $AB = 200, AC = 225$, and all of AE, AF, BD, and CD are positive integers. Find the sum of all possible values of BC.

在三角形 $\triangle ABC$ 中，D 在 $\overline{BC}$ 上使得 $\overline{AD}$ 平分 $\angle BAC$．令 ω 为经过点 A 且在点 D 处与线段 $\overline{BC}$ 相切的圆．令 $E \neq A$ 和 $F \neq A$ 分别为 ω 与线段 $\overline{AB}$ 和 $\overline{AC}$ 的交点．假设 $AB = 200$，$AC = 225$，且 AE，AF，BD，CD 的长度均为正整数．求 BC 的长度的所有可能值之和.

11. Find the greatest integer n such that the cubic polynomial

$$x^3 - \frac{n}{6}x^2 + (n-11)x - 400$$

has roots α^2, β^2, and γ^2, where α, β, and γ are complex numbers, and there are exactly seven distinct possible values for $\alpha + \beta + \gamma$.

求最大整数 n，使得三次多项式

$$x^3 - \frac{n}{6}x^2 + (n-11)x - 400$$

的根为 α^2，β^2，γ^2，其中 α，β，γ 是复数，且 $\alpha + \beta + \gamma$ 恰好有七个不同的可能值.

12. Call finite sets of integers S and T *cousins* if

- S and T have the same number of elements,
- S and T are disjoint, and
- the elements of S can be paired with the elements of T so that the elements in each pair differ by exactly 1 .

For example, $\{1,2,5\}$ and $\{0,3,4\}$ are cousins. Suppose that the set S has exactly 2020 cousins. Find the least number of elements the set S can have.

如果两个有限整数集合 S 和 T 满足:

- S 和 T 具有相同数量的元素,
- S 和 T 不相交, 且
- S 中的元素可以与 T 中的元素一一配对, 使得每对元素之差的绝对值恰好为 1, 则称 S 和 T 为“堂兄”集合.

例如, $\{1,2,5\}$ 和 $\{0,3,4\}$ 是堂兄集合. 假设集合 S 恰好有 2020 个堂兄集合. 求集合 S 的元素个数的最小值.

13. Consider a tetrahedron with two isosceles triangle faces with side lengths $5\sqrt{10}, 5\sqrt{10}$, and 10 and two isosceles triangle faces with side lengths $5\sqrt{10}, 5\sqrt{10}$, and 18 . The four vertices of the tetrahedron lie on a sphere with center S, and the four faces of the tetrahedron are tangent to a sphere with center R. The distance RS can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

考虑一个四面体, 它有两个边长为 $5\sqrt{10}$, $5\sqrt{10}$ 和 10 的等腰三角形面, 以及两个边长为 $5\sqrt{10}$, $5\sqrt{10}$ 和 18 的等腰三角形面. 该四面体的四个顶点位于一个球心为 S 的球面上, 且该四面体的四个面均与一个球心为 R 的球面相切. 距离 RS 可以表示为 $\frac{m}{n}$, 其中 m 和 n 是互质的正整数. 求 $m+n$.

14. For integers a and b, let $a \circ b = a - b$ if a is odd and b is even, and $a+b$ otherwise. Find the number of sequences $a_1, a_2, a_3, \ldots, a_n$ of positive integers such that

$$a_1 + a_2 + a_3 + \cdots + a_n = 12 \quad \text{and} \quad a_1 \circ a_2 \circ a_3 \circ \cdots \circ a_n = 0$$

where the operations are performed from left to right; that is, $a_1 \circ a_2 \circ a_3$ means $(a_1 \circ a_2) \circ a_3$.

对于整数 a 和 b, 定义新运算 $a \circ b$ 为: 如果 a 是奇数且 b 是偶数, 则 $a \circ b = a - b$, 否则 $a \circ b = a + b$. 求满足以下条件的由正整数组成的序列 $a_1, a_2, a_3, \ldots, a_n$ 的个数:

$$a_1 + a_2 + a_3 + \cdots + a_n = 12 \quad \text{且} \quad a_1 \circ a_2 \circ a_3 \circ \cdots \circ a_n = 0$$

其中运算从左到右进行; 即 $a_1 \circ a_2 \circ a_3$ 表示 $(a_1 \circ a_2) \circ a_3$.

15. Find the number of ordered 7-tuples $(a_1, a_2, a_3, \ldots, a_7)$ having the following properties:

- $a_k \in \{1,2,3\}$ for all k.
- $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is a multiple of 3 .
- $a_1a_2a_4 + a_2a_3a_5 + a_3a_4a_6 + a_4a_5a_7 + a_5a_6a_1 + a_6a_7a_2 + a_7a_1a_3$ is a multiple of 3.

求满足以下性质的有序 7 元组 $(a_1, a_2, a_3, \ldots, a_7)$ 的个数:

- 对于所有 k, $a_k \in \{1,2,3\}$.
- $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ 是 3 的倍数.
- $a_1a_2a_4 + a_2a_3a_5 + a_3a_4a_6 + a_4a_5a_7 + a_5a_6a_1 + a_6a_7a_2 + a_7a_1a_3$ 是 3 的倍数.

AIME 答案留存页

*请认真填写报名手机号、姓名信息，及作答答案，完成后交由监考老师保存；
*此页仅限答题卡遗失时使用，其他时候均以答案以答题卡涂卡结果为准

报名手机号	
中文姓名	
英文姓名	
题号	**答案（0-999 之间的三位整数，缺位用 0 补充，如 001,010）**
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