2020 AMC10B

Problem 1

What is the value of

$$1-(-2)-3-(-4)-5-(-6)?$$

下列算式的值为多少

$$1-(-2)-3-(-4)-5-(-6)?$$

$\textbf{(A)}\ -20 \qquad \textbf{(B)}\ -3 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 21$

Problem 2

Carl has 5 cubes each having side length 1, and Kate has 5 cubes each having side length 2. What is the total volume of these 10 cubes?

Carl 有 5 个边长为 1 的立方体，Kate 有 5 个边长为 2 的立方体。这 10 个立方体的总体积是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 45$

Problem 3

The ratio of w to x is $4:3$, the ratio of y to z is $3:2$, and the ratio of z to x is $1:6$. What is the ratio of w to y?

w 比 *x* 的比值为 4:3，*y* 比 *z* 的比值为 3:2，*z* 比 *x* 的比值为 1:6。则 *w* 比 *y* 是多少?

$\textbf{(A)}\ 4:3 \qquad \textbf{(B)}\ 3:2 \qquad \textbf{(C)}\ 8:3 \qquad \textbf{(D)}\ 4:1 \qquad \textbf{(E)}\ 16:3$

Problem 4

The acute angles of a right triangle are a° and b°, where $a>b$ and both a and b are prime numbers. What is the least possible value of b?

一个直角三角形的两个锐角的度数分别为 a° 和 b° ，其中 $a>b$，且 a 和 b 均为质数。则 b 的最小值是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 11$

Problem 5

How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.)

把 1 块棕色的瓷砖，1 块紫色瓷砖，2 块绿色瓷砖和 3 块黄色瓷砖从左到右排成一排，一共有多少种不同的排列？（颜色相同的瓷砖不加以区分）

(A) 210 **(B)** 420 **(C)** 630 **(D)** 840 **(E)** 1050

Problem 6

Driving along a highway, Megan noticed that her odometer showed 15951 (miles). This number is a palindrome-it reads the same forward and backward. Then 2 hours later, the odometer displayed the next higher palindrome. What was her average speed, in miles per hour, during this 2-hour period?

当 Megan 在高速公路上驾驶时，她发现车上的里程表的读数为 15951（英里）。这个数是个回环数：一个从前往后读和从后往前读，读数都一样的数。2 小时后，里程表读数为下一个较大的回环数。那么她在这 2 小时驾驶中，平均速度为多少英里每小时？

(A) 50 **(B)** 55 **(C)** 60 **(D)** 65 **(E)** 70

Problem 7

How many positive even multiples of 3 less than 2020 are perfect squares?

小于 2020 的所有 3 的倍数且是偶数的正整数中，有多少个数是完全平方数？

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

Problem 8

Points P and Q lie in a plane with $PQ = 8$. How many locations for point R in this plane are there such that the triangle with vertices P, Q, and R is a right triangle with area 12 square units?

点 *P* 和 *Q* 在一平面内且 *PQ*=8。在这个平面内有多少个这样的点 *R*，使得以 *P*，*Q* 和 *R* 为顶点的三角形是个直角三角形，且面积为 12?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 12

Problem 9

For how many ordered pairs of integers (x, y) satisfy the equation $x^{2020} + y^2 = 2y$?

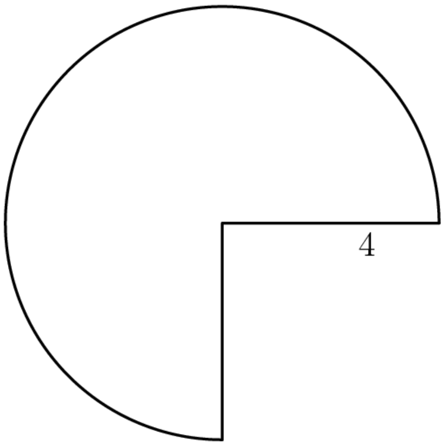
有多少对有序整数对（x，y）满足下面方程 $x^{2020} + y^2 = 2y$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

Problem 10

A three-quarter sector of a circle of radius 4 inches along with its interior is the 2-D net that forms the lateral surface area of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches?

如下图所示，一个半径为 4 英寸的圆的四分之三部分形成的扇形及其内部，可以通过将它的两个半径粘在一起卷成一个正圆锥体的侧面。则这个圆锥体的体积为多少立方英寸？



(A) $3\pi\sqrt{5}$ **(B)** $4\pi\sqrt{3}$ **(C)** $3\pi\sqrt{7}$ **(D)** $6\pi\sqrt{3}$ **(E)** $6\pi\sqrt{7}$

Problem 11

Ms. Carr asks her students to select 5 of the 10 books to read on her classroom reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?

Carr 女士要求她的学生们从一张列有 10 本书的书单上任选 5 本阅读。Harold 从书单上随机选择了 5 本，Betty 也是如此。那么，恰好有 2 本书被他俩都选中的概率是多少？

(A) $\frac{1}{8}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{14}{45}$ **(D)** $\frac{25}{63}$ **(E)** $\frac{1}{2}$

Problem 12

The decimal representation of $\frac{1}{20^{20}}$ consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?

$\frac{1}{20^{20}}$的小数表示在小数点后是一连串的 0，紧接着是 1 个 9，再接着是几个其他数字。则在小数点后遇到第一个 9 之前，一共有多少个初始的 0?

$\textbf{(A) } 23 \qquad \textbf{(B) } 24 \qquad \textbf{(C) } 25 \qquad \textbf{(D) } 26 \qquad \textbf{(E) } 27$

Problem 13

Andy the Ant lives on a coordinate plane and is currently at $(-20, 20)$ facing east (that is, in the positive x-direction). Andy moves 1 unit and then turns 90° left. From there, Andy moves 2 units (north) and then turns 90° left. He then moves 3 units (west) and again turns 90° left. Andy continues his progress, increasing his distance each time by 1 unit and always turning left. What is the location of the point at which Andy makes the 2020th left turn?

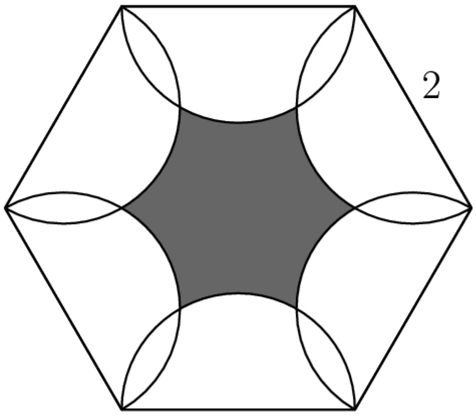
蚂蚁 Andy 住在坐标平面上，目前位于（−20，20）面朝东（即，面朝 *x* 轴正半轴方向）。Andy 首先向前移动一个单位，然后向左转了 90°。从所处的地点开始又移动了 2 个单位（向北），然后向左转了 90°。之后他又向前移动了 3 个单位（向西），然后再次向左转了 90°。Andy 以此方式不停地移动和转动，每次移动的距离都增加 1，且总是向左转动，当 Andy 第 2020 次向左转时，他所在的点的位置是多少？

$\textbf{(A) } (-1030, -994) \qquad \textbf{(B) } (-1030, -990) \qquad \textbf{(C) } (-1026, -994) \qquad \textbf{(D) } (-1026, -990) \qquad \textbf{(E) } (-1022, -994)$

Problem 14

As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region — inside the hexagon but outside all of the semicircles?

如下图所示，6 个半圆位于一个边长为 2 的正六边形的内部，且半圆的直径和六边形的边重合，那么，阴影部分（位于六边形内部但在半圆外部的区域）的面积是多少？



(A) $6\sqrt{3}-3\pi$ **(B)** $\frac{9\sqrt{3}}{2}-2\pi$ **(C)** $\frac{3\sqrt{3}}{2}-\frac{\pi}{3}$ **(D)** $3\sqrt{3}-\pi$ **(E)** $\frac{9\sqrt{3}}{2}-\pi$

Problem 15

Steve wrote the digits 1, 2, 3, 4, and 5 in order repeatedly from left to right, forming a list of $10,000$ digits, beginning $123451234512\ldots.$ He then erased every third digit from his list (that is, the 3rd, 6th, 9th, $\ldots$ digits from the left), then erased every fourth digit from the resulting list (that is, the 4th, 8th, 12th, $\ldots$ digits from the left in what remained), and then erased every fifth digit from what remained at that point. What is the sum of the three digits that were then in the positions $2019, 2020, 2021$?

Steve 从左往右依次重复地写 1，2，3，4，5，最终形成一个有 10，000 个数字的序列，以 123451234512...开始，然后他把这个序列里的每第 3 个数字擦除（即，从左往右依次擦除第 3，第 6，第 9，…个数字），然后把得到的序列里的每第 4 个数字擦除（即，从第一次擦除后所得的序列中，从左往右依次擦除第 4，第 8，第 12，…个数字），之后再从剩下的序列里，擦除每第 5 个数字，操作完之后，位于第 2019，2020 和第 2021 个位置上的 3 个数字之和为多少？

(A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Problem 16

Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

Bela 和 Jenn 在实数轴上［0，*n*］这个区间内做游戏，这里 *n* 是一个大于 4 的固定的整数。他们轮流做游戏，Bela 先开始。首先，Bela 从［0，*n*］区间内任意选择一个实数，接下来轮到某人开始，那个人选择的这个实数必须要和之前两人已经选过的所有实数的距离都大于 1。若有选手无法选出这样的数，那么这个选手就输了。若两人都使用最佳策略，问哪个选手会赢得这场游戏？

（A）Bela will always win | 总是 Bela 赢

（B）Jenn will always win | 总是 Jenn 赢

（C）Bela will win if and only ifn is odd | 当且仅当 n 是奇数时，Bela 会赢

（D）Jenn will win if and only ifn is odd | 当且仅当 n 是奇数时，Jenn 会赢

（E）Jenn will win if and only if n>8 | 当且仅当 n>8 时，Jenn 会赢

Problem 17

There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

10 个人站成一个圆形，相互之间间距相等。每个人都认识其余 9 个人中的 3 个人，在他两旁的 2 个人，以及他对面的那个人。若把这 10 个人分成 5 对，要求每对的两个人都相互认识，一共有多少种方法？

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

Problem 18

An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?

一个罐子里装着一个红球和一个蓝球。旁边还有一个盒子，里面装有若干红球和蓝球。George 做如下操作 4 次：他随机的从罐子里抽取一个球，然后从盒子里拿一个同色的球，最后把这 2 个匹配的球放回罐子里。经过 4 次这样的重复操作后，罐子里总共有 6 个球，问罐子里恰好有 3 个红球和 3 个蓝球的概率是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 19

In a certain card game, a player is dealt a hand of 10 cards from a deck of 52 distinct cards. The number of distinct (unordered) hands that can be dealt to the player can be written as $158A00A4AA0$. What is the digit A?

在某场棋牌游戏中，一个选手从一叠 52 张不同的扑克牌中抽取 10 张牌。这个选手抽取这 10 张牌的不同的（不考虑抽牌顺序）方法数可以被写成 158*A*00*A*4*AA*0，则数字 *A* 是多少?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 20

Let B be a right rectangular prism (box) with edges lengths $1, 3,$ and 4, together with its interior. For real $r \geq 0$, let $S(r)$ be the set of points in 3-dimensional space that lie within a distance r of some point in B. The volume of $S(r)$ can be expressed as $ar^3 + br^2 + cr + d$, where $a, b, c,$ and d are positive real numbers. What is $\frac{bc}{ad}$?

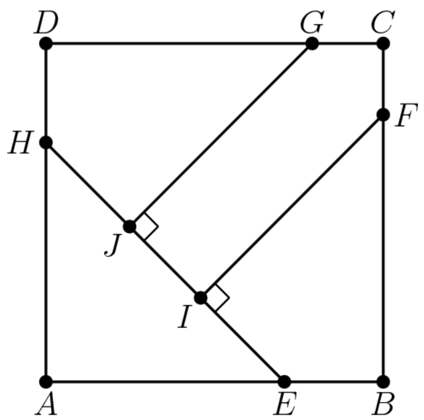
B 是一个棱长分别为 1，3 和 4 的长方体（盒子）及其内部区域。对于实数 *r*⩾0，定$S(r)$为三维空间中，到 *B* 中某些点的距离不超过 *r* 的所有点组成的集合。$S(r)$的体积可以表示为$ar^3 + br^2 + cr + d$，这里 *a*，*c* 和 *d* 都是正实数，求$\frac{bc}{ad}$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 19 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 38$

Problem 21

In square $ABCD$, points E and H lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH.$ Points F and G lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points I and J lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle AEH, quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area $1.$ What is FI^2?

在正方形 *ABCD* 中，点 *E* 和 *H* 分别位于线段$\overline{AB}$和$\overline{DA}$上，满足 *AE*=*AH*。点 *F* 和 *G* 分别位于线段$\overline{BC}$和$\overline{CD}$上，且点 *I* 和 *J* 位于线段$\overline{EH}$上，满足$\overline{FI} \perp \overline{EH}$，$\overline{GJ} \perp \overline{EH}$。如下图所示，三角形 *AEH*，四边形 *BFIE*，四边形 *DHJG* 和五边形 *FCGJI* 面积均为 1。问FI^2是多少?



$\textbf{(A)}\ \frac{7}{3} \qquad \textbf{(B)}\ 8-4\sqrt{2} \qquad \textbf{(C)}\ 1+\sqrt{2} \qquad \textbf{(D)}\ \frac{7}{4}\sqrt{2} \qquad \textbf{(E)}\ 2\sqrt{2}$

Problem 22

What is the remainder when $2^{202}+202$ is divided by $2^{101}+2^{51}+1$?

$2^{202}+202$除以$2^{101}+2^{51}+1$所得的余数是多少？

(A) 100 **(B)** 101 **(C)** 200 **(D)** 201 **(E)** 202

Problem 23

Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1), B(-1,1), C(-1,-1),$ and $D(1,-1)$. Consider the following four transformations:

- $L,$ a rotation of 90° counterclockwise around the origin;
- $R,$ a rotation of 90° clockwise around the origin;
- $H,$ a reflection across the x-axis; and
- $V,$ a reflection across the y-axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1,1)$ to $(-1,-1)$ and would send the vertex B at $(-1,1)$ to itself. How many sequences of 20 transformations chosen from $\{L,R,H,V\}$ will send all of the labeled vertices back to their original positions? (For example, R,R,V,H is one sequence of 4 transformations that will send the vertices back to their original positions.)

在坐标平面内，正方形 *ABCD* 的 4 个顶点的坐标分别为 *A*（1，1），*B*（−1，1），*C*（−1，−1），*D*（1，−1）。考虑如下 4 个变换：*L*，绕着原点逆时针旋转 90°；*R*，绕着原点顺时针旋转 90°；*H*，关于 *x* 轴对称；*V*，关于 *y* 轴对称。

这 4 个变换中的每一个都将正方形映射回自身，但是顶点的位置会发生改变，例如，依次使用变换 *R* 和 *V* 后，顶点 *A* 的位置就从（1，1）变到了（−1，−1），同时顶点 *B*（−1，1）最终又回到了它原来的位置。从集合｛*L*，*R*，*H*，*V*｝中依次选择 20 个变换组成一个序列，问有多少种这样的序列，在依次经过这个序列中的 20 次变换后，正方形的顶点又回到它们各自原来的位置？（例如，*R*，*R*，*V*，*H* 就是这样一组满足条件的 4 次变换的序列，能够把顶点都映射回它们各自原来的位置）

(A) 2^{37} **(B)** $3\cdot 2^{36}$ **(C)** 2^{38} **(D)** $3\cdot 2^{37}$ **(E)** 2^{39}

Problem 24

How many positive integers n satisfy $\dfrac{n+1000}{70} = \lfloor \sqrt{n} \rfloor$? (Recall that $\lfloor x \rfloor$ is the greatest integer not exceeding x.)

有多少个正整数 n 满足$\dfrac{n+1000}{70} = \lfloor \sqrt{n} \rfloor$?（$\lfloor x \rfloor$表示不超过 x 的最大整数.）

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Problem 25

Let $D(n)$ denote the number of ways of writing the positive integer n as a product

$$n = f_1 \cdot f_2 \cdots f_k,$$

where $k \ge 1$, the f_i are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6, $2 \cdot 3$, and $3 \cdot 2$, so $D(6) = 3$. What is $D(96)$?

$D(n)$表示把正整数 n 写成以下乘积形式的方法数：

$$n = f_1 \cdot f_2 \cdots f_k,$$

其中 k≥1，f_i都是严格大于 1 的整数，并且这些因子列出的先后顺序是需要考虑的（即，两种表示方法若只是因子的顺序不同，那么这两种表示方法被看作是不一样的表示方法）。例如，数字 6 可以写成 6，$2 \cdot 3$和$3 \cdot 2$，因此 D（6）=3。问$D(96)$是多少？

$\textbf{(A)}\ 112 \qquad \textbf{(B)}\ 128 \qquad \textbf{(C)}\ 144 \qquad \textbf{(D)}\ 172 \qquad \textbf{(E)}\ 184$