2020 AMC 12A

Problem 1

Carlos took 70% of a whole pie. Maria took one third of the remainder. What portion of the whole pie was left?

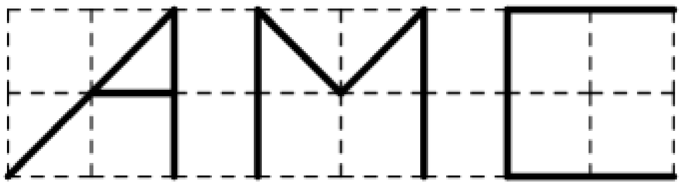
Carlos 拿走了馅饼的 70%，Maria 拿走了剩下的三分之一。问最后剩下原来馅饼的百分之多少？

$\textbf{(A)}\ 10\%\qquad\textbf{(B)}\ 15\%\qquad\textbf{(C)}\ 20\%\qquad\textbf{(D)}\ 30\%\qquad\textbf{(E)}\ 35\%$

Problem 2

The acronym AMC is shown in the rectangular grid below with grid lines spaced 1 unit apart. In units, what is the sum of the lengths of the line segments that form the acronym AMC$?$

缩略词 AMC 如下显示在直角网格里，网格线之间的距离是 1 个单位。那么形成缩略词 AMC 的所有线段长度之和是多少？



$\textbf{(A)}\ 17\qquad\textbf{(B)}\ 15+2\sqrt{2}\qquad\textbf{(C)}\ 13+4\sqrt{2}\qquad\textbf{(D)}\ 11+6\sqrt{2}\qquad\textbf{(E)}\ 21$

Problem 3

A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid $\$0.50$ per mile, and her only expense is gasoline at $\$2.00$ per gallon. What is her net rate of pay, in dollars per hour, after this expense?

一位司机以 60 英里每小时的速度开了 2 小时，在此期间她的车每消耗 1 加仑的汽油可以开 30 英里。她开 1 英里的收入是 0.5 美元，并且她的唯一支出是汽油支出，为每加仑 2 美元，在扣除掉此支出后，她的单位时间净收入是每小时多少美元？

$\textbf{(A)}\ 20\qquad\textbf{(B)}\ 22\qquad\textbf{(C)}\ 24\qquad\textbf{(D)}\ 25\qquad\textbf{(E)}\ 26$

Problem 4

How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

有多少个各个位上数字为偶数的 4 位正整数（即 1000 和 9999 之间的整数，包含 1000 和 9999）能被 5 整除？

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 200 \qquad \textbf{(E)}\ 500$

Problem 5

The 25 integers from -10 to $14,$ inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

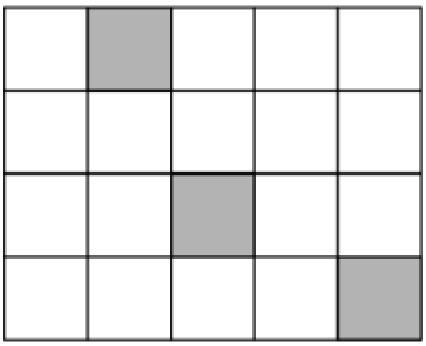
从-10 到 14（包含-10 和 14）之间的 25 个整数可以放进一个 5×5 的正方形中，满足正方形的每一行，每一列以及每个主对角线的数字之和都相等，则这个共同的和为多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 50$

Problem 6

In the plane figure shown below, 3 of the unit squares have been shaded. What is the least number of additional unit squares that must be shaded so that the resulting figure has two lines of symmetry?

在如下所示的平面图中，其中有 3 个单位正方形被涂成阴影。还至少需要把多少个正方形涂成阴影以保证形成的图案有两条对称轴？



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 7

Seven cubes, whose volumes are 1, 8, 27, 64, 125, 216, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

7 个体积分别为 1，8，27，64，125，216 和 343 的立方体竖直堆叠起来形成一座塔，且从塔底往上，立方体的体积依次减少，除了底部的立方体，其余立方体的底面恰好完全位于它下面的立方体的顶面上。这座塔的表面积（包括塔的底面）是多少？

$\textbf{(A)}\ 644 \qquad \textbf{(B)}\ 658 \qquad \textbf{(C)}\ 664 \qquad \textbf{(D)}\ 720 \qquad \textbf{(E)}\ 749$

Problem 8

What is the median of the following list of 4040 numbers?
$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$

下列 4040 个数的中位数是多少？

$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$

$\textbf{(A)}\ 1974.5 \qquad \textbf{(B)}\ 1975.5 \qquad \textbf{(C)}\ 1976.5 \qquad \textbf{(D)}\ 1977.5 \qquad \textbf{(E)}\ 1978.5$

Problem 9

How many solutions does the equation $\tan(2x) = \cos(\frac{x}{2})$ have on the interval $[0, 2\pi]$?

方程$\tan(2x) = \cos(\frac{x}{2})$在区间 $[0, 2\pi]$ 上有多少个解？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 10

There is a unique positive integer n such that $\log_2{(\log_{16}{n})} = \log_4{(\log_4{n})}$. What is the sum of the digits of n?

存在一个唯一的正整数 n，满足$\log_2{(\log_{16}{n})} = \log_4{(\log_4{n})}$.问 n 的各个位上的数字之和是多少？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Problem 11

A frog sitting at the point $(1, 2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0,0), (0,4), (4,4),$ and $(4,0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

一只位于点（1，2）处的青蛙开始一系列跳跃，每一次跳跃都和坐标轴平行且跳跃长度为1，跳跃的方向（向上，向下，向右或向左）随机选择。当青蛙到达以（0，0），（0，4），（4，4），和（4，0）为顶点的正方形的一条边上时，跳跃停止。问跳跃停止时，青蛙落在正方形的一条竖直的边上的概率为多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{5}{8} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{3}{4} \qquad \textbf{(E)}\ \frac{7}{8}$

Problem 12

Line ℓ in the coordinate plane has the equation $3x - 5y + 40 = 0$. This line is rotated 45° counterclockwise about the point $(20, 20)$ to obtain line k. What is the x-coordinate of the x-intercept of line k?

坐标平面内直线的方程为$3x - 5y + 40 = 0$，这条线绕着（20，20）逆时针旋转 45°得到直线 k，那么直线 k 的 x 截距的 x 坐标是多少？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 30$

Problem 13

There are integers a, b, and c, each greater than 1, such that $\sqrt[a]{N\sqrt[b]{N\sqrt[c]{N}}} = \sqrt[36]{N^{25}}$ for all $N > 1$. What is b?

大于 1 的整数 a，b 和 c 满足$\sqrt[a]{N\sqrt[b]{N\sqrt[c]{N}}} = \sqrt[36]{N^{25}}$，对于所有的$N > 1$都成立，那么 b 是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 14

Regular octagon $ABCDEFGH$ has area n. Let m be the area of quadrilateral $ACEG$. What is $\frac{m}{n}$?

正八边形 $ABCDEFGH$ 的面积为 n，令 m 为四边形 $ACEG$ 的面积。那么$\frac{m}{n}$是多少？

$\textbf{(A)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{3\sqrt{2}}{5} \qquad \textbf{(E)}\ \frac{2\sqrt{2}}{3}$

Problem 15

In the complex plane, let A be the set of solutions to $z^3 - 8 = 0$ and let B be the set of solutions to $z^3 - 8z^2 - 8z + 64 = 0$. What is the greatest distance between a point of A and a point of B?

在复平面内，集合 A 是方程$z^3 - 8 = 0$的解所形成的点集，集合 B 是方程$z^3 - 8z^2 - 8z + 64 = 0$的解所形成的点集，那么 A 中的点到 B 中的点的距离的最大值是多少？

$\textbf{(A)}\ 2\sqrt{3} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 2\sqrt{21} \qquad \textbf{(E)}\ 9 + \sqrt{3}$

Problem 16

A point is chosen at random within the square in the coordinate plane whose vertices are $(0,0), (2020,0), (2020,2020),$ and $(0,2020)$. The probability that the point is within d units of a lattice point is $\frac{1}{2}$. (A point (x,y) is a lattice point if x and y are both integers.) What is d to the nearest tenth?

从坐标平面内以（0，0），（2020，0），（2020，2020）和（0，2020）为顶点的正方形内部随机选择一个点，这个点和格点距离不超过 *d* 的概率为$\frac{1}{2}$（若 *x* 和 *y* 都是整数，那么点（*x*，*y*）称作格点）。那么 d 的值是多少（保留小数点后一位）？

(A) 0.3 **(B)** 0.4 **(C)** 0.5 **(D)** 0.6 **(E)** 0.7

Problem 17

The vertices of a quadrilateral lie on the graph of $y=\ln x$, and the x-coordinates of these vertices are consecutive positive integers. The area of the quadrilateral is $\ln\frac{91}{90}$. What is the x-coordinate of the leftmost vertex?

一个四边形的四个顶点位于函数$y=\ln x$的图像上，并且这四个顶点的 *x* 坐标是连续的正整数。已知这个四边形的面积是$\ln\frac{91}{90}$，那么最左边的那个顶点的 *x* 坐标是多少？

(A) 6 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

Problem 18

Quadrilateral $ABCD$ satisfies $\angle ABC=\angle ACD=90^\circ, AC=20$, and $CD=30$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E, and $AE=5$. What is the area of quadrilateral $ABCD$?

四边形 *ABCD* 满足∠*ABC*=∠*MCD*=90°，*AC*=20，且 *CD*=30，对角线 *AC* 和 *BD* 交于点 E，且 *AE*=5，问四边形 ABCD 的面积为多少？

(A) 330 **(B)** 340 **(C)** 350 **(D)** 360 **(E)** 370

Problem 19

There exists a unique strictly increasing sequence of nonnegative integers $a_1 < a_2 < \ldots < a_k$ such that $\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}$. What is k?

存在唯一一个严格递增的非负整数组成的数列$a_1 < a_2 < \ldots < a_k$，满足

$$\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}.$$

则 k 是多少？

(A) 117　**(B)** 136　**(C)** 137　**(D)** 273　**(E)** 306

Problem 20

Let T be the triangle in the coordinate plane with vertices $(0,0)$, $(4,0)$, and $(0,3)$. Consider the following five isometries (rigid transformations) of the plane: rotations of 90°, 180°, and 270° counterclockwise around the origin, reflection across the x-axis, and reflection across the y-axis. How many of the 125 sequences of three of these transformations (not necessarily distinct) will return T to its original position? (For example, a 180° rotation, followed by a reflection across the x-axis, followed by a reflection across the y-axis will return T to its original position, but a 90° rotation, followed by a reflection across the x-axis, followed by another reflection across the x-axis will not return T to its original position.)

T 是坐标平面内以（0，0），（4，0）和（0，3）为顶点的三角形。考虑如下 5 种平面变换：统着原点逆时针旋转 90°，180°和 270°，关于 x 轴作对称，关于 y 轴作对称。从这 5 种变换中选择 3 种变换（不必要不同），一共可以组成 125 种这样的 3 次变换，这 125 种变换中，有多少种可以将 T 再变换回它原来的位置？（例如，180°旋转后关于 x 轴对称，接着是关于 y 轴对称，将把 T 再变换回它原来的位置，但是 90°旋转后关于 x 轴对称，接着再关于 x 轴作对称，将不能把 T 变换回它原来的位置）

(A) 12　**(B)** 15　**(C)** 17　**(D)** 20　**(E)** 25

Problem 21

How many positive integers n are there such that n is a multiple of 5, and the least common multiple of $5!$ and n equals 5 times the greatest common divisor of $10!$ and n?

正整数 *n* 是 5 的倍数，并且 5！ 和 *n* 的最小公倍数等于 10！ 和 *n* 的最大公约数的 5 倍。问这样的 *n* 有多少个？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 72$

Problem 22

Let (a_n) and (b_n) be the sequences of real numbers such that $(2+i)^n = a_n + b_n i$ for all integers $n \geq 0$, where $i = \sqrt{-1}$. What is $\sum_{n=0}^{\infty} \frac{a_n b_n}{7^n}$?

(a_n)和(b_n)均为实数数列，满足$(2+i)^n = a_n + b_n i$，对于所有的整数$n \geq 0$都成立，这里$i = \sqrt{-1}$，那么下式的值是多少

$$\sum_{n=0}^{\infty} \frac{a_n b_n}{7^n}?$$

$\textbf{(A)}\ \frac{3}{8} \qquad \textbf{(B)}\ \frac{7}{16} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{9}{16} \qquad \textbf{(E)}\ \frac{4}{7}$

Problem 23

Jason rolls three fair standard six-sided dice. Then he looks at the rolls and chooses a subset of the dice (possibly empty, possibly all three dice) to reroll. After rerolling, he wins if and only if the sum of the numbers face up on the three dice is exactly 7. Jason always plays to optimize his chances of winning. What is the probability that he chooses to reroll exactly two of the dice?

Jason 扔了 3 个标准的六面骰子。当他看到投出的结果后，再决定选择其中一部分骰子（可能都不选，可能 3 个骰子都选）重新再扔一次。他第二次扔完后，当且仅当 3 个骰子面朝上的数字之和恰好为 7 时他才能赢。在总是设法使得自己赢的概率最大的情况下，.Jason 恰好选择其中 2 个骰子重新扔的概率是多少？

$\textbf{(A)}\ \frac{7}{36} \qquad \textbf{(B)}\ \frac{5}{24} \qquad \textbf{(C)}\ \frac{2}{9} \qquad \textbf{(D)}\ \frac{17}{72} \qquad \textbf{(E)}\ \frac{1}{4}$

Problem 24

Suppose that $\triangle ABC$ is an equilateral triangle of side length s, with the property that there is a unique point P inside the triangle such that $AP = 1$, $BP = \sqrt{3}$, and $CP = 2$. What is s?

假设△ABC 是个边长为s的等边三角形，并具有这样的性质：这个三角形的内部有唯一一个点 P，满足 *AP*=1，*BP*=3，*CP*=2，求s是多少？

$\textbf{(A)}\ 1+\sqrt{2} \qquad \textbf{(B)}\ \sqrt{7} \qquad \textbf{(C)}\ \frac{8}{3} \qquad \textbf{(D)}\ \sqrt{5+\sqrt{5}} \qquad \textbf{(E)}\ 2\sqrt{2}$

Problem 25

The number $a = \frac{p}{q}$, where p and q are relatively prime positive integers, has the property that the sum of all real numbers x satisfying $\lfloor x \rfloor \cdot \{x\} = a \cdot x^2$ is 420, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and $\{x\} = x - \lfloor x \rfloor$ denotes the fractional part of x. What is $p+q$?

有一个数字$a = \frac{p}{q}$，其中p和q是互质的正整数，*a* 具有如下性质：满足方程$\lfloor x \rfloor \cdot \{x\} = a \cdot x^2$的所有实数解 *x* 之和为 420，这里$\lfloor x \rfloor$表示小于或等于 *x* 的最大整数，并且$\{x\} = x - \lfloor x \rfloor$表示 *x* 的分数部分，那么$p$+$q$是多少？

$\textbf{(A)}\ 245 \qquad \textbf{(B)}\ 593 \qquad \textbf{(C)}\ 929 \qquad \textbf{(D)}\ 1331 \qquad \textbf{(E)}\ 1332$