2020 AMC 12B

Problem 1

What is the value in simplest form of the following expression?

$$\sqrt{1}+\sqrt{1+3}+\sqrt{1+3+5}+\sqrt{1+3+5+7}$$

下面表达式化简后的值为多少？

$$\sqrt{1}+\sqrt{1+3}+\sqrt{1+3+5}+\sqrt{1+3+5+7}$$

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 4+\sqrt{7}+\sqrt{10} \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 4+3\sqrt{3}+2\sqrt{5}+\sqrt{7}$

Problem 2

What is the value of the following expression?

$$\frac{100^2-7^2}{70^2-11^2}\cdot\frac{(70-11)(70+11)}{(100-7)(100+7)}$$

下面表达式的值是多少?

$$\frac{100^2-7^2}{70^2-11^2}\cdot\frac{(70-11)(70+11)}{(100-7)(100+7)}$$

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{9951}{9950} \qquad \textbf{(C)}\ \frac{4780}{4779} \qquad \textbf{(D)}\ \frac{108}{107} \qquad \textbf{(E)}\ \frac{81}{80}$

Problem 3

The ratio of w to x is $4:3$, the ratio of y to z is $3:2$, and the ratio of z to x is $1:6$. What is the ratio of w to y?

w 比 *x* 的比值为 4:3，*y* 比 *z* 的比值为 3:2，*z* 比 *x* 的比值为 1:6，则 *w* 比 *y* 的比值是多少？

$\textbf{(A)}\ 4:3 \qquad \textbf{(B)}\ 3:2 \qquad \textbf{(C)}\ 8:3 \qquad \textbf{(D)}\ 4:1 \qquad \textbf{(E)}\ 16:3$

Problem 4

The acute angles of a right triangle are a° and b°, where $a > b$ and both a and b are prime numbers. What is the least possible value of b?

一个直角三角形的两个锐角的度数分别为 *a*°和 *b*°，其中 *a*>*b*，且 *a* 和 *b* 均为质数，则 *b* 的最小值是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 11$

Problem 5

Teams A and B are playing in a basketball league where each game results in a win for one team and a loss for the other team. Team A has won $\tfrac{2}{3}$ of its games and team B has won $\tfrac{5}{8}$ of its games. Also, team B has won 7 more games and lost 7 more games than team A. How many games has team A played?

A 队和 B 队在一个篮球联盟内打比赛，每场比赛后，结果都是一个队伍赢，另一个队伍输。A 队赢了它所参加比赛总数的$\tfrac{2}{3}$，B 队赢了它所参加比赛总数的$\tfrac{5}{8}$，且 B 队比 A 队多赢了 7 场比赛，也比 A 队多输了 7 场比赛。问 A 队参加了多少场比赛？

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 63$

Problem 6

For all integers $n \geq 9$, the value of $\frac{(n+2)!-(n+1)!}{n!}$ is always which of the following?

对于$n \geq 9$,的所有整数，表达式$\frac{(n+2)!-(n+1)!}{n!}$的值总是下面哪个？

（A）A multiple of 4 | 4 的倍数

（B）A multiple of10 | 10 的倍数

（C）A prime number |一个质数

（D）A perfect square |一个完全平方数

（E）A perfect cube | 一个完全立方数

Problem 7

Two nonhorizontal, non vertical lines in the xy-coordinate plane intersect to form a 45° angle. One line has slope equal to 6 times the slope of the other line. What is the greatest possible value of the product of the slopes of the two lines?

xy 坐标平面内的两条非水平也非竖直的直线相交形成 45°的角。其中一条直线的斜率是另外一条直线斜率的 6 倍。那么这 2 条直线的斜率的乘积的最大可能值是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{2} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 6$

Problem 8

How many ordered pairs of integers (x, y) satisfy the equation $x^{2020} + y^2 = 2y?$

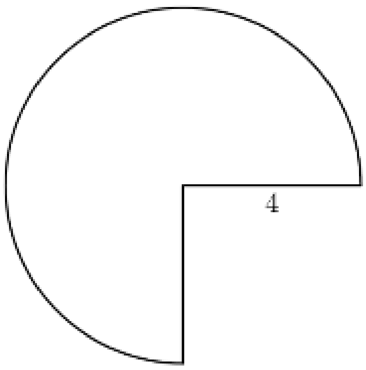
有多少对有序对（*x*，*y*）满足方程 $x^{2020} + y^2 = 2y?$

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ \text{infinitely many}$

Problem 9

A three-quarter sector of a circle of radius 4 inches together with its interior can be rolled up to form the lateral surface of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches?

如图所示，一个半径为 4 英寸的圆的四分之三部分形成的扇形及其内部，可以通过将它的两个半径粘在一起卷成一个正圆锥体的侧面。则这个圆锥体的体积为多少立方英寸？



$\textbf{(A)}\ 3\pi\sqrt{5} \qquad \textbf{(B)}\ 4\pi\sqrt{3} \qquad \textbf{(C)}\ 3\pi\sqrt{7} \qquad \textbf{(D)}\ 6\pi\sqrt{3} \qquad \textbf{(E)}\ 6\pi\sqrt{7}$

Problem 10

In unit square $ABCD,$ the inscribed circle ω intersects $\overline{CD}$ at $M,$ and $\overline{AM}$ intersects ω at a point P different from $M.$ What is $AP?$

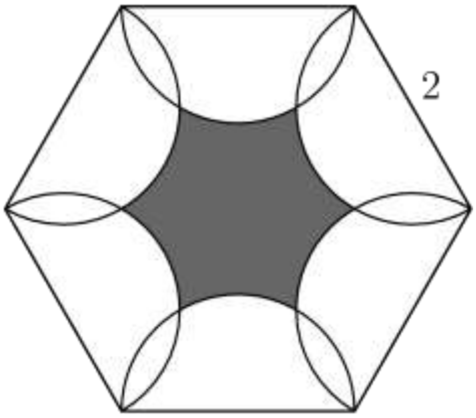
圆 *w* 内切于单位正方形 *ABCD*，且和线段$\overline{CD}$交于 *M* 点，线段$\overline{AM}$交 *W* 于和 *M* 不同的点 *P*。那么 *AP* 是多长？

$\textbf{(A) } \frac{\sqrt{5}}{12} \qquad \textbf{(B) } \frac{\sqrt{5}}{10} \qquad \textbf{(C) } \frac{\sqrt{5}}{9} \qquad \textbf{(D) } \frac{\sqrt{5}}{8} \qquad \textbf{(E) } \frac{2\sqrt{5}}{15}$

Problem 11

As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region—inside the hexagon but outside all of the semicircles?

如图所示，6 个半圆位于一个边长为 2 的正六边形的内部，且半圆的直径和六边形的边重合，那么阴影部分（位于六边形内部但在半圆外部的区域）的面积是多少？



$\textbf{(A) } 6\sqrt{3} - 3\pi \qquad \textbf{(B) } \frac{9\sqrt{3}}{2} - 2\pi \qquad \textbf{(C) } \frac{3\sqrt{3}}{2} - \frac{\pi}{3} \qquad \textbf{(D) } 3\sqrt{3} - \pi \qquad \textbf{(E) } \frac{9\sqrt{3}}{2} - \pi$

Problem 12

Let $\overline{AB}$ be a diameter in a circle of radius $5\sqrt{2}$. Let $\overline{CD}$ be a chord in the circle that intersects $\overline{AB}$ at a point E such that $BE = 2\sqrt{5}$ and $\angle AEC = 45^\circ$. What is $CE^2 + DE^2$?

$\overline{AB}$是半径为$5\sqrt{2}$.的圆的直径。$\overline{CD}$是圆内的弦且和$\overline{AB}$交于点 E，满足$BE = 2\sqrt{5}$, $\angle AEC = 45^\circ$.求$CE^2 + DE^2$?

(A) 96 **(B)** 98 **(C)** $44\sqrt{5}$ **(D)** $70\sqrt{2}$ **(E)** 100

Problem 13

Which of the following is the value of $\sqrt{\log_2 6 + \log_3 6}$?

下面哪个是下列表达式的值

$$\sqrt{\log_2 6 + \log_3 6}?$$

(A) 1 **(B)** $\sqrt{\log_5 6}$ **(C)** 2 **(D)** $\sqrt{\log_2 3} + \sqrt{\log_3 2}$ **(E)** $\sqrt{\log_2 6} + \sqrt{\log_3 6}$

Problem 14

Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

Bela 和 Jenn 在实数轴上[0，n]这个区间内做游戏，这里 *n* 是一个大于 4 的固定的整数。他们轮流做游戏，Bcla 先开始。首先，Bela 从[0，n]区间内任意选择一个实数，接下来轮到某人开始，那个人选择的这个实数必须要和之前两人已经选过的所有实数的距离都大于 1，若有选手无法选出这样的数，那么这个选手就输了。若两人都使用最佳策略，问哪个选手会赢得这场游戏？

(A)Bela will always win | 总是 Bela 赢

(B)Jenn will always win | 总是 Jenn 赢

(C)Bela will win if and only if *n* is odd | 当且仅当 *n* 是奇数时，Bela 会赢

(D)Jenn will win if and only if *n* is odd | 当且仅当 *n* 是奇数时，Jenn 会赢

(E)Jenn will win if and only if *n*>8 | 当且仅当 *n*>8 时，Jenn 会赢

Problem 15

There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

10 个人站成一个圆形，相互之间间距相等，每个人都认识其余 9 个人中的 3 个人：在他两旁的 2 个人，以及他对面的那个人，若把这 10 个人分成 5 对，要求每对的两个人都相互认识，一共有多少种方法？

(A) 11　**(B)** 12　**(C)** 13　**(D)** 14　**(E)** 15

Problem 16

An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?

一个罐子里装着一个红球和一个蓝球，旁边还有一个盒子，里面装有若干红球和蓝球，George 做如下操作 4 次：他随机的从罐子里抽取一个球，然后从盒子里拿一个同色的球，最后把这 2 个匹配的球放回罐子里，经过 4 次这样的重复操作后，罐子里总共有 6 个球，问罐子里恰好有 3 个红球和 3 个蓝球的概率是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 17

How many polynomials of the form $x^5 + ax^4 + bx^3 + cx^2 + dx + 2020$, where a, b, c, and d are real numbers, have the property that whenever r is a root, so is $\frac{-1+i\sqrt{3}}{2} \cdot r$? (Note that $i = \sqrt{-1}$)

多项式具有如下形式

$$x^5 + ax^4 + bx^3 + cx^2 + dx + 2020$$

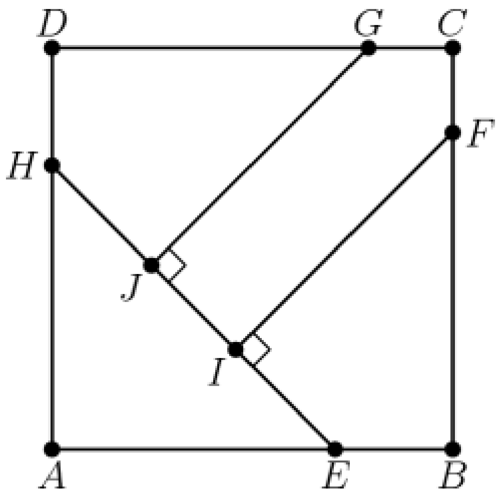
这里 *a*，*b*，*c* 和 *d* 都是实数，且满足只要 *r* 是多项式的根，那么 $\frac{-1+i\sqrt{3}}{2} \cdot r$ 也是多项式的根。那么这样的多项式有多少个？（注意$i = \sqrt{-1}$）

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 18

In square $ABCD$, points E and H lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH$. Points F and G lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points I and J lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle AEH, quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area 1. What is FI^2?

如图所示，在正方形 *ABCD* 中，点 *E* 和 *H* 分别位于线段$\overline{AB}$和$\overline{DA}$上，满足 *AE*=*AH*，点 *F* 和 *G* 分别位于线段$\overline{BC}$和$\overline{CD}$上，且点 *I* 和 *J* 位于线段$\overline{EH}$上，满足$\overline{FI} \perp \overline{EH}$，$\overline{GJ} \perp \overline{EH}$，三角形 *AEH*，四边形 *BFIE*，四边形 *DHJG* 和五边形 *FCGJI* 面积均为 1，问FI^2是多少？



(A) $\frac{7}{3}$ **(B)** $8 - 4\sqrt{2}$ **(C)** $1 + \sqrt{2}$ **(D)** $\frac{7}{4}\sqrt{2}$ **(E)** $2\sqrt{2}$

Problem 19

Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1), B(-1,1), C(-1,-1),$ and $D(1,-1).$ Consider the following four transformations:

- $L,$ a rotation of 90° counterclockwise around the origin;
- $R,$ a rotation of 90° clockwise around the origin;
- $H,$ a reflection across the x-axis; and
- $V,$ a reflection across the y-axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1,1)$ to $(-1,-1)$ and would send the vertex B at $(-1,1)$ to itself. How many sequences of 20 transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, R, R, V, H is one sequence of 4 transformations that will send the vertices back to their original positions.)

在坐标平面内，正方形 *ABCD* 的 4 个顶点的坐标分别为$A(1,1), B(-1,1), C(-1,-1),$ $D(1,-1).$考虑如下 4 个变换：*L*，绕着原点逆时针旋转 90°；*R*，绕着原点顺时针旋转 90°；*H*，关于 *x* 轴对称；*V*，关于 *y* 轴对称。这 4 个变换中的每一个都将正方形映射回自身，但是顶点的位置会发生改变。例如，依次使用变换 *R* 和 *V* 后，顶点 *A* 的位置就从（1，1）变到了（-1，-1），同时顶点 *B*（-1，1）最终又回到了它原来的位置。从集合{*L*，*R*，*H*，*V*}中依次选择 20 个变换组成一个序列，问有多少种这样的序列，在依次经过这个序列中的 20 次变换后，正方形的顶点又回到它们各自原来的位置？（例如，*R* ，*R*，*V*，*H* 就是这样一组满足条件的 4 次变换的序列，能够把顶点都映射回它们各自原来的位置）

$\textbf{(A)}\ 2^{37} \qquad \textbf{(B)}\ 3\cdot 2^{36} \qquad \textbf{(C)}\ 2^{38} \qquad \textbf{(D)}\ 3\cdot 2^{37} \qquad \textbf{(E)}\ 2^{39}$

Problem 20

Two different cubes of the same size are to be painted, with the color of each face being chosen independently and at random to be either black or white. What is the probability that after they are painted, the cubes can be rotated to be identical in appearance?

两个同样大小的正方体的每个面都要独立且随机地涂成黑色或者白色。那么涂色完成后，可以通过旋转使得这两个正方体一模一样的概率是多少？

$\textbf{(A)}\ \frac{9}{64} \qquad \textbf{(B)}\ \frac{289}{2048} \qquad \textbf{(C)}\ \frac{73}{512} \qquad \textbf{(D)}\ \frac{147}{1024} \qquad \textbf{(E)}\ \frac{589}{4096}$

Problem 21

How many positive integers n satisfy $\dfrac{n+1000}{70} = \lfloor \sqrt{n} \rfloor$? (Recall that $\lfloor x \rfloor$ is the greatest integer not exceeding x.)

有多少个正整数 *n* 满足

$$\frac{n+1000}{70} = \lfloor \sqrt{n} \rfloor?$$

（注意：$\lfloor x \rfloor$表示不超过 *x* 的最大整数。）

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Problem 22

What is the maximum value of $\dfrac{(2^t-3t)t}{4^t}$ for real values of t?

已知 *t* 为实数，那么表达式$\dfrac{(2^t-3t)t}{4^t}$的最大值是多少？

$\textbf{(A)}\ \frac{1}{16} \qquad \textbf{(B)}\ \frac{1}{15} \qquad \textbf{(C)}\ \frac{1}{12} \qquad \textbf{(D)}\ \frac{1}{10} \qquad \textbf{(E)}\ \frac{1}{9}$

Problem 23

How many integers $n \geq 2$ are there such that whenever $z_1, z_2, ..., z_n$ are complex numbers such that

$|z_1| = |z_2| = ... = |z_n| = 1 \text{ and } z_1 + z_2 + ... + z_n = 0,$ then the numbers $z_1, z_2, ..., z_n$ are equally spaced on the unit circle in the complex plane?

有多少个整数 n≥2 满足，只要$z_1, z_2, ..., z_n$是复数且有下面的关系式：

$|z_1| = |z_2| = ... = |z_n| = 1 \text{ and } z_1 + z_2 + ... + z_n = 0,$ 那么$z_1, z_2, ..., z_n$就均匀等距的分布在复平面内的单位圆上？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 24

Let $D(n)$ denote the number of ways of writing the positive integer n as a product $n = f_1 \cdot f_2 \cdots f_k$,where $k \geq 1$, the f_i are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6, $2 \cdot 3$, and $3 \cdot 2$, so $D(6) = 3$. What is $D(96)$?

$D(n)$表示把正整数 *n* 写成以下乘积形式的方法数：$n = f_1 \cdot f_2 \cdots f_k$,其中 *k*≥1，$f_i$都是严格大于 1 的整数，并且这些因子列出的先后顺序是需要考虑的（即，两种表示方法若只是因子的顺序不同，那么这两种表示方法被看作是不一样的表示方法）。例如，数字 6 可以写成 6，$2 \cdot 3$和$3 \cdot 2$，因此$D(6) = 3$，问$D(96)$是多少？

$\textbf{(A)}\ 112 \qquad \textbf{(B)}\ 128 \qquad \textbf{(C)}\ 144 \qquad \textbf{(D)}\ 172 \qquad \textbf{(E)}\ 184$

Problem 25

For each real number a with $0 \leq a \leq 1$, let numbers x and y be chosen independently at random from the intervals $[0, a]$ and $[0, 1]$, respectively, and let $P(a)$ be the probability that

$\sin^2{(\pi x)} + \sin^2{(\pi y)} > 1$What is the maximum value of $P(a)$?

对于$0 \leq a \leq 1$内的每一个实数 *a*，令 *x* 和 *y* 分别独立且随机地从区间$[0, a]$和$[0, 1]$取值，$P(a)$表示$\sin^2{(\pi x)} + \sin^2{(\pi y)} > 1$的概率。问$P(a)$的最大值是多少？

$\textbf{(A)}\ \frac{7}{12} \qquad \textbf{(B)}\ 2 - \sqrt{2} \qquad \textbf{(C)}\ \frac{1+\sqrt{2}}{4} \qquad \textbf{(D)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(E)}\ \frac{5}{8}$