# 2022 AMC 10B

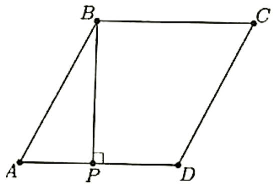
**1.** Define x◊ y to be $|x-y|$ for all real numbers x and y. What is the value of (1◊ (2◊3)) − ((1◊2) ◊3)?

对于所有的实数 x 和 y, x◊ y 定义为$|x-y|$.问(1◊ (2◊3)) − ((1◊2) ◊3)的值是多少?

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

**2.** In rhombus ABCD, point Plies on segment $\overline{AD}$ so that $\overline{BP} \perp \overline{AD}$, AP = 3, and PD =2. What is the area of ABCD? (Note: The figure is not drawn to scale.)

在菱形 ABCD 中,点 P 位于线段$\overline{AD}$上使得$\overline{BP} \perp \overline{AD}$, AP=3,PD=2 问 ABCD 的面积是多少?(注:图形未按比例绘制.)




(A) $3\sqrt{5}$ (B) 10 (C) $6\sqrt{5}$ (D) 20 (E) 25

**3.** How many three-digit positive integers have an odd number of even digits?

包含奇数个偶数数码的三位正整数有多少个?

(A) 150 (B) 250 (C) 350 (D) 450 (E) 550

**4.** A donkey suffers an attack of hiccups and the first hiccup happens at 4:00 one afternoon. Suppose that the donkey hiccups regularly every 5 seconds. At what time does the donkey's 700th hiccup occur?

一头驴因病连续打嗝，第一次打嗝发生在下午 4:00.假设驴每隔 5 秒打嗝一次.那么驴第 700

次打嗝发生在什么时间?

(A) 15 seconds after 4:58|4:58 之后的 15 秒

(B) 20 seconds after 4:58|4:58 之后的 20 秒

(C) 25 seconds after 4:58|4:58 之后的 25 秒

(D) 30 seconds after 4:58|4:58 之后的 30 秒

(E) 35 seconds after 4:58|4:58 之后的 35 秒

**5.** What is the value of the below expression?

下面表达式的值是多少?

$$\frac{\left(1+\frac{1}{3}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{7}\right)}{\sqrt{\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{7^2}\right)}}$$

(A) $\sqrt{3}$ (B) 2 (C) $\sqrt{15}$ (D) 4 (E) $\sqrt{105}$

**6.** How many of the first ten numbers of the sequence 121, 11211, 1112111, are prime numbers?

在数列 121，11211，1112111，...的前十项中有多少个数是素数?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

**7.** For how many values of the constant k will the polynomial $x^2+kx+36$ have two distinct integer roots?

使得多项式$x^2+kx+36$有两个不同的整数根的常数 k 的取值有多少种?

(A) 6 (B) 8 (C) 9 (D) 14 (E) 16

**8.** Consider the following 100 sets of 10 elements each:

{1,2,3,...,10},

{11,12,13,....20},

{21,22,23,…30},

…

{991,992,993,...,1000}.

How many of these sets contain exactly two multiples of 7?

考虑以下的 100 个集合，每个集合中有 10 个元素:

{1,2,3,....10},

{11,12,13,...,20},

{21,22,23....,30},

…

{991,992,993, .,1000}

在这些集合中，恰好包含两个 7 的倍数的集合有多少个?

(A) 40 (B) 42 (C) 43 (D) 49 (E) 50

**9.** The sum $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{2021}{2022!}$

can be expressed as $a-\frac{1}{b!}$, where a and b are positive integers. What is a+b?

和式$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{2021}{2022!}$

可以表达成$a-\frac{1}{b!}$的形式，其中 a 和 b 是正整数，问 a+ b 的值是多少?

(A) 2020 (B) 2021 (C) 2022 (D) 2023 (E) 2024

**10.** Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

Camila 写下了五个正整数，这些整数的唯一众数比它们的中位数大 2，而中位数比它们的算

术平均数大 2.问众数的最小可能值是多少?

(A) 5 (B) 7 (C) 9 (D) 11 (E) 13

**11.** All the high schools in a large school district are involved in a fundraiser selling T-shirts. Which of the choices below is logically equivalent to the statement "No school bigger than Euclid HS sold more T-shirts than Euclid HS"?

(A) All schools smaller than Euclid HS sold fewer T-shirts than Euclid HS.

(B) No school that sold more T-shirts than Euclid HS is bigger than Euclid HS.

(C) All schools bigger than Euclid HS sold fewer T-shirts than Euclid HS

(D) All schools that sold fewer T-shirts than Euclid HS are smaller than Euclid HS.

(E) All schools smaller than Euclid HS sold more T-shirts than Euclid HS.

一个很大的学区的所有高中都参与了一项出售 T 恤的筹款活动。以下哪个选项在逻辑上等同于陈述“没有比 Euclid 高中更大的学校卖出了比 Euclid 高中更多的 T 恤"?

(A)所有比 Euclid 高中更小的学校都卖出比 Euclid 高中更少的 T 恤

(B)没有比 Euclid 高中卖出更多的 T 恤的学校比 Euclid 高中更大

(C)所有比 Euclid 高中更大的学校都卖出比 Euclid 高中更少的 T 恤

(D)所有卖出比 Euclid 高中更少的 T 恤的学校都比 Euclid 高中更小

(E)所有比 Euclid 高中更小的学校都卖出比 Euclid 高中更多的 T 恤

**12.** A pair of fair 6-sided dice is rolled n times. What is the least value of n such that the probability that the sum of the numbers face up on a roll equals 7 at least once is greater than $\frac{1}{2}$?

一对公平的 6 个面的骰子被抛掷 n 次，如果至少有一次掷出的两个数之和等于 7 的概率大于 $\frac{1}{2}$，那么 n 的最小值是多少?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

**13.** The positive difference between a pair of primes is equal to 2, and the positive difference between the cubes of the two primes is equal to 31106. What is the sum of the digits of the least prime that is greater than those two primes?

一对素数之间形成的是正数的差等于2,这两个素数的立方之间形成的是正数的差等于31106.问大于这两个素数的最小素数的各位数字之和是多少?

(A) 8 (B) 10 (C) 11 (D) 13 (E) 16

**14.** Suppose that S is a subset of {1,2,3,...,25} such that the sum of any two (not necessarily distinct) elements of S is never an element of S. What is the maximum number of elements S may contain?

设 S 是{1,2,3,...,25}的子集，使得 S 中的任何两个(不一定互异)元素的总和不是 S 中的元素.问 S 中最多可以包含多少个元素?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

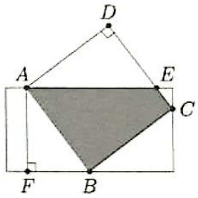
**15.** Let $S_n$, be the sum of the first n terms of an arithmetic sequence that has a common difference of 2. The quotient $\frac{S_{3n}}{S_n}$ does not depend on n. What is $S_{20}$?

设 $S_n$ 是一个公差为 2 的等差数列的前 n 项之和，商 $\frac{S_{3n}}{S_n}$ 的值不依赖于 n 问 S20 是多少?

(A) 340 (B) 360 (C) 380 (D) 400 (E) 420

**16.** The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?

下图显示了一个长为 8，宽为 4 的矩形以及一个边长为 5 的正方形.如图所示，正方形的三个顶点位于矩形的三条不同的边上。同正方形和矩形公共部分的面积是多少?




(A) $15\frac{1}{8}$ (B) $15\frac{3}{8}$ (C) $15\frac{1}{2}$ (D) $15\frac{5}{8}$ (E) $15\frac{7}{8}$

**17.** One of the following numbers is not divisible by any prime number less than 10. Which is it?

下列各数中有一个数不能被任何小于 10 的素数整除，问这是哪个数?

(A) $2^{606}-1$ (B) $2^{606}+1$ (C)$2^{607}-1$ (D) $2^{607}+1$ (E) $2^{607}+3^{607}$

**18.** Consider systems of three linear equations with unknowns x, y, and z,

$$a_1x+b_1y+c_1z=0$$
$$a_2x+b_2y+c_2z=0$$
$$a_3x+b_3y+c_3z=0$$

where each of the coefficients is either O or 1 and the system has a solution other than x =y = z = 0. For example, one such system is $\langle 1x+1y+0z=0,\ 0x+1y+1z=0,\ 0x+0y+0z=0\rangle$ with a nonzero solution of (x, y, z)=(1,-1, 1). How many such systems of equations are there? (The equations in a system need not be distinct, and two systems containing the same equations in a different order are considered different.)

考虑由三个线性方程组成的，来知数为 x，y 和 z 的方程组:

$$a_1x+b_1y+c_1z=0$$
$$a_2x+b_2y+c_2z=0$$
$$a_3x+b_3y+c_3z=0$$

其中的每个系数为 0 或 1，并且方程组有除去 x=y=z=0 之外的解，例如，一个这样的方程组是$\langle 1x+1y+0z=0,\ 0x+1y+1z=0,\ 0x+0y+0z=0\rangle$，它有非零解(x, y, z)=(1,-1,1).问共有多少个这样的方程组? (一个方程组中的方程允许是相同的，并且两个包含同样的方程，但排列顺序不同的方程组被认为是不同的.)

(A) 302 (B) 338 (C) 340 (D) 343 (E) 344

**19.** Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

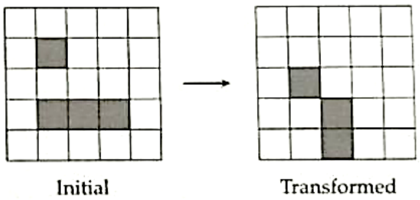
- Any filled square with two or three filled neighbors remains filled.
- Any empty square with exactly three filled neighbors becomes a filled square
- All other squares remain empty or become empty.

A sample transformation is shown in the figure below.

5×5 方格表中的每个方格要么是灰色的，要么是空白的，并且最多有八个相邻的方格，这里有公共边或者公共顶点的方格认为是相邻的、方格表按以下规则进行转换:

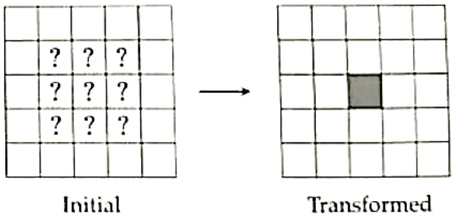
- 任何有两个或三个相邻的方格是灰色的灰色方格继续保持是灰色的
- 任何恰好有三个相邻的方格是灰色的空白方格将成为灰色方格,
- 所有其他的方格要么继续保持是空白的，要么变成空白方格

下图显示了一个转换的示例




Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)

假设 5×5 方格表周界上的方格都是空白的，它们围绕着中间的 3×3 子方格表，那么经过一次转换后，方格表变为只有中心方格是灰色的初始构型有多少个?(一个构型经过旋转和反射后而形成的构型被认为是不同的.)




(A) 14　　(B) 18　　(C) 22　　(D) 26　　(E) 30

**20.** Let ABCD be a rhombus with $\angle ADC = 46^\circ$. Let E be the midpoint of $\overline{CD}$, and let F be the point on $\overline{BE}$ such that $\overline{AF}$ is perpendicular to $\overline{BE}$. What is the degree measure of $\angle BFC$?

在菱形 ABCD 中,∠ADC=46° .设 E 为$\overline{CD}$的中点,而 F 为$\overline{BE}$上的点, 使得$\overline{AF}$垂直于$\overline{BE}$. 问∠BFC 的度数是多少?

(A) 110 (B) 111 (C) 112 (D) 113 (E) 114

**21.** Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial $x^2+x+1$, the remainder is $x+2$, and when $P(x)$ is divided by the polynomial $x^2+1$, the remainder is $2x+1$. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

设 P(x)是一个有理系数的多项式，使得当 P(x)除以多项式$x^2+x+1$时,余式为$x+2$，而当 P(x)除以多项式$x^2+1$时，余式为 2x+1.满足上述两个条件的次数最低的多项式是唯一的.问该多项式的各项系数的平方和是多少?

(A) 10 (B) 13 (C) 19 (D) 20 (E) 23

**22.** Let S be the set of circles that are tangent to each of the three circles in the coordinate plane whose equations are $x^2+y^2=4, x^2+y^2=64$, and $(x-5)^2+y^2=3$. What is the sum of the areas of all the circles in S?

设 S 是坐标平面中，与方程为$x^2+y^2=4, x^2+y^2=64, (x-5)^2+y^2=3$的三个圆中的每一个都相切的圆组成的集合，问 S 中所有圆的面积之和是多少?

(A) 48π (B) 68π (C) 96π (D) 102π (E) 136π

**23.** Ant Amelia starts on the number line at O and crawls in the following manner. For n = 1,2, 3, Amelia chooses a time duration $t_n$ and an increment $x_n$ independently and uniformly at random from the interval (0, 1). During the nth step of the process, Amelia moves $x_n$ units in the positive direction, using up $t_n$ minutes. If the total elapsed time has exceeded I minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?

蚂蚁 Amelia 在数轴上从 0 开始,按以下方式爬行.对于 n=1,2,3,Amelia 从区间(0,1)中随机独立且均匀地选择持续时间 t 切和步长 $x_n$.在爬行过程的第 n 步，Amelia 沿正向移动 $x_n$ 个单位，用时 $t_n$ 分钟，如果在第 n 步移动期间，所经过的总时间超过 1 分钟，则她在该步结束时停止;否则，她会继续下一步，最多一共走 3 步。问 Amelia 停止在大于 1 的数所对应的位置处的概率是多少?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{5}{6}$

**24.** Consider functions *f* that satisfy $|f(x)-f(y)|\le\frac{1}{2}|x-y|$ for all real numbers x and y. Of all such functions that also satisfy the equation $f(300)=f(900)$, what is the greatest possible value of

$$f(f(800))-f(f(400))?$$

考虑满足具有以下性质的函数 f:对于所有的实数 x 和 y，$|f(x)-f(y)|\le\frac{1}{2}|x-y|$.在所有这样的函数中,如果还要求满足等式 $f(300)=f(900)$,那么 $f(f(800))-f(f(400))$ 的最大可能值是多少?

(A) 25 (B) 50 (C) 100 (D) 150 (E) 200

**25.** Let $x_0, x_1, x_2, \ldots$ be a sequence of numbers, where each $x_k$ is either 0 or 1.

For each positive integer n, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k$$

Suppose $7S_n \equiv 1 \pmod{2^n}$ for all n⩾1. What is the value of the sum

$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}?$$

在数列$x_0, x_1, x_2, \ldots$中，每个 x 项均为 0 或 1.对于每个正整数 n，定义$S_n = \sum_{k=0}^{n-1} x_k 2^k$假设对所有 n⩾1，有$7S_n \equiv 1 \pmod{2^n}$问和式$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}$的值是多少?

(A) 6 (B) 7 (C) 12 (D) 14 (E) 15