2022 AMC 12A

1. What is the value of the below expression?

问下式的值是多少?

$$3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}}}$$

(A) $\frac{31}{10}$ (B) $\frac{49}{15}$ (C) $\frac{33}{10}$ (D) $\frac{109}{33}$ (E) $\frac{15}{4}$

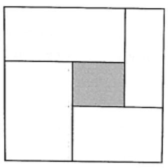
2. The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?

三个数之和为 96.第一个数是第三个数的 6 倍,第三个数比第二个数少 40.问第一个数与第二个数之差的绝对值是多少?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. Five rectangles, A, B, C, D, and E, are arranged in a square as shown below. These rectangles have dimensions 1×6, 2×4, 5×6, 2×7, and 2×3, respectively. (The figure is not drawn to scale.) Which of the five rectangles is the shaded one in the middle?

如下图所示，五个矩形，A、B、C、D、E，排列在一个正方形中.这些矩形的尺寸分别为 1×6, 2×4, 5×6, 2×7, 2×3.(图形未按比例绘制.)问五个矩形中哪一个是中间的阴影矩形?



(A) A (B) B (C) C (D) D (E) E

4. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n?

正整数 n 与 18 的最小公倍数是 180，并且 n 与 45 的最大公约数是 15.那么 n 的各位数字之和是多少?

(A) 3 (B) 6 (C) 8 (D) 9 (E) 12

5. The taxicab distance between points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by $|x_1 - x_2| + |y_1 - y_2|$. For how many points P with integer coordinates is the taxicab distance between P and the origin less than or equal to 20?

定义坐标平面上点(x_1, y_1)和(x_2, y_2)之间的出租车距离为$|x_1 - x_2| + |y_1 - y_2|$.与原点之间的出租车距离小于或等于 20 的，坐标为整数的点 P 有多少个?

(A) 441 (B) 761 (C) 841 (D) 921 (E) 924

6. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X?

一个数据集由 6 个(可以相同的)正整数组成:1、7、5、2、5、X.6 个数的(算术)平均值等于数据集中的某个数，问 X 的所有可能值的总和是多少?

(A) 10 (B) 26 (C) 32 (D) 36 (E) 40

7. A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color-red, orange, yellow, blue, or green-so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?

如图所示，一个矩形被划分为 5 个区域.每个区域都将被用红色、橙色、黄色、蓝色或绿色中的一种颜色涂实，使得有接触的区域被涂上不同的颜色，每种颜色可以使用多于一次.问有多少种可能的染色方式?

(A) 120 (B) 270 (C) 360 (D) 540 (E) 720

8. The infinite product $\sqrt[3]{10}\cdot\sqrt[3]{\sqrt[3]{10}}\cdot\sqrt[3]{\sqrt[3]{\sqrt[3]{10}}}\ldots$ evaluates to a real number. What is that number?

无穷乘积$\sqrt[3]{10}\cdot\sqrt[3]{\sqrt[3]{10}}\cdot\sqrt[3]{\sqrt[3]{\sqrt[3]{10}}}\ldots$的计算结果是一个实数，问这个数是多少?

(A) $\sqrt{10}$ (B) $\sqrt[3]{100}$ (C) $\sqrt[4]{1000}$ (D) 10 (E) $10\sqrt[3]{10}$

9. On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

“Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

“Are you an alternater?” The principal gave a piece of candy to each of the 15 children who answered yes.

“Are you a liar?” The principal gave a piece of candy to each of the9children who answered yes.

How many pieces of candy in all did the principal give to the children who always tell the truth?

万圣节那天，31 个孩子走进校长办公室要糖果.孩子们有三种类型:有些人总说谎话;有些人总说真话;有些人交替说谎话和真话.对于那些交替说谎话和真话的孩子，每个人首先对自己的第一个回答做出是说谎话还是说真话的选择，而随后的每次陈述是否说真话则与前一次情况相反.校长按如下的顺序问了每个人同样的三个问题。

“你是总说真话吗?”校长给回答“是”的 22 个孩子每人一块糖果，

“你是交替说谎话和真话吗?”校长给回答“是”的 15 个孩子每人一块糖果。

“你是总说谎话吗?”校长给回答“是”的 9 个孩子每人一块糖果，

问校长给了总说真话的孩子们一共多少块糖果?

(A) 7　(B) 12　(C) 21　(D) 27　(E) 31

10. How many ways are there to split the integers 1 through 14 into 7 pairs so that in each pair the greater number is at least 2 times the lesser number?

将从 1 到 14 的整数分成 7 对，使得每一对中较大的数至少是较小的数的 2 倍，这样的分法一共有多少种?

(A) 108　(B) 120　(C) 126　(D) 132　(E) 144

11. What is the product of all real numbers x such that the distance on the number line between $\log_6 x$ and $\log_6 9$ is twice the distance on the number line between $\log_6 10$ and 1?

数轴上 $\log_6 x$ 与 $\log_6 9$ 之间的距离是数轴上 $l0g_6 10$ 与 1 之间距离的两倍，所有满足该条件的实数 x 的乘积是多少?

(A) 10　(B) 18　(C) 25　(D) 36　(E) 81

12. Let M be the midpoint of $\overline{AB}$ in regular tetrahedron ABCD. What is cos(∠CMD)?

设 M 是正四面体 ABCD 的边$\overline{AB}$的中点.问 cos(∠CMD)的值是多少?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{3}}{2}$

13. Let R be the region in the complex plane consisting of all complex numbers z that can be written as the sum of complex numbers z_1 and z_2, where z_1 lies on the segment with endpoints 3 and 4i, and z_2 has magnitude at most 1. What integer is closest to the area of R?

考虑可以写成复数 z_1 与 z_2 之和形式的复数 z，其中 z_1 在端点为 3 和 4i 的线段上，z_2 的模长不超过 1.R 是复平面中所有这样的复数构成的区域.问最接近 R 的面积的整数是多少?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

14. What is the value of the below expression, where all logarithms have base 10?

问下式的值是多少，其中所有的对数均以 10 为底?

$$(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$$

(A) $\frac{3}{2}$ (B) $\frac{7}{4}$ (C) 2 (D) $\frac{9}{4}$ (E) 3

15. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

多项式$10x^3-39x^2+29x-6$的根是一个长方体盒子(底面是矩形的正棱柱)的高度、长度和宽度.如果一个新的长方体盒子是通过将原始盒子的每条边延长 2 个单位而形成.问新盒子的体积是多少?

(A) $\frac{24}{5}$ (B) $\frac{42}{5}$ (C) $\frac{81}{5}$ (D) 30 (E) 48

16. A triangular number is a positive integer that can be expressed in the form t_n =1+2+3+...+ n, for some positive integer n. The three smallest triangular numbers that are also perfect squares are $t_1 = 1 = 1^2$, $t_8 = 36 = 6^2$, and $t_{49} = 1225 = 35^2$. What is the sum of the digits of the fourth smallest triangular number that is also a perfect square?

如果一个正整数可以表达成 t_n =1+2+3+...+ n 的形式，其中 n 是某个正整数，则称其为三角形数，亦是完全平方数的三个最小的三角形数是 $t_1 = 1 = 1^2$, $t_8 = 36 = 6^2$, $t_{49} = 1225 = 35^2$.问亦是完全平方数的第四小的三角形数的各位数字之和是多少?

(A) 6 (B) 9 (C) 12 (D) 18 (E) 27

17. Suppose a is a real number such that the equation

$$a\cdot(\sin x+\sin(2x))=\sin(3x)$$

has more than one solution in the interval (0, π). The set of all such a can be written in the form $(p,q)\cup(q,r)$, where p, q, and r are real numbers with $p<q<r$. What is $p+q+r$?

设 a 是实数，使得方程

$$a\cdot(\sin x+\sin(2x))=\sin(3x)$$

在区间(0,π)上有多于一个解.所有这样的 a 组成的集合可以写成$(p,q)\cup(q,r)$ 的形式，其中 p、q、r 是满足 $p<q<r$ 的实数.问 p+q+r 是多少?

(A) −4 (B) −1 (C) 0 (D) 1 (E) 4

18. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \ldots, T_n$ returns the point (1,0) back to itself?

设 T_k 为坐标平面的变换，它首先将平面绕原点逆时针旋转 k 度，然后再将平面沿 y 轴反射.通过变换序列 $T_1, T_2, T_3, \ldots, T_n$ 使得点(1,0)重新回到自身的最小正整数 n 是多少?

(A) 359 (B) 360 (C) 719 (D) 720 (E) 721

19. Suppose that 13 cards numbered 1, 2, 3, ..., 13 are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards 1, 2, 3 are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, 7, 8, 9, 10 on the fourth pass, and 11, 12, 13 on the fifth pass. For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?

假设编号为 1、2、3、 13 的 13 张卡片排成一行.现在的任务是按数值递增的顺序拾取它们，从左到右重复工作.在下面的示例中，第一轮时拿起编号为 1、2、3 的卡片，第二轮时拿起编号为 4 和 5 的卡片，第三轮时拿起编号为 6 的卡片，第四轮时拿起编号为 7、8、9、10 的卡片，第五轮时拿起编号为 11、12、13 的卡片.

在所有 13!种可能的卡片排列中，有多少种排列使得 13 张卡片可通过恰好两轮被拾取?

$\boxed{7}$ $\boxed{11}$ $\boxed{8}$ $\boxed{6}$ $\boxed{4}$ $\boxed{5}$ $\boxed{9}$ $\boxed{12}$ $\boxed{1}$ $\boxed{13}$ $\boxed{10}$ $\boxed{2}$ $\boxed{3}$

(A) 4082 (B) 4095 (C) 4096 (D) 8178 (E) 8191

20. Isosceles trapezoid ABCD has parallel sides $\overline{AD}$ and $\overline{BC}$, with BC < AD and AB = CD. There is a point P in the plane such that PA=1, PB =2, PC =3,and PD =4. What is $\frac{BC}{AD}$?

在等腰梯形 ABCD 中，$\overline{AD}$ 和 $\overline{BC}$ 是平行的两边,BC<AD,并且 AB=CD 平面上有一个点 P，使得 PA=1、PB=2、PC=3、PD=4.问 $\frac{BC}{AD}$ 是多少?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

21. Let $P(x)=x^{2022}+x^{1011}+1$. Which of the following polynomials is a factor of $P(x)$?

假设 $P(x)=x^{2022}+x^{1011}+1$.问以下哪个多项式是 P(x)的因式?

(A) x^2-x-1 (B) x^2+x+1 (C) x^4+1 (D) x^6-x^3+1 (E) x^6+x^3+1

22. Let c be a real number, and let z_1 and z_2 be the two complex numbers satisfying the equation $z^2-cz+10=0$. Points z_1, z_2 $\frac{1}{z_1}$, and $\frac{1}{z_2}$ are the vertices of (convex) quadrilateral Q in the complex plane. When the area of Q obtains its maximum possible value, c is closest to which of the following?

设 c 为实数，z_1 和 z_2 是满足方程 $z^2-cz+10=0$ 的两个复数.复平面上的点 z_1，z_2 $\frac{1}{z_1}$，$\frac{1}{z_2}$ 是(凸)四边形 Q 的顶点.当 Q 的面积取得最大可能值时，c 最接近以下哪个数?

(A) 4.5 (B) 5 (C) 5.5 (D) 6 (E) 6.5

23. Let h_n and k_n be the unique relatively prime positive integers such that

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}=\frac{h_n}{k_n}.$$

Let L_n denote the least common multiple of the numbers 1, 2, 3, n. For how many integers n with $1 \le n \le 22$ is $k_n < L_n$?

设 h_n 和 k_n 是满足$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}=\frac{h_n}{k_n}$的唯一互素正整数对.设 L_n 为 1、2、3、...n 的最小公倍数.在满足 1≤n≤22 的整数 n 中，有多少个数使得$k_n < L_n$?

(A) 0 (B) 3 (C) 7 (D) 8 (E) 10

24. How many strings of length 5 formed from the digits 0,1, 2, 3, 4 are there such that for each $j \in \{1,2,3,4\}$, at least j of the digits are less than j? (For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

考虑由数字 0、1、2、3、4 组成的长度为 5 的字符串，满足对于每个 $j \in \{1,2,3,4\}$，字符串中至少有 j 个小于 j 的数字，问这样的字符串有多少个?(例如，02214 满足此条件，因为它包含至少 1 个小于 1 的数字、至少 2 个小于 2 的数字、至少 3 个小于 3 的数字、至少 4 个小于 4 的数字.字符串 23404 不满足条件，因为它不包含至少 2 个小于 2 的数字.)

(A) 500 (B) 625 (C) 1089 (D) 1199 (E) 1296

25. A circle with integer radius r is centered at (r, r). Distinct line segments of length c_i connect points $(0, a_i)$ to $(b_i, 0)$ for $1 \leqslant i \leqslant 14$ and are tangent to the circle, where a_i, b_i, and c_i are all positive integers and $c_1 \le c_2 \le \ldots \le c_{14}$. What is the ratio $\frac{c_{14}}{c_1}$ for the least possible value of r?

一个圆，具有整数半径 r，以(r, r)为圆心.对于 $1 \leqslant i \leqslant 14$，连接点$(0, a_i)$与$(b_i, 0)$的线段互不相同,长度为 c_i,并且与圆相切,这里 a_i, b_i, c_i 都是正整数, $c_1 \le c_2 \le \ldots \le c_{14}$.

当 r 取最小可能值时，比率 $\frac{c_{14}}{c_1}$ 是多少?

(A) $\frac{21}{5}$ (B) $\frac{85}{13}$ (C) 7 (D) $\frac{39}{5}$ (E) 17