2022 AMC 12B

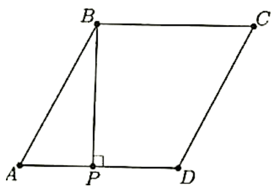
1. Define x◊ y to be $|x-y|$ for all real numbers x and y. What is the value of (1◊ (2◊3)) − ((1◊2) ◊3)?

对于所有的实数 x 和 y, x◊ y 定义为$|x-y|$.问(1◊ (2◊3)) − ((1◊2) ◊3)的值是多少?

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

2. In rhombus ABCD, point Plies on segment $\overline{AD}$ so that $\overline{BP} \perp \overline{AD}$, AP = 3, and PD =2. What is the area of ABCD? (Note: The figure is not drawn to scale.)

在菱形 ABCD 中,点 P 位于线段$\overline{AD}$上使得$\overline{BP} \perp \overline{AD}$, AP=3,PD=2 问 ABCD 的面积是多少?(注:图形未按比例绘制.)



(A) $3\sqrt{5}$ (B) 10 (C) $6\sqrt{5}$ (D) 20 (E) 25

3. How many of the first ten numbers of the sequence 121, 11211, 1112111,… are prime numbers?

在数列 121，11211，1112111，...的前十项中有多少个数是素数?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. For how many values of the constant k will the polynomial $x^2+kx+36$ have two distinct integer roots?

使得多项式$x^2+kx+36$有两个不同的整数根的常数 k 的取值有多少种?

(A) 6 (B) 8 (C) 9 (D) 14 (E) 16

5. The point $(-1, -2)$ is rotated $270°$ counterclockwise about the point (3,1). What are the coordinates of its new position?

点$(-1, -2)$绕点(3,1)逆时针旋转 $270°$.问它所在的新位置坐标是多少?

(A) $(-3,4)$ (B) (0, 5) (C) $(2,-1)$ (D) (4, 3) (E) $(6,-3)$

6. Consider the following 100 sets of 10 elements each:

{1,2,3,...,10},

{11,12,13,....20},

{21,22,23,…30},

…

{991,992,993,...,1000}.

How many of these sets contain exactly two multiples of 7?

考虑以下的 100 个集合，每个集合中有 10 个元素:

{1,2,3,....10},

{11,12,13,...,20},

{21,22,23....,30},

…

{991,992,993, .,1000}

在这些集合中，恰好包含两个 7 的倍数的集合有多少个?

(A) 40 (B) 42 (C) 43 (D) 49 (E) 50

7. Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

Camila 写下了五个正整数.这些整数的唯一众数比它们的中位数大 2，而中位数比它们的算术平均数大 2.问众数的最小可能值是多少?

(A) 5 (B) 7 (C) 9 (D) 11 (E) 13

8. What is the graph of $y^4+1=x^4+2y^2$ in the coordinate plane?

在坐标平面上，$y^4+1=x^4+2y^2$所表示的图形是怎样的?

(A) two intersecting parabolas | 两条相交的抛物线

(B) two nonintersecting parabolas | 两条不相交的抛物线

(C) two intersecting circles | 两个相交的圆

(D) a circle and a hyperbola | 一个圆和一条双曲线

(E) a circle and two parabolas | 一个圆和两条抛物线

9. The sequence $a_0, a_1, a_2, \ldots.$ is a strictly increasing arithmetic sequence of positive integers such that

$$2^{a7}=2^{27}\cdot a_7$$

What is the minimum possible value of a_2?

数列$a_0, a_1, a_2, \ldots$是由正整数组成的严格递增的等差序列，并且满足$2^{a7}=2^{27}\cdot a_7$问 a_2 的最小可能值是多少?

(A) 8 (B) 12 (C) 16 (D) 17 (E) 22

10. Regular hexagon ABCDEF has side length 2. Let G be the midpoint of $\overline{AB}$, and let H be the midpoint of $\overline{DE}$. What is the perimeter of quadrilateral GCHF?

正六边形 ABCDEF 的边长为 2.设 G 为$\overline{AB}$的中点，H 为$\overline{DE}$的中点.问四边形 GCHF 的周长是多少?

(A) $4\sqrt{3}$ (B) 8 (C) $4\sqrt{5}$ (D) $4\sqrt{7}$ (E) 12

11. Let $f(n)=\left(\frac{-1+i\sqrt{3}}{2}\right)^{n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{n}$, where $i=\sqrt{-1}$. What is $f(2022)$?

设 $f(n)=\left(\frac{-1+i\sqrt{3}}{2}\right)^{n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{n}$，其中$i=\sqrt{-1}$. 问$f(2022)$ 的值是多少?

(A) -2 (B) -1 (C) 0 (D) $\sqrt{3}$ (E) 2

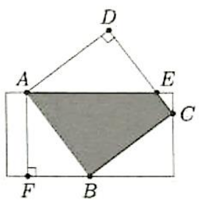
12. Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2?

Kayla 抛掷四个公平的 6 个面的骰子.在 Kayla 掷出的数中，至少有一个数大于 4，并且至少有两个数大于 2 的概率是多少?

(A) $\frac{2}{3}$ (B) $\frac{19}{27}$ (C) $\frac{59}{81}$ (D) $\frac{61}{81}$ (E) $\frac{7}{9}$

13. The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?

下图显示了一个长为 8，宽为 4 的矩形以及一个边长为 5 的正方形.如图所示，正方形的三个顶点位于矩形的三条不同的边上。同正方形和矩形公共部分的面积是多少?



(A) $15\frac{1}{8}$ (B) $15\frac{3}{8}$ (C) $15\frac{1}{2}$ (D) $15\frac{5}{8}$ (E) $15\frac{7}{8}$

14. The graph of $y=x^2+2x-15$ intersects the x-axis at points A and C and the y-axis at point B. What is tan(∠ABC)?

$y=x^2+2x-15$的图像与 x 轴相交于点 A 和点 C，与 y 轴相交于点 B.问 tan(∠ABC)是多少?

(A) $\frac{1}{7}$ (B) $\frac{1}{4}$ (C) $\frac{3}{7}$ (D) $\frac{1}{2}$ (E) $\frac{4}{7}$

15. One of the following numbers is not divisible by any prime number less than 10. Which is it?

下列各数中有一个数不能被任何小于 10 的素数整除，问这是哪个数?

(A) $2^{606}-1$ (B) $2^{606}+1$ (C)$2^{607}-1$ (D) $2^{607}+1$ (E) $2^{607}+3^{607}$

16. Suppose x and y are positive real numbers such that

$$x^y=2^{64} \text{ and } (\log_2 x)^{\log_2 y}=2^7$$

What is the greatest possible value of 1 $\log_2 y$?

假设 x 和 y 是正实数，满足$x^y=2^{64}$ 和 $(\log_2 x)^{\log_2 y}=2^7$问$\log_2 y$的最大可能值是多少?

(A) 3 (B) 4 (C) $3+\sqrt{2}$ (D) $4+\sqrt{3}$ (E) 7

17. How many 4×4 arrays whose entries are 0s and 1s are there such that the row sums (the sum of the entries in each row) are 1, 2, 3, and 4, in some order, and the column sums (the sum of the entries in each column) are also 1, 2, 3, and 4, in some order? For example, the below array satisfies the condition.

考虑 4×4 的数阵，其中的每个数均为 0 和 1，行和(即每行中的各数之和)按某种顺序排列后是 1、2、3、4，而列和(即每列中的各数之和)按某种顺序排列后也是 1、2、3、4.问这样的数阵一共有多少个?例如，下面的数阵就满足要求，

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(A) 144 (B) 240 (C) 336 (D) 576 (E) 624

18. Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

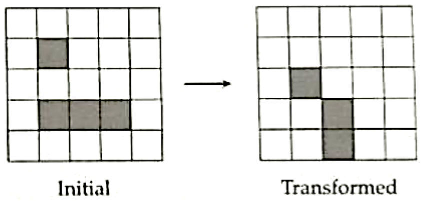
- Any filled square with two or three filled neighbors remains filled.
- Any empty square with exactly three filled neighbors becomes a filled square
- All other squares remain empty or become empty.

A sample transformation is shown in the figure below.

5×5 方格表中的每个方格要么是灰色的，要么是空白的，并且最多有八个相邻的方格，这里有公共边或者公共顶点的方格认为是相邻的、方格表按以下规则进行转换:

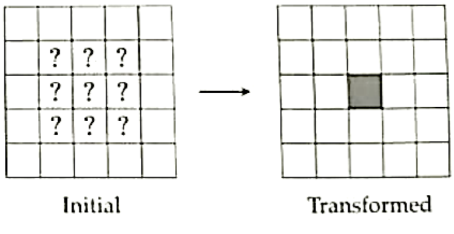
- 任何有两个或三个相邻的方格是灰色的灰色方格继续保持是灰色的
- 任何恰好有三个相邻的方格是灰色的空白方格将成为灰色方格,
- 所有其他的方格要么继续保持是空白的，要么变成空白方格

下图显示了一个转换的示例



Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)

假设 5×5 方格表周界上的方格都是空白的，它们围绕着中间的 3×3 子方格表，那么经过一次转换后，方格表变为只有中心方格是灰色的初始构型有多少个?(一个构型经过旋转和反射后而形成的构型被认为是不同的.)



(A) 14　　(B) 18　　(C) 22　　(D) 26　　(E) 30

19. In $\triangle$ABC medians $\overline{AD}$ and $\overline{BE}$ intersect at G and $\triangle$AGE is equilateral. Then cos(C) can be written as $\frac{m\sqrt{p}}{n}$, where m and n are relatively prime positive integers and p is a positive integer not divisible by the square of any prime. What is m +n + p?

在△ABC 中，中线$\overline{AD}$和中线$\overline{BE}$相交于 G，并且△AGE 是等边三角形.已知 cos(C)可以写成$\frac{m\sqrt{p}}{n}$的形式，其中 m 和 n 是互素的正整数，p 是正整数，并且不能被任何素数的平方整除.问 m+ n+ p 是多少?

(A) 44 (B) 48 (C) 52 (D) 56 (E) 60

20. Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial x^2+x+1, the remainder is $x+2$, and when $P(x)$ is divided by the polynomial x^2+1, the remainder is $2x+1$. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

设 P(x)是一个有理系数的多项式，使得当 P(x)除以多项式x^2+x+1时,余式为$x+2$，而当 P(x)除以多项式x^2+1时，余式为 2x+1.满足上述两个条件的次数最低的多项式是唯一的.问该多项式的各项系数的平方和是多少?

(A) 10 (B) 13 (C) 19 (D) 20 (E) 23

21. Let S be the set of circles that are tangent to each of the three circles in the coordinate plane whose equations are $x^2+y^2=4, x^2+y^2=64$, and $(x-5)^2+y^2=3$. What is the sum of the areas of all the circles in S?

设 S 是坐标平面中，与方程为$x^2+y^2=4, x^2+y^2=64, (x-5)^2+y^2=3$的三个圆中的每一个都相切的圆组成的集合，问 S 中所有圆的面积之和是多少?

(A) 48π (B) 68π (C) 96π (D) 102π (E) 136π

22. Ant Amelia starts on the number line at O and crawls in the following manner. For n = 1,2, 3, Amelia chooses a time duration t_n and an increment x_n independently and uniformly at random from the interval (0, 1). During the nth step of the process, Amelia moves x_n units in the positive direction, using up t_n minutes. If the total elapsed time has exceeded I minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?

蚂蚁 Amelia 在数轴上从 0 开始,按以下方式爬行.对于 n=1,2,3,Amelia 从区间(0,1)中随机独立且均匀地选择持续时间 t 切和步长 x_n.在爬行过程的第 n 步，Amelia 沿正向移动 x_n 个单位，用时 t_n 分钟，如果在第 n 步移动期间，所经过的总时间超过 1 分钟，则她在该步结束时停止;否则，她会继续下一步，最多一共走 3 步。问 Amelia 停止在大于 1 的数所对应的位置处的概率是多少?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{5}{6}$

23. Let $x_0, x_1, x_2, \ldots$ be a sequence of numbers, where each x_k is either 0 or 1.

For each positive integer n, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k$$

Suppose $7S_n \equiv 1$ (mod 2^n) for all n⩾1. What is the value of the sum

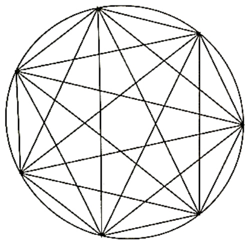
$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}?$$

在数列 $x_0, x_1, x_2, \ldots$ 中，每个 x 项均为 0 或 1.对于每个正整数 n，定义 $S_n = \sum_{k=0}^{n-1} x_k 2^k$ 假设对所有 n⩾1，有 $7S_n \equiv 1$ (mod 2^n)问和式 $x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}$ 的值是多少?

(A) 6 (B) 7 (C) 12 (D) 14 (E) 15

24. The figure below depicts a regular 7-gon inscribed in a unit circle. What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?

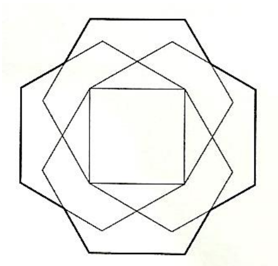
下图所示是一个内接在单位圆中的正七边形.它的所有边和对角线共有 21 条，问这些线段长度的 4 次方之和是多少?



(A) 49　　(B) 98　　(C) 147　　(D) 168　　(E) 196

25. Four regular hexagons surround a square with side length 1, each one sharing an edge with the square, as shown in the figure below. The area of the resulting 12-sided outer nonconvex polygon can be written as $m\sqrt{n}+p$, where m, n, and p are integers and n is not divisible by the square of any prime. What is m +n + p?

如下图所示，四个正六边形环绕着一个边长为 1 的正方形，每个正六边形与正方形有一条公共边.由此形成的图形的边界是非凸的 12 边多边形，它的面积可以写成$m\sqrt{n}+p$，其中 m，n 和 p 是整数，n 不能被任何素数的平方整除.问 m+n+p 是多少?



(A) -12　　(B) -4　　(C) 4　　(D) 24　　(E) 32