British Physics Olympiad 2024-25

Round 2 Competition Paper

Thursday 6th February 2025

Instructions

Time: 3 hours (approximately 45 minutes per question).

Questions: All four questions should be attempted.

Marks: The four questions carry similar marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets.
Students should ensure their name and school is clearly written on all answer sheets.
A new question should be started on a new page.
Pages must be numbered.

Instructions: Graph paper should be provided.
A standard formula booklet with standard physical constants should be supplied.
To accommodate students sitting the paper at different times, please do not discuss any aspect of the paper on the internet until 8am Saturday 15th February.

This paper must not be taken out of the exam room. The paper and any scrap paper and notes must be collected in by the invigilator.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode".

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will definitely not be marked and overall clarity is an important aspect of this competition paper.

Training Dates and the International Physics Olympiad

Following this round, students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the **Training Camp** *to be held in the Physics Department at the University of Oxford,* **(Monday 7th April to Friday 11th April 2024)**. *Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training. From May there will be mentoring by email to cover some topics and problems. There may be (depends on availability) a weekend* **Experimental Training Camp in Oxford during the first weekend of half term in May (Friday evening - Sunday afternoon)**, followed by a **training camp in Cambridge beginning in early July.**

The IPhO this year will be held in Paris in July 2025.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Mass of an electron	m_{e}	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of a neutron	m_{n}	$1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of a proton	m_{p}	$1.67 \times 10^{-27}\,\mathrm{kg}$
Radius of a nucleon	r_0	$1.2 \times 10^{-15}\,\mathrm{m}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Molar gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Specific heat capacity of water	c_{w}	$4.19 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Mass of the Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_{E}	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth	R_{E}	$6.38 \times 10^{6}\,\mathrm{m}$

Qu 1. General Questions

(a) Half the length of a 1.0m rope is stretched out in a straight line on a frictionless table. The other half of the rope hangs vertically from the edge of the table. The rope is released from rest. Calculate the speed of the rope as it leaves the table.

(b) Stoodley Pike Monument in West Yorkshire stands $37\,\mathrm{m}$ tall at high elevation above exposed moorland (Fig. 1). As such, in winter, it is subject to ferocious wind speeds v.

Roughly speaking, the tower is cylindrical in shape and solid throughout with an average diameter of $7.5\,\mathrm{m}$. The local sandstone, which used as construction material, has a density of $3.0\,\mathrm{g\,cm^{-3}}$. A horizontal wind exerts an inertial force $F = \rho A v^2$, where A is the projected area of the tower and $\rho = 1.3\,\mathrm{kg\,m^{-3}}$ is the density of air.

Assuming that it has negligible foundations (below-ground construction), determine the wind speed that would topple the monument. Suggest the significance of the actual shape of the monument.

Figure 1: Stoodley Pike Monument. Source: Alex Noble https://commons.wikimedia.org/wiki/User:Alex_Noble. Accessed Dec 2024.

(c) The *angular momentum* L of a particle about a specified axis A is defined as

$$L = rp, \tag{1}$$

where r is the perpendicular distance from A to the line of motion of the particle and p is the linear momentum of the particle.

(i) Calculate the angular momentum of a $25\,\mathrm{g}$ rubber bung on a $50\,\mathrm{cm}$ light string being whirled in a circle 4.0 times per second. Give your answer in the standard (S.I.) unit for angular momentum.

(ii) A cyclist is travelling at $12\,\mathrm{m\,s^{-1}}$. Her front wheel has a mass of $1.3\,\mathrm{kg}$ and a diameter of $62\,\mathrm{cm}$. If the wheel does not slip with respect to the ground, find its angular momentum.

(iii) We can crudely model a uniform rod of mass M and length ℓ as a system of N equally spaced particles, each of mass M/N. If the rod is pivoted at one end and rotates with angular velocity ω, determine an expression for its angular momentum in terms of N.

Hint: the sum of the first n square numbers is given by

$$\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$$

(iv) By taking an appropriate limit, find an exact expression for the angular momentum of the rotating rod from part **(iii)** in terms of M, ℓ and ω only.

(v) Determine the kinetic energy of the rotating rod from part **(iii)** in terms of M, ℓ and ω only.

Qu 2. Random walks

This question explores the motion of individual particles within a gas. This is an example of a *random walk*.

(a) Calculate the typical (root mean-square) speed of nitrogen molecules (N_2) at room temperature. Assume that N_2 acts as an ideal gas. The molar mass of $\mathrm{N_2}$ is $28\,\mathrm{g\,mol^{-1}}$.

(b) Show that the density ρ of any substance composed of N particles occupying a total volume V is given by

$$\rho = \frac{M_R N}{N_A V}, \tag{2}$$

where M_R is the molar mass and $N_A = 6.022 \times 10^{23}$ mol^{-1} is Avogadro's constant. Hence, find an expression (in terms of density) for the average volume occupied by each particle.

(c) Under standard conditions, liquid nitrogen (N_2) has a density of 807 kg m^{-3}. Estimate (crudely) the diameter of a molecule of $\mathrm{N_2}$, stating any assumptions you make.

For an ideal gas of N particles occupying a total volume V, the typical distance moved between collisions by molecules of diameter d is known as the *mean free path* λ. This is given by:

$$\lambda = \frac{V}{\sqrt{2}\pi d^2 N}. \tag{3}$$

We can also define the *collision time* Δt as the typical time from one collision to the next.

(d) Determine Δt for nitrogen molecules (N_2) at room temperature and pressure $p = 1.01 \times 10^5$ Pa.

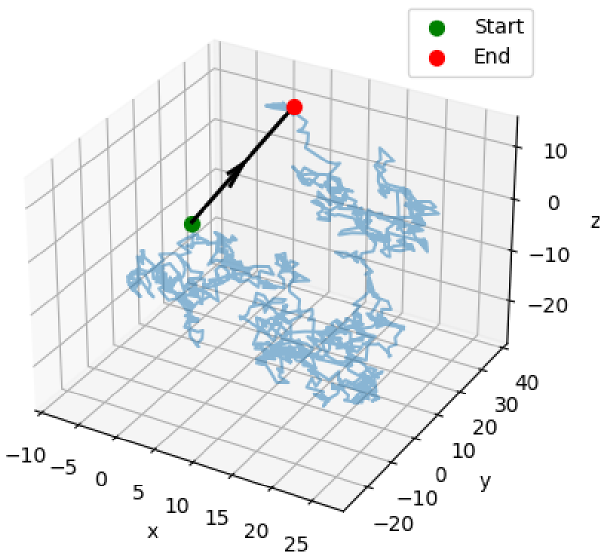

Figure 2: The random walk of a gas molecule in three dimensions. Each collision randomises the direction of travel. The black vector indicates the final displacement of the molecule from its starting point after undergoing 1000 collisions.

With each collision the direction of travel of a given molecule is effectively *randomised*. As such, any individual molecule executes a *random walk* through the gas, an example of which is shown in Fig. 2.

For simplicity, consider gas molecules moving in two dimensions (2D) and make the approximation that the speed v of any given molecule is constant in time. Moreover, assume that

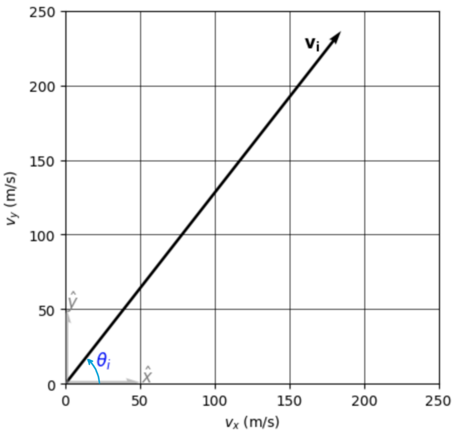

Figure 3: An example of a velocity vector of a gas molecule moving in 2 dimensions. The unit vectors are not drawn to scale.

the time between collisions Δt is constant from one collision to the next. Due to the exchange of momentum and the random nature of collisions, these assumptions are clearly false in detail. Nevertheless, they will yield instructive results.

(e) Suppose a given molecule undergoes M collisions in a total time t. After collision i, its velocity vector $\mathbf{v}_i$ points at an angle θ_i measured anticlockwise from the positive x axis, as shown in Fig. 3. Write down an expression for $\mathbf{v}_i$ in terms of θ_i, the molecular speed v and the conventional unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, which are aligned with the positive horizontal and vertical axes, respectively.

(f) Briefly explain why, after time t and M collisions, the vector displacement of the molecule $\mathbf{r}(t)$ is given by

$$\mathbf{r}(t) = \sum_{i=1}^{M} \mathbf{v}_i \Delta t. \tag{4}$$

(g) With reference to your answer to (e), argue why on average the vector displacement is zero.

(h) By considering the average value of $\mathbf{r} \cdot \mathbf{r}$, determine an expression for the expected magnitude of the molecular displacement after M collisions.

(i) Assuming the expression you have just derived also applies in three dimensions (3D), estimate the time taken for a nitrogen molecule to diffuse a distance of $5.00\,\mathrm{m}$ from its starting point. Assume room temperature and pressure $p = 1.01 \times 10^5$ Pa, and leave your answer in a sensible unit.

Qu 3. The blast radius of *The Gadget*

This question is about the world's first atomic bomb test, which took place in New Mexico, USA, in 1945. The operation - part of the Manhattan Project - was code-named *Trinity* and the bomb itself was nicknamed *The Gadget*.

The radius R of the shockwave from an atomic bomb depends upon the density of the atmosphere ρ, the time after the explosion t and the energy E released in the explosion.

(a) Using arguments of dimensional homogeneity (i.e., consistency of units), determine the values of the dimensionless unknowns α, β and γ if

$$R \propto \rho^{\alpha} t^{\beta} E^{\gamma}. \tag{5}$$

It is reasonable to assume the dimensionless constant of proportionality k implied in Eq. 5 is of order unity. Its precise value depends upon the circumstances of the explosion. Assume for the remainder of the question, and for simplicity, that $k = 1$.

(b) Prove that the atmospheric density ρ is given by

$$\rho = \frac{M_R p}{RT}, \tag{6}$$

where M_R is the (representative) molar mass of an air molecule. Show that, under typical sea-level conditions, $\rho \approx 1.3$ kg m^{-3}.

(c) The speed of sound c_s is related to the ambient pressure p and density ρ by a simple formula:

$$c_s = \sqrt{\frac{\gamma p}{\rho}}, \tag{7}$$

where $\gamma = 1.40$ is the adiabatic index for air. Calculate the speed of sound under typical sea-level conditions.

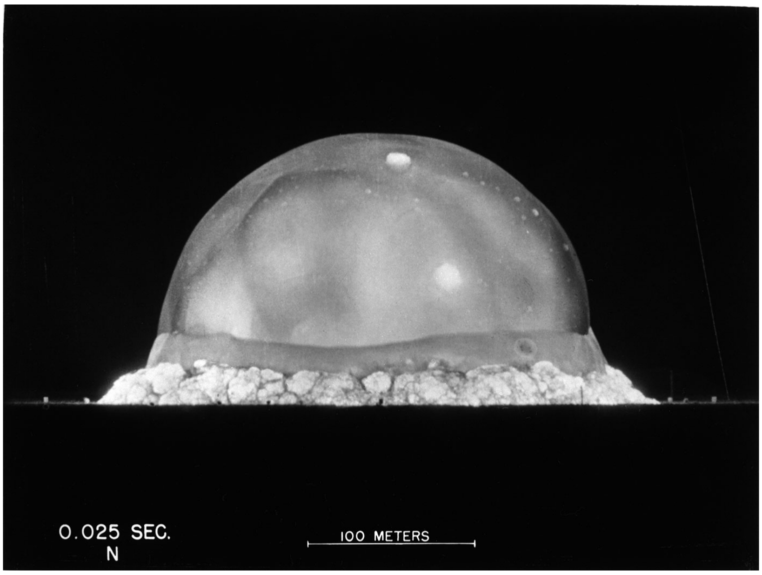

Figure 4: The hemispherical shockwave 25 ms after detonation of The Gadget. Source: https://www.atomicarchive.com/media/photographs/trinity/fireball-2.html. Accessed Dec 2024.

(d) Use information in Fig. 4 to show the the energy released by The Gadget was approximately 100 TJ.

(e) Calculate the radius of the shockwave generated by The Gadget after (i) $1\,\mathrm{ms}$, (ii) $10\,\mathrm{ms}$, and (iii) $0.1\,\mathrm{s}$.

(f) Calculate the speed of the shockwave after (i) $1\,\mathrm{ms}$, (ii) $10\,\mathrm{ms}$, and (iii) $0.1\,\mathrm{s}$.

(g) Eventually, the shockwave stalled due to dissipation of the shockwave energy to the surroundings. Briefly explain why energy dissipation increases dramatically when the shockwave speed drops below the speed of sound in the external atmosphere.

(h) Estimate the radius at which The Gadget's shockwave stalled and the corresponding time, after detonation, that this occurred.

(i) Following the test, scientists recorded structural damage to buildings at a distance of $3\,\mathrm{km}$ from the detonation site. Broken glass damage was observed as far as $30\,\mathrm{km}$ away. Reconcile these figures with your answer to part (h).

Qu 4. The Pendulum

A mass m is suspended from a frictionless pivot P by a light string of length l. It is released from rest at an angle θ_0 from the vertical.

(a) Prove, given certain assumptions that you should clearly state, that the time period τ of the subsequent oscillations is

$$\tau = 2\pi\sqrt{\frac{l}{g}}. \tag{8}$$

(b) By considering the principle of conservation of energy, prove that the angular velocity $\frac{d\theta}{dt}$ at angular displacement $\theta < \theta_0$ satisfies

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{l}\Big(\cos\theta - \cos\theta_0\Big). \tag{9}$$

(c) Hence argue that the time period of the subsequent oscillations can be written in integral form as

$$T = \sqrt{\frac{8l}{g}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}. \tag{10}$$

(d) By introducing an auxillary angle ϕ such that

$$\sin\phi = \frac{\sin(\theta/2)}{\sin(\theta_0/2)}, \tag{11}$$

prove that

$$T = \sqrt{\frac{16l}{g}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{1 - k^2\sin^2\phi}}, \tag{12}$$

and express the constant k in terms of θ_0. *Hint:* $\cos\theta = 1 - 2\sin^2(\theta/2)$.

(e) By inserting a relevant value for k into the integral expression above, show clearly how the result from part (a) can be recovered.

(f) For a general value of θ_0 (and therefore k) there is no closed-form solution for the integral above. However, the result *can* be expressed in terms of the arithmetic-geometric mean function, agm:

$$T = \frac{\tau}{\mathrm{agm}[\cos(\theta_0/2), 1]}, \tag{13}$$

where τ is the result derived in part (a). The arithmetic-geometric mean function agm is recursively defined as the large n limit of two simple sequences a_n, b_n: to find agm$[a, b]$, set $a_0 = a, b_0 = b$ and then:

$$\begin{aligned} a_{n+1} &= \frac{1}{2}(a_n + b_n) \\ b_{n+1} &= \sqrt{a_n b_n} \end{aligned} \tag{14}$$

for $n = 0, 1, 2, ...$ If $\ell = 1.00$m and the mass is released from the same level as the pivot, with the string taut, determine the oscillatory time period to 3 significant figures.

(g) A more complex pendulum is composed of two masses, each $m/2$, respectively at distances $\ell(1-\alpha)$ and $\ell(1+\alpha)$ from the pivot, where $0 < \alpha < 1$. The masses are connected with the pivot and to one another by co-linear, light rods. Draw the arrangement and explain, for a given value of θ_0, whether the oscillatory time period is shorter than, equal to, or longer than the simple pendulum of mass m and length ℓ.

END OF PAPER

Questions set by: Rupert Allison, Giggleswick School, North Yorkshire.